7. CELESTIAL MECHANICS (MÉCANIQUE CÉLESTE)

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This report is organized according to twelve subjects of interest in modern celestial mechanics. Emphasis is placed on the results of an exhaustive literature search in which assistance was received from several members of Commission 7. The impetus on celestial mechanics by space projects, by new observational techniques and by the use of high speed electronic computers continued to be of considerable importance. The fact that celestial mechanics became an experimental science is still too recent to be fully evaluated. The high accuracy of radar and laser observations, the new analytical and numerical techniques, the recent sophisticated statistical orbit determination methods and many other results achieved during the past three years suggest that celestial mechanics is an exploding field of science in which new and exciting results emerge. Celestial mechanics is influencing other fields of scientific endeavor and is intimately connected and embedded in almost all areas of modern astronomy and space research.

The main topics are: (1) orbits of artificial satellites and the Earth's gravitational field; (2) natural satellites of the Solar system; (3) lunar theory and lunar probes; (4) planetary theory and planetary probes; (5) orbits of asteroids, comets and minor planets; (6) the problem of two bodies; (7) the problem of three bodies and periodic orbits; (8) the many-body gravitational problem; (9) integrals and stochasticity of dynamical systems; (10) resonance in celestial mechanics; (11) varying gravitational constant; (12) analytical and numerical techniques.

The following abbreviations are used: AA=Astron. and Astrophys.; AIAA=Am. Inst. Aeronaut. and Astronaut.; AJ=Astron. J.; AN=Astron. Nachr.; AZ=Astron. Zh. USSR.; BAAS=Bull. Am. Astron. Soc.; BAIC=Bull. Astron. Inst. Czech.; BITA=Bjull. Inst. Teor. Astron., Leningrad; CM=Celest. Mech.; CR=Compt. Rend. Acad. Sci. Paris; IAF=Internat. Astronautical Fed.; JPL=Jet Prop. Lab.; MN=Monthly Not. Roy. Astron. Soc.; PASJ=Publ. Astron. Soc. Japan; SAO=Smithsonian Astroph. Obs.; SSAI= Soobtcheniya, Sternberg Astron. Inst.; TITA=Transaction, Inst. Teor. Astron. Leningrad; TSAI=Trudy, Sternberg Astron. Inst. When available, references are made to the listings in Astronomy and Astrophysics Abstracts in the form: volume.section. reference. For instance, 18.042.031 refers to vol. 18, section 042 and publication 031.

(1) The subject of finding the Earth's potential from observations of artificial satellites was still very much in the forefront, and the recognition of some fundamental problems concerning the non-uniqueness and the persistent divergence of the solution of this problem resulted in several new descriptions of the potential. Modifications of classical elements, the use of the extended phase space, elimination of the parallax allowed integration of the main problem to the first order. Luni-solar perturbations, drag, aerodynamic lift and radiation effects received analytical treatment but prediction of the long-time behavior of artificial satellites and of their re-entry still challenge the profession. The detection of tectonic plate movements, of ocean surface motions, of relativity effects and of gravitational waves became subjects of satellite technology. The problem of relative motion of non-interacting satellites and the effect of rigid body motion on the orbital motion of satellites received considerable interest. See: Aksenov, E., Vashkoviak, S. N. and Emelianov, N. V. (1978), TSAI 49; Aksnes, K., 17.052.002; Alfriend, K. T. (1977), CM 16, 441; Amin, M., 17.081.002; Andrus, J. F.

https://doi.org/10.1017/S0251107X00010592 Published online by Cambridge University Press

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Richardson, J. A., 18.081.038; Wagner, C. A. and Klocko (1977), CM 16, 143; Walker, D. M. C., 19.081.019; Yurkina, M. I., 17.081.034; Zagars, J., 19.052.025; Zlatoustov, V. A., 18.052.004.

(2) The natural satellites of the solar system received considerable attention. The orbits of Jupiter's satellites were studied in detail and a second order theory for the construction of quasi-periodic solutions of the Galilean satellites was presented. The dynamics and stability of Saturn's and Uranus' rings were investigated. Improved ephemerides of the satellites of Mars, Jupiter and Saturn have been presented and Saturn's oblateness parameter was studied. Theories of the rigid body (and not-so-rigid-body) motion of planets and satellites were developed and unified theories of spin-orbit coupling were offered. An example of success along these lines was the establishment of Mercury's 3:2 spin-orbit resonance. Some of the results discussed under topics (1) and (10) are applicable to the above subject. For references see: Aksnes, K. and Marsden, B. G., 18.099.212; Brown, B. (1977), CM 16, 229; Davis, D. R., 18.042.054; Demchenko, B. J. (1978), SSAI 208, 209; Ferraz-Mello, S. (1978), Proc. IAU Coll. 41, 209; Horedt, G. P., 18.042.029; Kozai, Y. (1976), PASJ 28, 675; Lieske, J. H., 18.042.049; Orlov, A. A., Solovaya, N. A. and Chepurova, V. M. (1978), TSAI 49 and SSAI 208, 209; Seidelmann, P. K. (1975), CM 12, 59; Singer, S. F. (1977), Proc. IAU Coll. 39; Szebehely, V. (1978), CM 18, 383; Szebehely, V. and McKenzie, R. (1978), CM 18, 391.

(3) Precise measurements of the orbit and the motion of the moon furnished impetus to further analytical and semi-analytical lunar theories. Once again the accuracy of orbit calculations fall behind that of observations. Several new algorithms for constructing literal formal solutions of the lunar problem were presented. A coordinate transformation has been introduced which circumvents Hill's infinite determinants and Hill's variational orbit has been re-determined. New attacks have been made on the main problem. Corrections due to the Earth's gravitational harmonics and due to planetary perturbations were introduced. Lunar libration was redetermined experimentally and recomputed by several workers. In general perturbation studies, the non-numerical use of computers proved to be of considerable help. Algebraic manipulations in general, computerized Poisson and Fourier series and compression and storage of ephemerides by Chebyshev polynomials were some of the significant accomplishments. Methods of computing orbits of lunar and interplanetary probes were developed with high reliability. The difficult problem of lunar satellite theory was attacked with considerable success. See: Abu el Ata, N. (1978), Proc. IAU Coll. 41, 113; Baker, D., 17.053.012; Borsenberger, M. (1978), Thesis, Paris; Bursa, M., 20.042.033; Calame, O. and Mulholland, J. D., 20.094.006; Calame, O. and Mulholland, J. D. (1978), Science 199, 875 and 977; Chapront, J. and Abu el Ata, N. (1977), AA 55, 83; Counselman, C. C., III, King, R. W. and Shapiro, I. I. (1976), J. Geoph. Res. 81; Counselman, C. C., III, Cappallo, R. and Shapiro, I. I. (1977), Sci. Rept. #2, U.S. Air Force Geophy. Lab; Elsmore, B., 18.043.004; Eckert, W. J. and Smith, H. F. (1976), Am. Ephem. 19; Finkelstein, A. M. and Kreinovich, V. Y. (1976), CM 13, 151; Gusev, L. I. and Nicalin, A. M., 20.052.046; Hartung, J. B., 18.094.411; Henrard, J. (1978), CM 17, 195; Henrard, J. and Moons, M. (1978), Proc. IAU Coll. 41, 125; Heppenheimer, T. A. and Kaplan, D. (1977), AIAA J. 15 No. 4 (in press); Kovalevsky, J., 20.042.029; Migus, A. (1977), Thesis, Paris; Mulholland, J. D., 19.94.161; Perko, L. M., 19.042.039; Shiryaeo, A. A., 18.043.009; Singer, S. F. (1977), Earth-Science Rev. 13, 171; Szebehely, V. and McKenzie, R. (1977), AJ 82, 303 and BAAS 9, 435; Tufekcioglu, T., 18.042.042; Van Flandern, T. (1976), CM 13, 511.

(4) In the field of planetary theories new numerical and analytical results allowed the preparation of high accuracy ephemerides, secular terms to high orders were studied analytically, secular trends were investigated numerically, and long-time numerical integrations were performed. Indications that the solar system is stable according to Hill's definition, attempts to apply a modified KAM theory and the finding that secular terms in the semi-major axes of the planets do not appear

https://doi.org/10.1017/S0251107X00010592 Published online by Cambridge University Press

to any order, may be considered important developments. Trigonometric expressions for the geocentric and heliocentric positions of the Sun, Moon and planets to one minute of arc precision and valid for hundreds of years have been derived. New planetary theories have been developed up to the third order for the four large planets with internal precisions of a few second of arcs over 1000 years. A second order theory was prepared for the inner planets. The effects of solar oblateness were studied. The profession responded with considerable interest when the orbit of (2060) Chiron=1977UB (with strong influence by Saturn), the orbit of a new satellite of Jupiter and the orbit (and existence) of a satellite of Pluto were to be established from a few observations. New theories of the rotation of the Earth and planets were presented. Stellar perturbations on the motion of the major planets of the Solar system were established. See: Ajtekeeva, Z. A., 18.042.036; Baker, D., 17.053.003; Berger, A. L., 18.043.003; Blackwell, K. C., 20.043.001; Blitzer, L. (1977), CM 16, 87; Bretagnon, P. and Chapront, J. (1978), Proc. IAU Symp. 81; Brumberg, V. A., Evdokimova, L. S. and Skripnichenko, V. I. (1978), Proc. IAU Coll. 41, 33; Chapront, J. and Dvorak, R., 18.042.109; Chapront, J., 19.042.061 and 20.042.017; Citrynell, H., 20.042.028; Clark, P. S., 17.053.005; Dallas, S. S. (1977), CM 15, 111; Duncombe, R. L. and Seidelmann, P. K. (1977), Navigation 24, 2; Duncombe, R. L. and Van Flandern, T. C., 17.043.002; Duriez, L., 19.042.009 and (1978), AA 68, 199; Dvorak, R., 17.042.052; Edgar, R., 17.052.001; Heppenheimer, T. A., 18.042.082; Hoenselaers, C., 18.042.130; Hooke, A. J., 17.053.006; Horedt, G. P. (1976), AJ 81, 675; Horedt, G. P., Pop, P. and Ruck, H. (1977), CM 16, 209; Ivanov, N. M., Martynov, A. I. and Belykh, V. D., 18.052.005, Jefferys, W. and Szebehely, V. (1978), Comm. on Astrophy. 8, 9; Khentov, A. A., 18.042.028; Kinoshita, H., 20.042.046; Kowal, C., Liller, W. and Marsden, B. G. (1978), Proc. IAU Symp. 81; Kozai, Y. (1976), Tokyo Astron. Obs. Rept. 501, 675; Krasinsky, G. A., Pit'eva, E. V., Sveshnikov, M. L. and Syeshnikova, E. S. (1978), BITA 17; Laubscher, R. E., 18.043.001, 18.043.002 and 18.043.007; Labunskij, A. V. and Leshchemco, A. V., 20.052.044; Lieske, J. (1976), AA 58, 1; Mardus, F., 18.042.121; Marsden, B. G. (1977), IAU Circ. 3215; Melbourne, W. G., 17.053.017; Monin, I. F., 18.081.035; Mucket, J. D. and Treder, H.-J., 19.042.002; Nacozy, P. E. (1977), CM 16, 77; Nacozy, P. E. and Diehl, R. E. (1978), CM 17, 405; Nezinsky, E. M. (1976), AZ 53, 1137; Petrovskaya, M. S., 20.042.041; Petrovskaya, M. S. and Ivanova, T. V. (1978), BITA 16, 288; Pit'eva, E. V. (1978), BITA 14; Plakhov, Y. V., 17.042.040; Popovič, B., 20.042.067; Rochester, M. G., 18.043.008; Sajdov, P. I., Vcherashnij, R. I. and Sajdov, Y. P., 17.042.049; Sato, A., 18.042.056; Schubart, J., 20.042.076; Seidelmann, P. K., 18.043.006, (1977), CM 15, 165 and (1978), CM 17, 103; Sharaf, S. G. and Budnikova, N. A. (1977), TITA 16, 88; Shoemaker, E. M. and Helin, E. F., 18.042.047; Simon, J. L. and Bretagnon, P. (1976), AA Suppl. 22, 107; Simon, J. L. and Bretagnon, P. (1976), AA Suppl. 22, 107 and (1976), AA 42, 259; Soffen, G. A., 18.051.045; Stellmacher, I., 20.042.002; Strel'tsov, V. A., 19.042.023; Stumpff, P., 19.042.032; Szebehely, V. and McKenzie, R., 19.042.038; Szebehely, V. (1978), Proc. IAU Coll. 41, 53; Van Flandern, T. and Harrington, R. (1976), Icarus 28, 435.

(5) The orbits of asteroids, comets and minor planets received considerable attention since it was pointed out that because of their large number they may be considered more representative of the solar system than the planets are. New theories of capture were proposed, the still unsolved problem of the Kirkwood gaps was treated extensively (see also item 10) and dynamical explanations of the origin of these members of the solar system were offered. Secular perturbations of asteroids and comets were studied with semi-analytical methods. New minimum distances between asteroids and their relative motions were determined. The effect of Jupiter's oblateness on cometary orbits was evaluated by means of the two fixed force center problem. Improved orbits for 110 long-period comets were computed. The long term motion of 1976 AA and 1976 UA were studied. These seem to be the first two minor planets with semi-major axes smaller than 1 a.u. See: Banfi, V., 18.042.031; Belbruno, E. A. (1977), CM 15, 467; Bien, R., 19.042.022, 20.042.077 and (1978), AA 68, 295; Chepurova, V. M. (1977), Vestnic Mosk. Gos. Univ. No. 1 & 2; Duboshin, G. N. (1976), CM 14, 239; Erdi, B., 20.042.049; Froeschle, C. and

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(6) The problem of two bodies received attention with a variety of interesting modifications. Bodies with variable masses, gravitationally interacting finite non-spherical bodies and relativity effects were studied. New techniques were presented for the solution of old problems (e.g., Lambert's problem and the solution of Kepler's equation). Techniques for accurate orbit computations at close approaches were presented. See: Averbukh, A. I. and Girshovich, B. V., 17.042.068; Barker, B. M., 19.042.057; Belbruno, E. A., 20.042.052; Chan, L. -H. and O'Connell, R. F., 20.042.031; Cid, R. and Palacios, M., 20.042.027; Correas, J. M., 18.042.012; Degtyarev, V. G. and Shablinskij, A. G., 18.052.008; Demin, V. G., 19.042.027; Devaney, R. L. (1978), Invent. Math. 45, 221; Duboshin, G. N., 18.042.084; Freeman, I. M., 20.042.032; Glikman, L. G., 17.042.006; Hameen-Anttila, K. A., 18.042.010; Iljuhin, A. C. and Bycan', E. N., 18.042.040; Kinoshita, H., 17.042.077 and 20.042.055; Kondurar, V. T. and Troitskaya, L. S., 18.042.091; Kondurar , V. T., 18.042.117; Kriz, J., 19.042.027; Mison, K., 18.042.111 and 19.053.030; Sidlichovsky, M. (1978), BAIC 29, 90; Verhulst, F., 19.042.056; Werner, A. and Schrader, H., 18.042.032.

(7) The fundamental unsolved and non-integrable problem of celestial mechanics, the problem of three bodies received the same attention as it has ever since Newton and Euler attacked it. Similarly to Elis Strömgren's work in the early nineteen hundreds on the restricted problem of three bodies, the recent efforts concentrated on establishing families of periodic orbits of the general problem of three bodies. The classification and systematic survey of these families and of their stability is far from complete at the present time. New families of symmetric, asymmetric, two and three-dimensional periodic orbits were found. Bifurcations were used to generate new families of the general problem. The method of the surface of section was utilized to study quasi-periodic orbits. The plane planetary three-body problem ($m_1 >> m_2 = \varepsilon \geq m_3 = k\varepsilon$, $0 \leq k \leq 1/5$) and its stability received considerable attention. The existence of mono-parametric families with fixed masses was shown. It was demonstrated that such orbits are linearly stable if resonance conditions of the type (2n-1)/(2n+1) are avoided. Triple stellar systems and their stability were studied by establishing zero velocity surfaces for the general problem of three bodies and by using classical perturbation methods. Integrals and particular solutions of the problem of three finite rigid bodies under non-Newtonian forces were investigated. Motion and stability of orbits near equilibrium points received special attention. See: Aarseth, S. J. and Heggie, D. C. (1976), AA 53, 259; Agekyan, T. A. and Anosova, Z. P., 19.042.034; Anderle, P., 17.052.005; Antonov, V. A. and Cholshevnikov, U. V. (1978), AN 299, 131; Arenstorf, R. F. and Bozeman, R. E. (1977), CM 16, 179; Arenstorf, R. F. (1978), CM 17, 331; Ash, M. E., 18.052.035; Barkham, P. G. D., Modi, V. J. and Soudack, A. C., 19.042.040; Benest, D., 17.042.044; 18.042.055 and 19.042.018; Bhatnagar, K. B. and Chawla, J. M. (1977), CM 16, 129; Bhatnagar, K. B. and Hallan, P. O. (1978), CM 18, 105; Bozis, G., 18.042.019; Bozis, G. and Hadjidemetriou, J. D., 17.042.041; Breakwell, J. V., 18.042.050; Brjuno, A. D. (1978), CM 18, 9 and 51; Broucke, R., 17.042.021; Choudhry, R. K. (1977), CM 16, 411; Contopoulos, G. (1976), Proc. NATO Adv. Study Inst., 43; Contopoulos, G. (1978), AA 64, 323; Delibaltas, P., 18.042.063; Demin, V. G. and Kurchanova, M. V., 20.042.021; Duboshin, G. N. (1978), CM 17, 357 and

TSAI 49; Dunham, J. B., 18.042.051; Dvorak, R., 20.042.075; Dzhanikashvili, G. V., 18.052.002; Efimov, A. A., 18.042.114; Elmabsout, B., 20.042.022; Evteev, V. P., 19.042.036; Evteev, V. P. and Nagorev, B. M., 20.042.062; Gavrilov, N. I., 19.042.031; Grebenikov, E. A. and Kiosa, M. N., 18.042.066; Grebenikov, E. A., 18.042.115; Hadjidemetriou, J. D., 17.042.022, 17.042.039 and (1977), CM 16, 61; Hadjidemetriou, J. D. and Michalodimitrakis, M. (1978), Proc. IAU Symp. 41, 263; Halioulias, A., Katsiaris, G. A. and Markellos, V. V., 17.042.067; Heggie, D. (1976), CM 14, 69; Hénon, M., 17.042.051; 20.042.014, 20.042.015 and 20.042.039; Henrard, J. and Renard J. (1978), CM 17, 325; Heppenheimer, T. A. and Porco, C., 19.042.003; Hitzl, D. L., 19.042.008; Hitzl, D. L. and Hénon, M., 20.042.050; Horedt, G. and Mioc, V., 18.042.074; Iljuhin, A. G., 18.042.041; Irigoyen, M., 18.042.039; Ivanov, Y. I., 19.042.006; Kammeyer, P. C. (1978), CM 17, 121; Khapaev, M. M., 19.042.047; Kholshevnikov, C. V. (1977), Prik. Mat. Mec. 41; Kirchgraber, U., 18.042.128; Kochiev, A. A., 19.042.013; Krasilnikov, P. S. and Kunitsyn, A. L., 20.042.037; Kunitsyn, A. L. and Perezhogin, A. A. (1978), CM 18, 395; Kurchanova, M. V., 19.042.016; Leimanis, E., 17.042.050; Lidov, M. L. and Vashkov'jak, M. A., 18.042.006; Lidov, M. L. and Ziglin, S. L., 18.042.078; Lidov, M. L., 20.042.023; Losco, L. (1978), CM 17, 251; Luk'yanov, L. G., 18.042.076, 20.042.054 and (1977), CM 15, 489; Marchal, C. (1978), Proc. IAU Symp. 81, in press; Markeev, A. P. and Sokol'skij, A. G., 20.042.003; Markellos, V. V., 18.042.020, 19.042.053, 20.042.018, (1978), AA 67, 229 and (1978), MN 184, 273; Markellos, V. V. and Kazantzis, P. G., 17.042.029; Markellos, V. V., Goudas, G., Katsiaris, G. and Georgantopoulos, G., 18.042.062; Markellos, V. V. and Halioulias, A. A., 20.042.057; Markellos, V. V. and Kazantzis, P. G., 20.042.036; Markellos, V. V. and Klimopoulos, S. (1977), Astroph. Space Sci. 48, 471; Markellos, V. V., Klimopoulos, S. and Halioulias, A. S. (1978), CM 17, 215; Markellos, V. V., Klimopoulos, S. and Goudas, C. L. (1978), CM 17, 233; Markellos, V. V. and Zagouras, C. G. (1977), CM 16, 123 and AA 61, 505; Matas, V. (1978), CM 17, 193 and BAIC 29, 5; Mather, J. N. (1976), Adv. Math. 20, 263; Merman, G. A. (1977), TITA 15, 44 and 16, 59; Michalodimitrakis, M. (1978), AA 64, 83; Monaghan, J. J., 18.042.004, 18.042.060 and 19.042.029; Moskovkina, L. A., 18.042.021; Nahon, F., 17.042.024, 17.042.073 and 19.042.025; Nariai, K., 20.042.004; Perezhogin, A. A., 19.042.017; Perko, L. M. (1976), CM 14, 395 and (1977), CM 16, 275; Radzievskij, V. V., 17.042.070; Rimmer, R. (1977), J. Math. Analy. Appl. 59, 342; Robe, H. A. G. (1977), CM 16, 343; Schmidt, D. S., 17.042.033; Schutz, B. E. (1977), AIAA 77-33; Sergysels, R., 17.042.037; Sharma, R. K. and Rao, P. V. S., 17.042.042 and (1978), CM 18, 185; Sharma, R. K. , 18.042.001 and 18.042.015; Shirmin, G. I., 18.042.110; Sidlichovsky, M. (1978), BAIC 29, 14; Singh, V., 18.042.081 and (1977), CM 16, 137; Sokolov, V. G., 18.042.070; Standish, E. M., 19.042.041; Starzhinskij, V. M., 18.042.035; Stellmacher, I., 18.042.011; Szczodrowska-Kosar, B., 18.042.105 and 20.047.071; Szebehely, V., 18.042.053, (1977), CM 15, 107; BAAS 9, 620 and Rev. Mex. Astron. Astroph. 13, 145; Szebehely, V. and Zare, K. (1977), AA 58, 145; Troitskaya, L. S., 17.042.007; Vagner, E. A., 19.042.037; Valtmen, M. J., 18.042.002; Verhulst, F., 17.042.035; Vikyakin, V. V., 17.042.056, (1976), CM 13, 325 and (1977), CM 16, 509; Vrcelj, Z. and deJonge, J. H. (1978), AJ 83, 514; Waldvogel, J., 17.042.023 and 18.042.085; Zagouras, C., 20.042.059; Zagouras, C. and Markellos, V. V., 19.042.052; Zare, K. (1977), CM 16, 35; Zeleny, M., 17.042.001 and 18.042.112; Zhuravlev, S. G., 20.081.003; Ziglin, S. L., 19.042.014; Ziglin, S. L. and Lidov, M. L. (1977), Prik. Mat. Mec. 41, 234 and 20.042.065.

(8) The increased speed and storage capacity of high-speed electronic computers allowed the efficient study of gravitational n-body problems. New approximate techniques were introduced to integrate numerically the differential equations of motion of 1000 gravitationally interacting point masses. On the other end of the spectrum, the motion and stability of four and five-body problems, resembling the solar system, were studied (see item 7). Systems of interacting finite non-spherical bodies and systems with several fixed and one free body were analyzed. First order perturbations were derived concerning the motion of a star in an ellipsoidal cluster. See: Arazov, G. T., 17.042.070, 19.042.019 and (1977), CM 16,

https://doi.org/10.1017/S0251107X00010592 Published online by Cambridge University Press

41; Arazov, G. T. and Gabibov, S. A., 19.042.035, 20.042.045, (1977), CM 15, 265 and (1978), CM 17, 49; Arenstorf, R. F., 18.042.093; Barkham, P. G. D., Modi, V. J. and Soudack, A. C., 20.042.034; Duboshin, G. N., Dolgachev, V. P., Kalimina, E. P., Rybakov, A. I. and Holopov, P. N. (1976), AZ 53; Ivanov, Y. I., 19.042.005; Kozlov, I. S., 19.042.012; Losco, L., 18.042.073; Marchal, C. (1976), J. Diff. Equ. 20, 150; Message, P. J., 17.042.025, Palmore, J. I., 18.042.129; Pascal, M. (1976), Acta Astron. 3, 481 and 19.042.059; Saari, D. G., 18.042.094 and (1977), J. Diff. Equ. 26, 1; Seidel, E., 18.042.018; Simo, C. (1978), CM 18, 165.

(9) The fundamental problem of integrability was attacked with new vigor. Since the pertinent differential equations of celestial mechanics are, in general, non-integrable, the establishment of local (or "third") integrals offer a never-ending challenge to workers in this field. On the other hand, because of the non-integrability of the system, the long-time behavior in the classical deterministic sense, is an unsolved problem. Relations between high order resonant periodic orbits, characteristic exponents and integrability were studied. The range of validity of local integrals in the elliptic restricted problem was established. The disappearance of integrals, the onset of stochasticity and the existence of ergodic seas was demonstrated in various dynamical systems. The utilization of techniques of statistical mechanics and the study of bundles of trajectories (corresponding to not precisely defined initial conditions) has been proposed. See: Contopoulos, G., 20.042.070, (1977), Proc. Int. Conf. on Plasma Phys., in press, (1978), CM 17, 167, (1978), Theor. Princ. in Astroph. and Relativity, Chicago Univ. Press, 93, (1978), Stochastic Behavior in Hamiltonian Syst., Springer Publ. and (1978), Proc. NATO Adv. Study Inst., in press; DeWitt-Morette, C. (1976), Proc. NATO Adv. Study Inst., Reidel Publ., 57; Giorgilli, A. and Galgani, L. (1978), CM 17, 267; Hameen-Anttila, K. A., 20.042.056 and 20.042.061; Losco, L. 20.042.053; Nahon, F., 18.042.095; Olson, P., 20.081.007; Prigogine, I., Grecos, A. and George, C. (1977), CM 16, 489; Reichl, L. (1976), Proc. NATO Adv. Study Inst., Reidel Publ., 71; Sarris, E. (1977), Thesis, Univ. Athens; Sergysels, R. (1976), AA 48, 257 and (1978), Proc. Cong. Diff. Equ., 109; Szebehely, V., 19.042.042.

(10) The motion of Trojan asteroids, the Kirkwood gaps, locked in planetary and some special satellite motions have the common property of being associated with resonance problems. The non-linear dynamical systems occurring in celestial mechanics show peculiar behavior in resonance conditions, especially when more than one low order commensurability occurs. The dynamical topology of this problem in the circular as well as in the elliptic restricted problem of three bodies received considerable attention. See: Bhatnagar, K. B. and Gupta, B., 19.042.033; Bien, R. (1978), AA 68, 295 and 19.042.022; Blitzer, L. (1977), CM 16, 87; Contopoulos, G. (1978), CM 18, 195; Contopoulos, G. and Mertzanides (1977), AA 61, 477; Dallas, S. S., 20.052.020; Froeschlé, C. and Scholl, H., 17.098.016 and 19.042.045; Garfinkel, B. (1976), CM 14, 301, 17.042.046 and 18.042.045; Giacaglia, G. E. O., 18.042.079; Greenberg, R., 20.042.016; Jefferys, W. H., 17.042.038; Khentor, A. A., 18.042.092; Klokocnik, J., 18.054.005; Lewin, L. and Vagners, J., 19.021.005; Osorio, J., 17.042.031; Peale, S. J., 20.042.019; Reigber, C. and Balmino, G., 18.081.021; Roels, J., 17.042.032; Romanowicz, B., 18.054.009; Sanders, J. (1977), CM 16, 421; Scholl, H. and Froeschlé, C., 20.098.102; Schubart, J. (1978), Proc. IAU Coll. 41, 137; Wiesel, W. E., 17.042.002.

(11) A special subject in celestial mechanics, known as the problem of varying gravitational constant also received renewed interest and attention during the past three years. See: McVittie, G. C. (1978), MN 183, 749; Reasenberg, R. D. and Shapiro, I. I., 18.043.005; Saari, D. (1977), CM 16, 407; Van Fladern, T. C. (1976), Sc. Am. 234, 44; Vinti, J. (1976), CM 14, 363 and (1977), CM 16, 391; Walter, H. G., 17.043.001.

(12) New analytical and numerical techniques always abound in celestial mechanics since this field of astronomy strongly attracks mathematicians and numerical

analysts. The use of Lie series, averaging methods, regularization, canonical operations in the extended phase space, applications of asymptotic expressions, etc., are becoming increasingly popular. High, variable order and variable step-size Runge-Kutta and recurrent power series and other numerical integration methods were developed allowing long-time numerical integration of the solar system (10^6 - 10^7 years). The relation between numerical and dynamical instabilites must be understood before numerical integration for arbitrary long time can be performed with meaningful results. The introduction of control terms and various stabilization techniques (often based on regularization) offer hopes in this direction. The staggering number of papers, pertinent to celestial mechanics and published in this field made choices of inclusion difficult but at the same time demonstrated that the spirit of Gauss is still with our field. Review papers form part of the following list of references: Aarseth, S. J., 17.042.019; Agekian, T. A. (1976), AZ 53, 1106; Andrus, J. F., 20.052.033; Babadzhanyants, L. K., 18.042.008 and 18.042.009; Baker, R. M. L. and Jacoby, N. H., 20.042.043; Barbanis, B. (1976), CM 14, 209; Bartlett, J. H., 17.042.014 and (1978), CM 17, 3; Baumgarte, J., 17.042.004, 17.042.018, 17.042.047 and 18.042.102; Belen'kij, I. M., 18.042.034; Bertin, G. and Radicati, L. A., 17.042.058; Bettis, D. and Horn, M. K. (1976), CM 14, 133; Birnbaum, D., 20.042.025; Boigey, F., 19.042.026; Bond, V. (1976), CM 13, 287, 18.042.069, (1977), NASA JSC-13128, 13786, 14281, and Proc. IAU Coll. 41, 159; Borderies, N., 17.052.010 and (1977), CM 16, 291; Boronenko, T. S., 18.042.022; Brahic, A., 19.042.060; Broucke, R., 20.021.002; Broucke, R., Lass, H. and Boggs, D., 18.042.090; Brumberg, V. A. (1978), CM 18, 319; Burns, J. A., 19.042.058; Calvo, M., 18.042.013 and 18.042.108; Chepurova, V. M., 20.042.064; Chernitsov, A. M., 18.042.037; Cid, R. and Camarena, V., 20.042.026; Contopoulos, G., 17.042.013; Deprit, A., 17.042.048; Donaldson, J. D. and Jezewski, D. J. (1977), CM 16, 367; Drozyner, A., 20.021.010; Dvorak, R., 17.042.008; Eichhorn, H., 18.042.127; Feagin, T., 17.042.028; Feagin, T. and Beauchet, P. R. (1976), CM 13, 111; Feagin, T. and Mikkilineni, R. P. (1976), CM 13, 491; Ferraz-Mello, S. (1978), CR 286, 969; Ferronsky, V. I., Denisik, S. A. and Ferronsky, S. V. (1978), CM 18, 113; Gavrilov, N. I., 19.042.011; Glebov, V. D., 18.021.025; Heggie, D. C., 18.042.097; Heintz, W. H., 18.042.126; House, F., Weiss, G. and Wiegandt, R. (1978), CM 18, 311; Howard, R. A., 18.042.043; Howland, R. A., 20.042.047; Jefferys, W. H., 17.042.075 and 18.042.044; Jezewski, D. J., 18.042.100; Kholshevnikov, K. V., 18.042.118; Kinoshita, H. (1978), CM 17, 131; Kirchgraber, U., 17.042.015 and 18.042.089; Kraige, L. G. and Junkins, J. L., 17.042.003; Kriz, J. (1978), CM 18, 371; Kurcheeva, I. V., 20.042.048; Lahulla, J. F., 17.042.076; Langlois, M. and Losco, L., 18.042.096; Laricheva, V. V., 20.042.063; Lass, H. and Georgevic, R. M. (1978), CM 18, 3; Lazovic, J., 20.042.068; Lounesto, P. (1978), CM 17, 207; Lyakhov, E. I., 19.042.010; Marchal, C., 17.042.020; Matas, V. (1978), CM 17, 193; Meffroy, J., 17.042.072, 17.042.074 and 18.042.038; Merman, G. A., 18.042.005 and 20.042.078; Mitchell, T. P. (1978), CM 17, 259; Moore, P. (1978), CM 17, 281; Myachin, V. F. and Sizova, O. A., 18.042.069 and (1978), BITA 14, 215; Nacozy, P. E. (1976), CM 13, 495, (1977), CM 16, 309, 17.042.030 and 18.042.103; Nacozy, P. E. and Dallas, S. S., 18.081.028; Niethammer, W. and Schweitzer, U., 18.042.017; Osipov, Y. S. (1977), CM 16, 191; Ovenden, M. W., 17.042.026; Petroskevicius, P. A., 18.042.025 and 18.052.026; Petruk, V. P., 20.042.058; Platonov, A. K. and Kozakova, R. K., 18.021.023 and 19.052.011; Russmann, H. (1976), CM 14, 19; Scarlet, W. (1978), CM 17, 299; Sigrist, N., 17.042.017; Sokolsky, A. G. (1978), CM 17, 373; Stickforth, J., 20.042.042; Stiefel, E., 17.042.011 and 18.042.099; Stokes, A. (1976), CM 16, 27, (1978), CM 17, 137 and 17.042.034; Szebehely, V., 17.042.012, 17.042.081 and J. Guidance and Contr. 1, 387; Szebehely, V. and Zare, K., 18.042.053; Titov, V. B., 18.042.026; Topan, G., 18.042.112; Vagner, E. A., 17.042.005; Velez, C. E. and Helinski, S. (1978), CM 17, 83; Velte, W. (1978), CM 17, 395; Verhulst, F., 17.042.016; Vey, J., 18.042.057; Vitins, M. (1978), CM 17, 173; Waldvogel, J.(1976), Zeitsch. Ang. Mat. Phys. 27, 867; Wierzbinski, S., 18.042.104; Zadunaisky, P. E. (1976), Numer. Math. 27, 21; Zagars, J., 19.052.026.

Some of the major books published during the past three years were:

Abalakin, V. K., Aksenov, E. P., Grebenikov, E. A., Demin, V. G. and Ryabov, Y-A.: "Manual on Celestial Mechanics & Astrophysics", Nauka, Moscow, 1976, (18.003.171).

Aksenov, E. P.: "Theory of Motion of Artificial Earth Satellites", Nauka, Moscow, 1977, (19.003.019).

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Szebehely, V. (ed.): "Dynamics of Planets and Satellites and Theory of Their Motion", Proc. IAU Coll. 41, D. Reidel Publ., 1978.

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Szebehely, V. and Tapley, B. D. (eds.): "Long-Time Predictions in Dynamics", 1976, (17.012.005).

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In summary, it may be stated that a significant progress made during the past three years in celestial mechanics, indeed in modern dynamics, was the recognition of the limitations of present day analytical and numerical techniques. When artificial earth satellite observations offer centimeter accuracy our theories seem to be quite unacceptable in comparison, even for high altitude orbits. The accuracy of our lunar theories are in order of magnitude below modern observations. But some limitations, in principle, are even more significant. Long-time predictions, both of qualitative and quantitative levels encounter serious difficulties. The often divergent series solutions of general perturbation methods are inadequate for long-time predictions or for stability research and even the most sophisticated high-order numerical integration techniques have similar limitations. These limitations offer challenges and an unquestionably bright future for workers in the field of celestial mechanics. A field which was certainly not closed in Newton's or Poincaré's time and is ready to enter one of its most exciting periods of expansion.

V. SZEBEHELY
President of the Commission.