multiply.' Life in this world is *self-renewing existence*. That is the essential pattern of it, whether it be the life of the lily or the life of the man. And that pattern is not changed by redemption in the grace of Christ, through the Church. Christ is the second Adam. Life in him is essentially human. Grace and ultimate happiness are to be won, not by turning away from the realities of our nature but by redeeming them. Marriage and family life are now fortified by a sacramental status.

The very process of redemption itself was a family affair. Christ is the Son of God and of our Lady. In this world he lived with his Mother, and with her husband, St Joseph, who in the designs of God was to be thought to be the father of our Lord. The truth that the marriage was chaste and virginal, far from detracting from the holiness of sex, surely only emphasises it the more.

INTRODUCTORY REMARKS TO MODERN LOGIC

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IT is a feature of much contemporary philosophical writing that an amateur of the literature finds himself unable even to think he understands it because of the extent to which the writers draw on the technicalities of modern logic. Usually such a reader does not know where to turn for enlightenment, and very frequently on being given some references to introductory books he finds himself baffled by an austere and technical exposition of the very technicalities that he wishes to understand. These few pages contain some preliminary remarks addressed only to such investigators.

Formal logic is, and so far as it has remained true to itself, always has been, an exact science. The syllogism, we once heard a theologian remark, is not an essay in *vers libre*. There are indeed degrees of exactness. Aristotle, who founded the science so far as concerns its European development, laid it down as a principle that phrases equivalent in significance should be interchangeable, but what phrases these might be is left to be discovered from his usage and forms no part of his system. We find for instance the sentence 'all medicine is science' treated as a substitution in the scheme for a sentence '*B* belongs to all *A*', it having been stated

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that '*A*' is to be replaced by 'medicine' and '*B*' by 'science'. Correct substitution produces 'science belongs to all medicine', and we are left to infer that Aristotle deemed this equivalent to 'all medicine is science'. Modern standards of exactness are more demanding. A modern logician would have stated the interchangeability of '*B* belongs to all *A*' with 'All *A* is *B*', if he desired to have both modes of expression in his system. Modern logic insists on the maximum of explicitness in the derivation of one formula from another—that is, on the maximum of exactness attainable in the lay-out of the primitive material of a deductive system, in the rules for meaningful combination of this material, in the rules for operating upon it so as to produce new combinations of it.

But there is a sense in which Aristotle's procedure is already exact, and requiring that special attention of mind to which the logically uninitiate are not accustomed in their reading of philosophy, and for which they commonly experience a certain repugnance. The laws of Aristotle's logic are stated in terms partly of words, partly of letters of the alphabet which the scolastics called 'transcendent terms' and which are now called 'variables'. In the last paragraph we described 'all medicine is science' as a sentence, '*B* belongs to all *A*' as a scheme for a sentence. The so-called scheme is not a sentence; it only becomes one when both the capital letters are replaced by words of the intended kind of which we have examples in 'medicine' and 'science'. Even in isolation from other such, this scheme exhibits some small degree of structure, form or pattern, of which the reader must have taken notice if he followed the meaning of the sentence which followed the scheme's initial appearanace. '*B*' comes first in it, '*A*' second, a fact of which we have to take account if we are to make a correct substitution of 'medicine' for '*A*' and 'science' for '*B*'. The order to make this substitution correctly tells us to copy out the scheme exactly, only wherever we come to '*A*' we are to write 'medicine' and wherever we come to '*B*' we are to write instead 'science'. Anyone who is unwilling to bring to his task that degree of attention which is necessary to take practical and effective recognition of the difference between '*B* belongs to all *A*' and '*A* belongs to all *B*' should abandon the attempt to understand anything about logic. A great deal of philosophical writing does not demand that kind of attention. Opinions are divided as to whether it should be of such a kind as always or never or only sometimes to demand it, but it is certain that it is one kind of attention which logical and highly logicised thought requires.

The point made in the preceding paragraph is emphasised when we come to deal with logical laws. 'All *A* is *B*' is not of course a law, for we can make substitutions in it which produce a false sentence, whereas a law must become a true sentence no matter what substitution is made. Here is a law from Aristotle's system:

(1) If both all *C* is *B* and all *A* is *C* then all *A* is *B*. In this we have three schemes, 'all *C* is *B*', 'all *A* is *C*', 'all *A* is *B*', of which the first two are united to form a scheme for a compound sentence by the connective 'both . . . and——', this compound then being united with the third by means of the connective 'if . . . then——'. The position of the variables in each of the simple schemes now becomes relevant not only to possible substitution requirements but to the position of similar variables in the remainder of the simple schemes. If we interchange '*C*' and '*B*' at their first occurrence only, we shall have altered not just an atomic scheme but the pattern of the whole expression. The result will be:

(2) If both all *B* is *C* and all *A* is *C* then all *A* is *B*. This last expression is not a law, and the reader will easily find terms to put for the variables which will turn the first two simple schemes into true sentences, the third into a false one, which will give us an 'if . . . then——' sentence with true antecedent and false consequent, all such being false. (If he cannot, 'book-binding' for '*B*', 'craft' for '*C*', 'printing' for '*A*' will achieve the desired result.)

We are going to stress the same point yet further by considering the derivation of another law from (1). It is a correct rule governing 'if . . . then——' sentences, or implications as they are called, that where the antecedent—the part between the 'if' and the 'then'—is a conjunction, i.e. a sentence unified by 'both . . . and——', either half of the conjunction may be interchanged with the consequent—the part following the 'then'—on condition that both interchanged parts are negated; the result of these operations has the same validity as the original. Applying this rule to (1) in respect of the second part of the conjunction, we obtain:

(3) If both all *C* is *B* and not all *A* is *B* then not all *A* is *C*. In this derivation the material on which we operate is a pattern, of which the operator must see the parts relevant to the rule, and to which he must apply the rule exactly to produce the new pattern.

It will be instructive to re-letter (3), interchanging '*C*' and '*B*' *throughout*. We obtain:

(4) If both all *B* is *C* and not all *A* is *C* then not all *A* is *B*. By so doing we have indeed changed the written expression, but this time we have kept the same pattern of interrelatedness among the variables, by contrast to the operation first effected on (1) where by interchanging '*C*' and '*B*' only at their first occurrence we altered the pattern. We still have similarly shaped variables in the first and sixth places, in the second and fourth, in the third and fifth, and differently shaped ones in each of these three pairs of places. In spite of the re-lettering we still have the same form exhibited. It may further be noted that so far as the variables alone are concerned we have in this law the same pattern as in the invalid (2). The difference lies in the presence of and position of the negators in (4). The *whole* context evidently makes up a formal unity. An understanding of logic requires not only exactness of reading and operation, but a recognition of similarities and contrasts between such forms and a knowledge of their derivabilities from one another.

A thoroughly modern treatment of the system from which we have been taking our examples will not have any ordinary words appearing at all. Let us see how they are to be translated into the notation of Łukasiewicz. First replace the upper case variables in (1) and (4) by the corresponding letters from the lower case. Secondly suppress every occurrence of 'is'. We obtain in place of (1):

(1)' If both all *c b* and all *a c* then all *a b*, and in place of (4):

(4)' If both all *b c* and not all *a c* then not all *a b*.

Thirdly, replace 'all' by '*A*' and 'not' by '*N*' giving:

(1)'' If both *Acb* and *Aac* then *Aab*,

(2)'' If both *Abc* and *NAac* then *NAab*.

Lastly, suppress 'and' and 'then', replacing 'if' by '*C*', 'both' by '*K*', obtaining (5) and (6):

(5) *CKAcbAacAab*

(6) *CKAbcNAacNAab*.

The reader should reverse these steps so as to obtain (1) and (4) again. The principle of the notation is evident. In (4) we have besides variables a recurrent 'all . . . is——' which unites two variables to form a scheme for a sentence; 'both . . . and——' which unites two schemes to form a compound one; 'if . . .

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then——' which again unites two schemes to form a compound; and we have the negator 'not' which affects just one scheme at each occurrence. Each of these four *operators* or *functors* is expressed by a single capital and written immediately in front of the term or terms, either variables or schemes, on which it operates. (5) and (6) are far more easily compared than (1) and (4), and it is far easier to manipulate (5) so as to derive (6) than it was to manipulate (1) so as to derive (4). In (5) and (6) materials have been pared to the minimum so as to exhibit structure to the maximum. Whereas it is relatively complicated to replace 'not all . . . is——' by 'some . . . is not——' as the system warrants us in doing, it is relatively simple to replace '*NA*' by '*O*', turning (6) into (7):

(7) *CKAbcOacOab*.

There are two other functors in this system comparable to *A* and *O* in that they unite two variables to form a sentential scheme, viz. *I*: 'some . . . is——', and *E*: 'not some . . . is——' or 'no . . . is——'. The Aristotelian logic is the theory of these four functors, *A*, *I*, *E*, *O*.

The functors *C*, *K*, *N*, are of a different kind to *A*, *I*, *E*, *O*, in that they govern sentential schemes. And evidently they will remain meaningful if they are prefixed to sentences that are not formed by application of the Aristotelian functors. For instance we may be dealing with variables replaceable by numerals and united by the functors $>$ (. . . is greater than ——) or $<$ (. . . is less than——). In this field we have the law:

(8) $CK > xy < yx$,

'if x is greater than y, y is less than x'. The theory of *C*, *K*, *N*, is known as the propositional calculus or theory of deduction, and is the most fundamental logical system. In it are used variables which are replaceable by sentences or sentential schemes of whatever structure, and it is to this theory that there belongs the rule which enabled us to derive (4) from (1) and (6) from (5), based on the law:

(9) *CCKpqrCKpNrNq*.

If in (9) we replace '*p*' by '*Acb*', '*q*' by *Aac*', '*r*' by '*Aab*' we shall obtain an expression beginning with '*C*' followed by an expression letter for letter the same as (5), followed by another expression which is exactly like (6) except that '*b*' and '*c*' are everywhere interchanged. Since a true implication (such as (9)) with a true antecedent (such as this substitution has given to (9)) always has a true consequent, this other expression may be asserted as a law. Re-lettering it by interchanging '*b*' and '*c*' everywhere we obtain (6).

The theory of deduction, like the Aristotelian system, can be based on axioms, and the significance of the functors is determined by these axioms, by the rules of operation which may be applied to them, and by the totality of laws concerning the functors which may be derived. In the normal development of the theory the functors turn out not to have quite the same significance as they do in ordinary speech. The axioms in fact abstract certain data given us by ordinary speech, and in so far as the data are incomplete the functor in the symbolic system will to some extent diverge from its usual behaviour. This is not conspicuously the case with the conjunctor *K*, but is so with the implicator *C*. Nowadays a great deal of research is in progress to develop in an equally exact way implicators that correspond to normal usage more closely. However, not only is the nature of the symbolic implicator rendered crystal clear by its axiomatic treatment, its very divergences help to clarify by contrast the vaguer normal usage. At the same time these exact methods have enabled much light to be shed on the structure of particular deductive systems, and on the nature of deductive systems in general.

The examples we have taken for discussion have been drawn from very ancient fields of research. The Aristotelian system had to wait more than two thousand years for its last secrets to be discovered. The more fundamental theory of deduction first began to be systematically investigated by Aristotle's Stoic successors and again has only been completed within this century. In this matter indeed the history of logic shows a remarkable sequence of discovery and oblivion, but the matter is now so well understood that it is hard to see how it can ever be lost again so completely as has happened in the past. But in spite of these and other greater gains in knowledge due to the modern methods, an unfortunate situation is developing in philosophy. Writers are becoming rapidly divided into those who have mastery of logical calculations and those who have not. The writings of the former demand that special attention and way of thought of which we have been speaking; those of the latter often seem to the others mere vague and unscientific ranting. A serious dissipation of energy and loss of collaboration is the result. It seems to us that the full powers and limits of the modern logical way of thought will only become apparent when it has become widely diffused as part of normal culture and higher education. Since it is being increasingly applied to science in its full generality, it would not seem unreasonable to hope for the day when no one is accepted at a university without preliminary training in the elementary logical disciplines.

One study which might be mentioned, though of an advancement out of all proportion with the elementary nature of the foregoing remarks, is Rudolf Carnap's *Logical Foundations of Probability*.[1] Here is a case in point where a traditionally vague and thorny notion is submitted to the exact methods of modern logic. In the midst of much purely technical development the author has a number of excellent things to say on the benefits to be derived from that kind of abstraction and resultant treatment to which we have alluded in the matter of the implicator (sections 43 and 45). He also draws a distinction, in the true scholastic manner, between two concepts of probability, for lack of which there has been much past contention and confusion. Indeed, those with scholastic training will find in the work of the best modern logicians a new and startlingly thorough exemplication of their own traditional ideals of exact definition, deduction from (in some sense) first principles, and dispassionate analysis of objects viewed for their own sake and for what they are.

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