

Solution by J. W. Moon, University of Alberta, Edmonton

It is clear that $n \geq k + 2$. We may suppose that G has some vertex x of valence at most k , for otherwise the result is certainly true. Then the graph obtained from G by removing x and its incident edges has $n - 1$ vertices and more than $k(n-k) + \binom{k}{2} - k = k[(n-1)-k] + \binom{k}{2}$ edges. The result now follows immediately by induction, since it is trivially true when $n = k + 2$.

Also solved by W. G. Brown and the proposer.

Editor's comment. The result is vacuously true for $n = k, k + 1$ since then no graph has as many edges as the problem requires; but as stated, it is false for $n < k$, the complete graph furnishing a counter-example.

P 90. Let $\log_s x$ be the log function iterated s times, and let m be the smallest positive integer such that $\log_4 m > 1$. Then show that the sum

$$\sum_{k=m}^{\infty} \frac{1}{k(\log k) (\log_2 k) (\log_3 k) (\log_4 k)^2}$$

is approximately 1 - correct to more than one million decimal places!

John D. Dixon, California Institute of Technology

Solution by S. Spital, California State Poly. College.

Since the series in question

$$S = \sum_{k=m}^{\infty} u(k) = \sum_{k=m}^{\infty} \frac{1}{k \log k \log_2 k \log_3 k (\log_4 k)^2}$$

is composed of positive decreasing terms, and since