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BOOK REVIEWS

Davis, J. F. and Kirk, P. *Lecture notes in algebraic topology* (Graduate Studies in Mathematics, no. 35, American Mathematical Society, 2001), xvi+367 pp., 0 8218 2160 1 (hardback), $55.00.

According to the authors, the book under review originated from the lecture notes for the second year algebraic topology course at Indiana University, and could have been titled 'What every young topologist should know'.

In recent years, some basic instruments and ideas of algebraic topology have been successfully translated into pure algebra and algebraic geometry (strong homotopy algebras, the homotopy theory of operads, etc.); moreover, physicists and geometers used to decorate such fundamental to algebraic topology objects as cohomology/homology of a space with solutions of some important integrable systems of differential equations (Painlevé VI, WDVV, etc.). Thus the potential audience for the book under review is much larger than 'young topologists', and the book might have a longer title as well: 'What every mathematician and theoretical physicist should know about algebraic topology'.

This book is one of the best introductions to algebraic topology, its main mathematical instruments and notions, of which the reviewer knows. The text is clear, clean, short, very informative and well illustrated with clever examples. Sometimes the proofs are sketchy or are just left as exercises. For some readers this can be a disadvantage (and they are advised to view this book merely as a first reading and then turn to thicker introductions to the subject), for others this can be an advantage as it offers relatively quick access to many treasures of this beautiful area of mathematics.

This book is also an excellent basis for a lecture course. Every chapter is equipped with a selection of course projects.

The contents are as follows.

Chapter 1. Chain complexes, homology and cohomology

Complexes associated to a space, tensor products, adjoint functors, singular cohomology.

Chapter 2. Homological algebra

Tor and Ext functors, projective and injective modules and resolutions, the fundamental lemma of homological algebra, universal coefficients theorems.

Chapter 3. Products

The Künneth Theorem, Eilenberg–Zilber maps, cross and cup products, the Alexander–Whitney diagonal approximation, relative cup and cap products.

Chapter 4. Fiber bundles

Group actions, principal and associated bundles, structure group reduction, maps of bundles and pullbacks.

Chapter 5. Homology with local coefficients

Its definition, basic properties and examples, its functoriality.

Chapter 6. Fibrations, cofibrations and homotopy groups

Compactly generated spaces, fibrations/cofibrations, path space fibrations, fibre homotopy, replacing a map by a fibration/cofibration, homotopy classes of maps and (relative) homotopy groups, adjoint of loops and suspension, smash products, fibration and cofibration sequences, examples, the action of the fundamental group on homotopy sets, the Hurewicz and Whitehead Theorems.

Chapter 7. Obstruction theory and Eilenberg–Maclane spaces

Basic problems of obstruction theory, the obstruction cycle and its construction, the extension theorem, obstructions to finding a homotopy, primary obstructions, Eilenberg–Maclane spaces, aspherical spaces, CW-approximations and Whitehead's Theorem, obstruction theory in fibrations, characteristic classes.

Chapter 8. Bordism, spectra, and generalized homology

Framed bordism and homotopy groups of spheres, suspension and the Freudenthal Theorem, stable tangential framings, spectra, more general bordism theories, classifying spaces, construction of the Thom spectra, generalized homology theories.

Chapter 9. Spectral sequences

Their definition, the Leray–Serre–Atiyah–Hirzebruch spectral sequence, the edge homomorphisms and the transgression, the homology/cohomology spectral sequences and their applications, homology of groups and covering spaces, relative spectral sequences.

Chapter 10. Further applications of spectral sequences

Serre classes of abelian groups, homotopy groups of spheres, suspension, looping and transgression, cohomology operations, the mod 2 Steenrod algebra, the Thom Isomorphism Theorem, intersection theory, Stiefel–Whitney classes, localization, construction of bordism invariants.

Chapter 11. Simple-homotopy theory

Invertible matrices and $K_1(R)$, torsion for chain complexes, Whitehead torsion and CW complexes, Reidemeister torsion, lens spaces, the s-cobordism theorem.

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