Matters for debate

A partial comparison between 1997 O level and GCSE mathematics papers

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Introduction

I was recently asked to compare the problems set on the 1997 O level (Syllabus D) papers set by the University of Cambridge Local Examination Syndicate (UCLES) for schools in Singapore and the Caribbean with the problems set on higher tier GCSE papers for English schools. At first I was inclined to make excuses: there are all sorts of reasons why such comparisons can be difficult! However, I agreed to look at the papers and to comment in whatever way seemed appropriate.

Knowing how exam boards work, I felt that it would be unfair, and misleading to focus on a single board by comparing the UCLES O level papers with GCSE papers form the same board; so I borrowed a set of 1997 GCSE papers from a successful local school (all of whose students take the higher tier papers – most doing so a year early). These happened to be Papers 5 and 6 for the ULEAC Mathematics 1384 Syllabus. The effort that goes into making sure that the GCSE papers from different boards are comparable would tend to suggest that these papers are unlikely to be markedly different from those of other boards.

When I looked at the two sets of questions in detail, it soon became clear that the contrast was sufficiently stark to justify making a more serious attempt to compare the two genres than I had originally envisaged.

Methodology

Comparisons are odious – and messy. To keep things manageable one has to simplify; however, I have tried to do this in a reasonably balanced way.

1. I have restricted my comments to the questions set, and have made no attempt to explore the important issue of mark schemes and ultimate mark distributions.

2. I have restricted my comments mainly to the easier material on each paper. For each syllabus, a specified topic could apparently be covered at any point on either of the two papers. However, earlier questions tend to be easier, and are worth fewer marks, than later questions, so I limit my comments to:

(a) the first ten questions on the ULEAC 1384 GCSE Paper 5, and any comparable questions on the UCLES (Syllabus D) O level papers, no matter where they occur;

(b) the first ten questions on the UCLES (Syllabus D) O level Paper 1,

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and any comparable questions on the ULEAC Papers 5 and 6, no matter where they occur.

This approach would seem to allow a reasonably fair comparison of the coverage of easier material on the two examinations.

3. There is a serious problem in trying to present the comparison in a way that allows the reader to keep track of the cumulative differences. The strategy I eventually adopted was to lay out the questions so that

- the GCSE problems, and my comments on them, appear on the left hand side of each page, and
- the O level problems and associated comments are printed on the right of the page.

I hope this layout will make it easier for the reader.

4. Similar questions on comparable material are grouped together printed in boxes. They are interspersed with my comments on the problems and my assessment of their relative cognitive demands (in ordinary type).

One might think that, since each syllabus is treated in the same way, the outcome is unbiased. In practice, all one can do is to structure one's approach so that the extent of any bias can be assessed: comparisons are never unbiased – but can still be highly instructive. (For example, apart from my failure to examine scripts and mark schemes, I have made no allowance for two important differences:

(i) the UCLES O level Paper 2 consists exclusively of longer, problem-solving questions of a kind current English students might find rather testing;

(ii) in at least some user countries the UCLES (Syllabus D) papers are sat by a considerably larger fraction of Year 11 pupils than sit the GCSE Higher Tier papers in England.)

Despite the need for caution, having completed the exercise I see no way to avoid the following conclusions.

In every instance examined, the syllabus which is only available for use in countries outside the UK sets questions

- which are better designed;
- which make tougher demands on candidates;
- which are 'more mathematical' (in the sense that they encourage teachers to lay more satisfactory foundations for candidates to be able subsequently to make use the mathematics they have learned);
- and which award comparable or fewer marks for the same material.

It is hard to state these conclusions too strongly.

Background on the papers

ULEAC 1384 GCSE Paper 5
23 questions - total 96 marks

UCLES O level Paper 1
24 questions - total 80 marks

The ULEAC 1384 GCSE papers
allow use of a calculator;
the UCLES O level papers do not.
The ULEAC GCSE papers provide a formula sheet;
the UCLES O level papers appear not to.

It follows that questions which at first sight seem to be comparable on the two papers often make dramatically different cognitive demands on candidates.

The questions

Q1 A graph (said to be about oil production) is given, with data points already plotted!
The candidate has to
- insert a 'line of best fit'; [2 marks]
- use this to find the 'y-value' for a given 'x-value'. [2 marks]

This challenge to 'find a y-value corresponding to a given x-value' is the easy way round for weak candidates, since this is the way graphs are plotted.
The question fills a complete page!
It appears to be about mathematics, but is really about something different.

The corresponding question on the UCLES papers is Paper 1, Q7:

1:Q7 Candidates are told simply '1 dollar = 3.56 Pula', without further explanation.
They must then
- draw the straight line graph to convert from dollars (x) to Pula (y); [1 mark]
- use their graph to find the x-value corresponding to a given y-value. [1 mark]

Note the combination of the simpler context, the greater abstraction, and the tougher demands (from y to x) in the second part.

Q2 '*x is an integer.*
Write down the greatest value of x for which $2x < 7$.' [1 mark]

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The wording positively encourages candidates to use guesswork rather than mathematics.
There is a more mathematical variation on the same theme later on in the same paper:

Q9 '*Solve the inequality* $7y > 2y - 3$.' [2 marks]

This is in one of the easier forms of an inequality – leading almost immediately to $5y > -3$.
The candidate is then required only to have the confidence to write $y > -3/5$.

The corresponding material on the UCLES papers is in

1:Q14	
'(*a*) *write down all the integer values of n for which* $-3 < n \leqslant 1$.	[1 mark]
(*b*) *Solve the inequality* $3 - 5x \geqslant 18$.'	[2 marks]

Part (a) does not spoon-feed the candidate by breaking the question into two separate sentences.
Moreover, it uses a compound inequality, involving both '<' and '⩽'.
The question is easy, but not trivial.
Part (b) requires the candidate to make a crucial extra step ('add $5x$ to both sides'), and so tests understanding at a higher level (for the same reward).

Q3 This gives a picture of a cylinder and all the measurements and asks the candidate to find the volume.

The formula is given on the formula sheet.
All the candidate has to do is to
(i) divide the given diameter by 2
(ii) use a calculator to evaluate the 'answer';
(iii) round the output to 3 significant figures.

The UCLES paper does not award marks for such things.

Q4 '*Here are the first five terms of a number sequence.* 5, 8, 11, 14, 17. *Write down an expression for the nth term of the sequence.*'

Unless the mark scheme gives credit for any nth term consistent with the first 5 terms, the question is plain wrong: one cannot give 'the first five terms of a sequence' and ask for 'the nth term' unless the sequence

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has an associated 'rule'!
Candidates are expected to play 'guess what's in the examiner's mind'.
This they will happily do: however, it is not mathematics.
The expected rule is presumably $5 + 3(n - 1) = 3n + 2$.
As though one example of this type of question were not already too much, there is a further question on the same lines (awarding yet more marks for what is an anti-mathematical activity).

> Paper 6: Q2 '*Work out an algebraic expression for the* n*th term of this sequence of numbers*
> 2, 8, 18, 32, 50, ...' [2 marks]

In mathematics the exhortation 'work out' should involve a deterministic calculation. What is intended here is for candidates to 'guess the nth term'.

The corresponding material on the UCLES Paper 1 is presented in a more acceptable – and far more testing – way.

> Q15 '*The sequence of numbers*
> $$1, 5, 11, 19, 29, \ldots$$
> *can also be expressed in the form*
> $$1^2 + 0,\ 2^2 + 1,\ 3^2 + 2,\ 4^2 + 3, \ldots \text{'}$$

By writing the question in this form, the examiner is implicitly specifying the rule whereby the sequence is generated. Hence the rest of the question becomes mathematically and educationally sound, with no need to play 'guess what's in the examiner's mind'.

> '*(a) Express the 5th term in the same form.* [1 mark]
> *(b) Write a formula for the nth term.* [1 mark]
> *(c) Calculate the value of the* 100*th term.*' [1 mark]

One cannot overstate how much better designed this is than the corresponding ULEAC GCSE questions. The content is at a much higher mathematical level; and the question is nicely graded to make it possible for those who have been reasonably taught to get started – with a lovely sequence of parts:
(a) do a small concrete example to show you understand and can use the notation in easy cases;
(b) imitate what you have just written with letters in place of numbers (still no guarantee that the candidate understands what he or she has written);

(c) show that you understand the meaning of the 'n' in the formula you have just written, that you can evaluate 100^2, and that you realise the extra summand is $(100 - 1)$ rather than 100.

This is how this sort of material should be assessed. (I would prefer 2 marks for this last part – but that is my only quibble.)

Q5 The question
- describes a context (throwing a 'dice' 200 times);
- gives a table of observed frequencies;
- asks candidates to read from the table and calculate an estimate of the probability of throwing a 3.

Is this question appropriate on Higher Tier papers? Candidates merely have
- to avoid being put off by four lines of dubious 'context';
- to extract from the given table the number '46' (namely the number of times a '3' was thrown); and
- use their calculators (!) to work out 46/200 = 0.23.

[2 marks]

There is no equivalent question on the UCLES papers.

Q6

'*Solve the simultaneous equations*

$$3x + y = 13$$
$$2x - 3y = 16.\text{'}$$

[4 marks]

At first sight this looks like a good old-fashioned algebra problem.

But this is misleading – and in two ways.

First, one equation has just 'y' (coefficient =1), so one only has to multiply one equation to eliminate a variable (y): (3 × (Eqn 1)) + (Eqn 2) yields $11x = 55$.

This then reveals the second 'unfortunate' feature: the solutions are small integers – $x = 5, y = -2$

Moreover, x (the natural unknown to go for first) is positive, so can easily be guessed – for example, by taking the very first term in the very first equation (namely $3x$) and making this close to the constant 13 on the right hand side, then choosing $y = -2$.

The corresponding UCLES question is

1:Q11

'*Solve the simultaneous equations*

$$7x - 5y = 17$$
$$3x - 2y = 7.\text{'}$$

[3 marks]

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The values of x, y are still small integers, but
- x is now negative;
- one has to form a genuine combination of the two equations to eliminate a variable (e.g. $3 \times$ (Eqn 1) $- 7 \times$ (Eqn 2)); thus the question provides a much stiffer test of whether the candidate understands the elimination procedure;
- this procedure forces the candidate to work fluently with negative numbers;
- the solution cannot be stumbled upon by the same sort of guesswork (weak candidates are unlikely to try $x = -1$).

This is again on a completely different cognitive level from the ULEAC question, yet earns only 3 marks.

Q7 *'Shreena put* £484 *in a new savings account.*
At the end of every year, interest of 4.3% *was added to the amount of her savings account at the start of that year.*
Calculate the total amount in Shreena's savings account at the end of 2 *years.'* [5 marks]

The corresponding question on the UCLES papers is

1:Q13 *'In a shop a bicycle is priced at* \$451. *The price includes Government Tax at* 10%.
How much is the tax?' [3 marks]

The contrast between these two questions is quite striking.

The ULEAC question
- is wordy;
- pretends to be realistic, but trips up on the difficulty of specifying exactly when the interest is added to the account (and, to make the question tractable, forces the account to operate in a curious way, with nothing deposited or withdrawn, and a fixed interest rate for two years);
- requires students merely to apply a superficial understanding of 'interest'.

All that is needed is to work out

$$484 \times 1.043 \times 1.043$$

using a calculator – though most candidates will laboriously work out

$$I = [(4.3)/100] \times 484$$

and add it to 484, before repeating the whole operation with the new amount.

The only scope for 'error' is that many will give the answer

$$484 + 2I.$$

One fears that these will score at least 3 marks – despite having demonstrated that they do not really understand.

In contrast, the UCLES question

- is short;
- to the point;
- tests genuine insight (since the answer is *not* \$45.10, since the '10%' is 10% of the pre-tax price); and
- earns only 3 marks.

Q8 The question gives a concave pentagon drawn on a grid. Candidates are given a 'starter line' – parallel to, and one third as long as, the corresponding side in the original pentagon – which they must then use to draw a shrunken copy.

They are asked

'*(a) Write down the scale factor of the enlargement.*

[1 mark]

(b) Complete the drawing.' [2 marks]

While this sort of question is fine as part of an education (at age 11-13), is it an appropriate way of testing enlargement and scale factors on a Higher Tier GCSE paper (at age 16)?

Q10 '*Matthew uses this formula to calculate the value of D.*

$$D = \frac{a - 3c}{a - c^2}.$$

(a) Calculate the value of D when $a = 19.9$, $c = 4.05$.

Write down all the figures in your calculator display.

[3 marks]

Matthew estimates the value of D without using a calculator.

(b) (i) Write down an approximate value for each of a and c that Matthew could use to estimate the value of D.

(ii) Work out the estimate that these approximations give for the value of D. Show all your working.'

[3 marks]

You are advised to re-read the question! (How can Matthew possibly 'estimate the value of D' in part (b) when it depends on two variables a and c whose values are not given in this part.)

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The question is appallingly worded, leaving the candidate to use the values from part (a), and the underlying idea is no better.
In part (a) the candidate is not expected to do anything with the given formula except to substitute the given values using a calculator and to copy the display.
[3 marks]
The curious additional instruction (to copy the whole display) is presumably intended to prepare candidates to do the hoped-for thing in part (b).
Part (b) is a completely artificial attempt to force candidates (with a calculator to hand!)
- to approximate the given *a* and *c*;
- to calculate a corresponding 'approximate' value of D.

Estimation may be worth testing – but not like this.

It may be worth explicitly highlighting part of the underlying problem here. Estimation depends on a fluency with exact calculation; and the fact that calculators are available for the ULEAC papers makes it difficult to test exact calculation. Thus one is trying to assess *second order* thinking while neglecting the relevant *first order* thinking.

The only question on the UCLES papers which seems to involve estimation is 2:Q11 – a remarkable 'problem-solving' question (worth 12 marks).
All the questions on the UCLES Syllabus D Paper 2 are of this demanding and enlightened kind – requiring sustained reasoning and calculation.
These questions represent the sort of 'problem-solving' we should be developing: that is, problems with mathematical content which test pupils' ability to use the exact techniques they have mastered (and which have been effectively tested by the questions on Paper 1).

We now look at the routine early questions (1-10) from the UCLES Syllabus D Paper 1 which do not cover material equivalent to early questions on the ULEAC 1384 GCSE Paper 5.

1:Q1
'*(a) Arrange* 1/5, 0.22, 0.033 *in order,* smallest first.'[1 mark]

Without a calculator, this is an excellent simple test: candidates
- must 'know' 1/5 =0.2; and
- must then avoid the trap of thinking that if 0.2 < 0.22, 'then' 0.22 < 0.033.

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'(b) *On a particular day the temperature varied by* 28°C. *The highest temperature recorded was* 22°C. *What was the lowest temperature that day?*' [1 mark]

A simple question requiring candidates to think about and use given information in a non-trivial way.

1:Q2 Candidates are given a partial reflection of the letters 'FIRE RESCUE' for them to complete. [2 marks]

In particular, they have to remember to reverse the 'C' and the 'F'.
An interesting test of geometrical sense.

1:Q3 '*Given that matrix* $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$,

(a) calculate the value of the determinant of A,

(b) write down A^{-1}.'

This is a simple test of technique prior to the assessment in Paper 2 of candidates' ability to use these ideas to solve sustained problems involving matrices.
The material requires a willingness to work with abstract ideas, and is a very important part of modern mathematics.

Elementary matrix calculations no longer appear in corresponding English syllabuses. This points up a weakness of both competing English trends:

- the progressive approach which advocates broader syllabuses and more 'less formal' content, and
- 'back to basics'.

The inclusion of such material in the UCLES Syllabus D papers reflects the concern to develop skills which will be important at higher levels.

In contrast, many of the problems on the ULEAC Higher Tier papers appear designed to satisfy the requirements to assess items on a backward-looking syllabus – items which frequently handicap rather than support subsequent progress.

1:Q4 '*The exterior angle of a regular polygon is* 15°. *How many sides does it have?*' [2 marks]

Working with angles in 2D diagrams is an excellent test of pupils' ability to make sense of formal geometrical figures.

Paper 2, Questions 2, 4, 8 all develop the use of 'angle' in harder problems.

Remarkably, there is not a single question on the two ULEAC papers which tests whether candidates understand simple properties of 'angle'.
The only appearance of angle on the ULEAC papers is Paper 6, Q18, and is at too high a cognitive level to assess most candidates' basic technique.

1:Q5 '*Evaluate (a)* 3^{-2}	[1 mark]
(b) $(\frac{1}{16})^{1/2}$.'	[1 mark]

These are excellent tests of candidates' understanding and ability to calculate reliably (no calculators!) – making sense of expressions involving index notation.

The corresponding question in the ULEAC papers (with calculators!) is

Paper 6:Q14	
'*Evaluate (a)* $27^{2/3}$	[1 mark]
(b) $(\frac{3}{2})^{-2}$.'	[1 mark]

This would be a perfectly sensible question
- if candidates were required to work without calculators, and
- if answers were not accepted in decimal form (so that the only acceptable answer to (b) were 4/9).

As it is, candidates will simply plug values into a calculator and copy whatever appears in the display.
Unlike the UCLES question, the marks candidates score here provide little indication whether they understand index notation.

While we are considering arithmetical fluency, it may be worth drawing attention to two other questions on the UCLES Syllabus D papers.

1:Q10 '*Given that* $87 \times 132 = 11484$,	
(a) complete the statement	
$88 \times 132 = 11484 + \ldots$	[1 mark]
(b) write down the exact value of	
(i) 0.087×13200,	[1 mark]
(ii) $0.11484 \div 0.0087$.'	[1 mark]

This is an excellent test of understanding of place-value, and of the relations between the different operations of elementary arithmetic.

There is no corresponding question on the ULEAC GCSE papers.

> 1:Q12 '*Evaluate*
> *(a)* $16 - 2 \times 3$ [1 mark]
> *(b)* $\frac{1}{4} \times 2\frac{1}{2}$ [1 mark]
> *(c)* $\frac{7}{8} \div \frac{1}{16}$.' [1 mark]

This is more than just a test of routine skill. Candidates have to interpret
- the precedence of operations in (a);
- the '$2\frac{1}{2}$' (as $\frac{5}{2}$) in (b); and
- exploit the fact that $16 = 2 \times 8$ in (c).

There is no corresponding question on the ULEAC papers.

> 1:Q6 '*The mass of the earth is*
> $$5.9763 \times 10^{27} \textit{ grams.}$$
> *Expressing your answers in standard form, correct to 3 significant figures, write this mass*
> *(a) in grams,* [1 mark]
> *(b) in kilograms.'* [1 mark]

A nice assessment question (without the distraction of a calculator!).
Part (a) is a simple test of
- standard form;
- rounding; and
- the candidate's willingness to make sense of frighteningly large numbers (they have to round up to 5.98×10^{27}).

Part (b) is a simple test of
- how many grams in a kilogram;
- interpreting this as 10^3;
- knowing how to extract this from 10^{27} to get 10^{24}, not 10^9).

Astonishingly (for a culture that has embraced the calculator so wholeheartedly) there is no attempt to test 'standard form' on the ULEAC papers.

> 1:Q8 '*Solve the equations*
> *(a)* $\frac{3}{x} = 4$, [1 mark]
> *(b)* $5y - 3(y - 1) = 23$ [2 marks]

Linear equations in one unknown constitute the simplest and most fundamental kind of equation. They allow pupils to master the essential principles which underpin all of algebra – namely
- that if one does the same to both sides of an equation, the equality is preserved;

- that the goal is to use this idea and simple arithmetic to work towards an equation of the form '$x = \dots$' or '$y = \dots$'.

Questions 8(a) and 8(b) are excellent tests of whether candidates have mastered these simplest ideas.

Part (a) is straightforward (in that it requires candidates to make two steps of the same arithmetical type – one multiply and one divide);

Part (b) requires candidates to combine

- addition/subtraction, and
- multiplication/division,

by first multiplying out, and collecting up terms, then rearranging (to get $2y = 20$), before dividing by 2.

The solution is an integer, but not such that candidates will guess the value – and one suspects that the mark scheme requires candidates to calculate exactly and algebraically.

There is no corresponding question on the two ULEAC papers.

(There are quadratics – even a cubic. But the method of so-called 'solution' recommended is often anti-mathematical:

For example, candidates are encouraged

- to guess (Paper 6:Q9); or
- to work experimentally on graph paper (Paper 5:Q19);

rather than to *solve analytically*.

There may be times when guessing and experimenting can be useful; but the priorities of English curricula and examiners are so distorted as to give a whole generation of candidates and their teachers a completely false impression of what it means to 'solve' an equation mathematically.

Paper 5: Q22 involves quadratics (though the problem is broken down to such an extent that candidates scarcely need to understand what they are doing).

In particular, by failing to test basic understanding at the level of linear equations in one unknown, one encourages teachers and candidates to depend on poorly comprehended rules for more complicated equations.

1:Q9 '*When it is* 07 00 *in New York, the time in London is* 12 00.

(a) *What is the time in London when it is* 22 00 *in New York?* [1 mark]

(b) *A flight from London departs at* 4.30*p.m. The flying time is* 6 *hours. What is the time in New York when it arrives?'* [2 marks]

A nice problem requiring candidates to use their imagination, to sort out two standard time conventions, and to compare different origins – with an easy part (a), and a more testing part (b).

There is no similar question on the ULEAC paper.

Conclusion

I suspect that most mathematics teachers will recognise the stark contrast between the simple but effective assessment problems in the right hand column, and those which we have come to accept for our own pupils in recent years – both at GCSE and at Key Stage 3.

The poor quality of questions set in public examinations and in national assessments has faced competent teachers with an impossible choice: to respect mathematics, or to train their pupils to jump through misshapen hoops. Some have resolved this dilemma by taking early retirement. Others have had to 'adjust' as best they can.

If we are to improve the mathematical preparation that we currently offer pupils, it is important that the National Curriculum be so structured as to allow English examining boards (and the overseeing Qualifications and Curriculum Authority) to rediscover the central importance of setting high quality assessment problems. Only then will teachers be free to teach well.

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Advert did not add up for girls

Pupils at a girls' school have forced a chip company to change its advertising. The girls calculated that the firm, McCain Feeds, had got its sums wrong.

Wanda Marshall, a maths teacher at St Dunstan's Abbey in Plymouth, saw tabletop advertisements in a British Home Stores restaurant claiming that the firm sold enough chips to stretch to the moon and back ten times, or circle the earth 293 times, if they were laid end-to-end. The suspicious teacher set her pupils at the independent girls' school the task of working out the figures.

Mrs Marshall said that the company claimed to sell 108,107,900 french fries to BhS customers. But as the distance to the moon is 382,240 kilometres, that would mean the chips, average length 10 centimetres, would need to be 70 metres long. For the chips to stretch nearly 300 times around the world's circumference, 40,074 kilometres, they would each need to be 109 metres long.

The firm has now withdrawn the adverts from 90 BhS stores and is to have them reprinted with the correct figures.

Spotted in *The Times* 3 April 98 sent in by Richard Crossley and A. R. Pargeter.