ON THE USE OF MIXTURE THEORIES TO MODEL HEAT AND MASS TRANSPORT IN MOUNTAIN SNOWPACK

(Abstract only)

by

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ABSTRACT

Snow is a mixture of ice, air, water vapor, and liquid water, and as such is a mechanically and thermodynamically very active material. In an attempt to gain a better understanding of the process of metamorphism and the interaction of mechanical and thermodynamic processes within mountain snowpack, modern mixture theory was used. Only dry snowpack was considered, so that the liquid phase could be neglected.

Each constituent (ice, vapor, and air) was assumed to have its own density ρ_α, temperature Θ_α, partial stress T_α, particle velocity $\grave{z}_\alpha$, internal energy ε_α, entropy η_α, free energy Ψ_α, and heat flux q_α. Each constituent was considered to have the ability of reacting with the other constituents, so that mass supplies $\hat{C}_\alpha$, energy supplies $\hat{\varepsilon}_\alpha$, and momentum supplies $\hat{p}_\alpha$ can be defined to characterize the manner in which each constituent receives mass, energy, and momentum from the other constituents due to chemical and mechanical interactions. In addition, a concentration c_α was defined in order to specify the extent to which the solid and vapor phases have exchanged mass.

The mass balance, momentum balance, and energy balance principles were then developed for each constituent, thereby giving a set of nine differential equations to solve for Θ_α, $\grave{z}_\alpha$, and ρ_α for each constituent. In addition the second law was defined for the mixture and used to place restrictions on the allowable forms for the constitutive equations for T_α, ε_α, η_α, Ψ_α, q_α, $\hat{p}_\alpha$, $\hat{\varepsilon}_\alpha$, and $\hat{c}_\alpha$ for each constituent. In this way the forms for these variables under nonequilibrium thermodynamic conditions could be specified.

For example the restrictions imposed by the second law require that, under nonequilibrium conditions, the stress, mass exchange rate, heat flux, and momentum supply may have the forms:

$$T_\alpha = C_\alpha E_\alpha + \gamma_\alpha \grave{E}_\alpha + B_\alpha(\Theta_\alpha - \Theta_o) + \Psi_{\alpha o}, \tag{1}$$

$$\hat{C}_\alpha = \sum_{\beta=1}^{3} h_{\alpha\beta} U_{\beta 3} + \gamma_{\alpha\beta} \frac{\partial \Theta_\beta}{\partial Z}, \tag{2}$$

$$q_\alpha = \kappa_\alpha \frac{\partial \Theta_\alpha}{\partial Z} + h_\alpha U_{\alpha 3}, \tag{3}$$

$$\hat{p}_a = \sum_{\beta=1}^{3} 1\, \xi_{\alpha\beta} \frac{\partial \Theta_\beta}{\partial Z} + \sum_{\beta=1}^{3} \beta_{\alpha\beta} \frac{\partial E_\beta}{\partial Z} + \sum_{\beta=1}^{3} \phi_{\alpha\beta} U_{\beta 3} \tag{4}$$

where E_α, $\partial E_\alpha/\partial Z$, $U_{\alpha 3}$, and $\partial\Theta_\beta/\partial Z$ are respectively the strain, strain gradient, diffusion velocity relative to the air phase, and temperature gradient. An alternate form for $\hat{c}_\alpha$ which was also found to be consistent with the second law is

$$\hat{C}_2 = C_1 = a(\mu_1 - \mu_2), \tag{5}$$

where $\hat{C}_2$ is the rate of sublimation from the solid to vapor phase and μ_1 and μ_2 are the chemical potentials for the ice and vapor phases.

As a special example, densification due only to the presence of surface energy was calculated. In this case, the material is unloaded, and the free energy Ψ_{10} for ice is dominated by its surface energy. Therefore $\Psi_{10} = \sigma_1 A_1$ where A_1 is the surface area per unit volume and σ_1 is the specific surface energy. If T_1 and Θ_1 are set to zero and Θ_0, the solution to Equation (1) yields the result illustrated in Figure 1 for the density ρ_1.C_{10} and $\gamma 10$ are the initial elastic modulus and viscosity. This case corresponds to that of new, very fragile snow. Denser, stronger snows would densify much less, but cracking may still occur in the material if constrained from densifying freely.

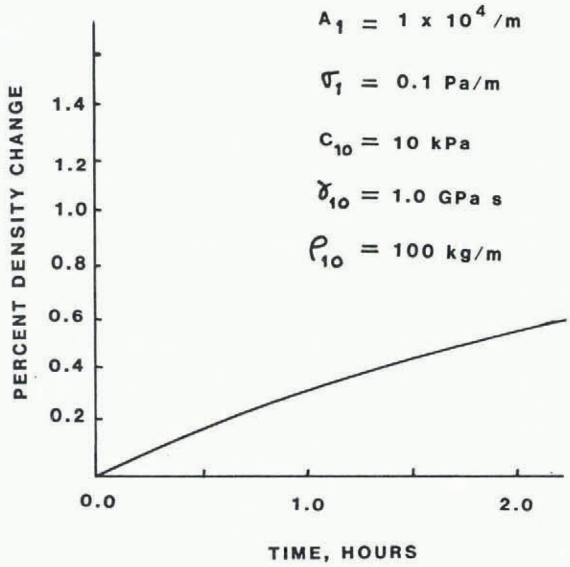

Fig.1. Densification of low-density snow due to effect of surface free energy.

Next, the added effect of equitemperature metamorphism on densification was considered. This involves the process of transfer of water molecules from small grains to large grains to produce a more efficient distribution of ice and hence some densification. The solution was found, and preliminary results indicate densification is enhanced by this process. However, final quantitative results must await a better means of relating density, grain size, and grain shape to specific surface area A_1. Current formulations for finding A_1 are of questionable accuracy.

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