SURVEY OF SOME RECENT STELLAR-EVOLUTION CALCULATIONS

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1. INTRODUCTION

A basic objective of stellar-evolution theory is to provide
detailed quantitative information on the evolutionary character-
istics of stars with different compositions and masses. For the
evolutionary phases from the zero-age main sequence (ZAMS) through
core-helium burning the important physical processes are suffi-
ciently well understood to justify a systematic investigation,
and, indeed, extensive grids of evolutionary sequences have been
constructed for these phases by many researchers. However,
several difficulties are sometimes encountered when using the
available calculations. For example, differences in numerical
techniques and physical assumptions can give rise to systematic
differences among the results of various investigations. Further-
more, the available calculations do not always explore the full
ranges of the input parameters, and in some cases they neglect
physical effects that are now believed to be important.

In an attempt to overcome some of these limitations, I have
undertaken a series of computational projects covering the main-
sequence, red giant and horizontal-branch phases. The immediate
aim has been to produce a self-consistent set of evolutionary
sequences for wide ranges in the composition and mass. The long-
term aim involves the detailed application of these results to
observational data on cluster HR diagrams and the integrated pro-
perties of galaxies. Other persons who have contributed signif-
icantly to various aspects of this work include P. Demarque,
J. Mengel and P. Gross.

TABLE I

MAIN-SEQUENCE-TURNOFF AND RED-GIANT SEQUENCES

Y	Z	$M/M_\odot$ (Turnoff)	$M/M_\odot$ (Red Giant)
0.10	0.00001	0.70 - 4.40	0.90
0.20	0.00001	0.70 - 3.50	0.70, 0.90
0.30	0.00001	0.55 - 3.50	0.70, 0.90
0.40	0.00001	0.55 - 3.50	0.70
0.10	0.0001	0.70 - 4.40	0.90
0.20	0.0001	0.70 - 3.50	0.70, 0.90, 1.40
0.30	0.0001	0.55 - 3.50	0.70, 0.90, 1.40
0.40	0.0001	0.55 - 3.50	0.70
0.20	0.0004	0.70 - 3.50	0.90
0.30	0.0004	0.55 - 3.50	0.70
0.10	0.001	0.70 - 4.40	0.90
0.20	0.001	0.70 - 3.50	0.70, 0.90
0.30	0.001	0.55 - 3.50	0.70, 0.90
0.40	0.001	0.55 - 3.50	0.70
0.20	0.004	0.70 - 4.40	0.90
0.30	0.004	0.70 - 3.50	0.70
0.10	0.01	0.90 - 4.40	0.90, 1.10
0.20	0.01	0.70 - 4.40	0.90, 1.10, 1.40, 1.75, 2.20
0.30	0.01	0.70 - 3.50	0.70, 0.90, 1.10, 1.40, 1.75, 2.20
0.40	0.01	0.55 - 3.50	0.70, 1.10
0.20	0.04	0.90 - 4.40	0.90, 1.75
0.30	0.04	0.70 - 4.40	0.70, 1.75
0.20	0.10	0.90 - 6.90	-
0.30	0.10	0.70 - 5.50	-

A listing of the computed sequences is provided in Tables I
and II for all values of the helium abundance Y and the heavy-
element abundance Z studied so far. The turnoff sequences extend-
ed from the ZAMS to the red giant branch except for a few sequences
with high mass and low Z. The range in mass M given in column 3
of Table I was covered by approximately 10 turnoff sequences having
roughly equally spaced values of log M. The red giant sequences
listed in column 4 were continued to the onset of helium burning
which in nearly all cases occurred under degenerate conditions.
The quantity M_c in Table II denotes the mass within the helium
core at the zero-age horizontal-branch (ZAHB) phase.

The stellar-evolution program developed by Sweigart (1972,
1973) from the Princeton program (Schwarzschild and Härm 1965)
was used for all computations to insure the self-consistency of
the results. A more complete description of this program can be
found in Sweigart and Gross (1974, 1978). Modifications were

TABLE II

HORIZONTAL-BRANCH SEQUENCES

Y	Z	$M_c/M_\odot$	$M/M_\odot$
0.10	0.0001	0.475	0.50(0.04)0.90
0.20	0.0001	0.475	0.50(0.04)0.90
0.30	0.0001	0.475	0.50(0.04)0.90
0.40	0.0001	0.475	0.50(0.04)0.98
0.20	0.0001	0.525	0.54(0.04)0.90
0.30	0.0001	0.525	0.54(0.02)0.58(0.04)0.90
0.40	0.0001	0.525	0.54(0.04)1.02
0.30	0.001	0.425	0.46(0.04)0.90
0.40	0.001	0.425	0.46(0.04)0.90
0.10	0.001	0.475	0.50(0.04)0.90
0.20	0.001	0.475	0.50(0.04)0.90
0.30	0.001	0.475	0.50(0.02)0.70(0.04)0.94
0.40	0.001	0.475	0.50(0.04)1.02
0.10	0.001	0.525	0.54(0.04)0.90
0.20	0.001	0.525	0.54(0.04)0.90
0.30	0.001	0.525	0.54(0.04)0.90
0.40	0.001	0.525	0.54(0.04)1.06
0.30	0.001	0.575	0.58(0.04)1.06
0.30	0.01	0.475	0.50(0.02)0.58(0.04)0.94

made to this program during the course of these projects in order
to incorporate those additional physical effects of particular
importance for each evolutionary phase. For example, relativistic
degeneracy and semiconvection were included in the red giant and
horizontal-branch sequences, respectively. It is well-known that
the CNO abundance can significantly influence the characteristics
of the main-sequence turnoff, and consequently the approach to
equilibrium of the ON cycle was followed during the turnoff
calculations. The CNO abundance has been assumed here to scale
with the Fe abundance, although there is observational evidence
indicating that CNO and Fe may not be so tightly correlated.

The present results, consisting in total of over 100,000
stellar models, have a number of immediate astrophysical appli-
cations. From the fitting of theoretical isochrones with observed
HR diagrams and from the relative red giant and horizontal-branch
lifetimes one can in principle determine the ages and helium abun-
dance of the globular clusters. Such information provides impor-
tant constraints on cosmological models and has direct implications
for the dynamical and chemical evolution of the Galaxy. The
number distribution of stars along the giant branch can be compar-
ed with theoretical luminosity functions in an attempt to
understand further the observed irregularities (Sandage et al.

1968; Sweigart 1978). The present red giant models can serve as
the starting point for subsequent studies of the helium-core
flash (Mengel 1978) and of the unusual C, N and s-process abun-
dances found in some globular-cluster stars (Mengel and Sweigart
1978). Furthermore it will be possible to consider again in more
detail some of the intriguing problems associated with the mor-
phology of the horizontal branch.

     In the following sections I will survey the various evolu-
tionary calculations. For conciseness the discussion must, how-
ever, be limited to a few representative cases. A more thorough
description of the available data is contained in the cited
references.

2.   TURNOFF SEQUENCES

     The turnoff tracks for Z = 0.01 and Y = 0.20 and 0.30 are
shown, respectively, in Figs. 1 and 2 (Mengel et al. 1978). In
the higher mass sequences central hydrogen exhaustion occurs
throughout a rather large region at the end of the main-sequence
phase. This leads to an abrupt contraction of the star and hence
to a blueward fluctuation in the track. Observationally this
fluctuation would appear as a gap in a cluster HR diagram.
Although not plotted in Figs. 1 and 2, a few intermediate mass
tracks have been constructed in order to define the development
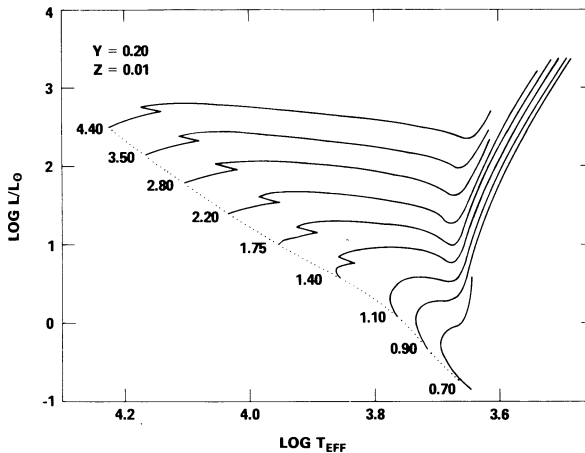of the gap more precisely. As expected, an increase in Y shifts

Fig. 1. Main-sequence-turnoff and red-giant tracks for (Y, Z) =
(0.20, 0.01). The location of the ZAMS is indicated by the
dotted line. Each track is labeled by the mass in solar units.
The tracks for $0.90 \leq M \leq 2.20 \, M_{\odot}$ have been evolved to the onset
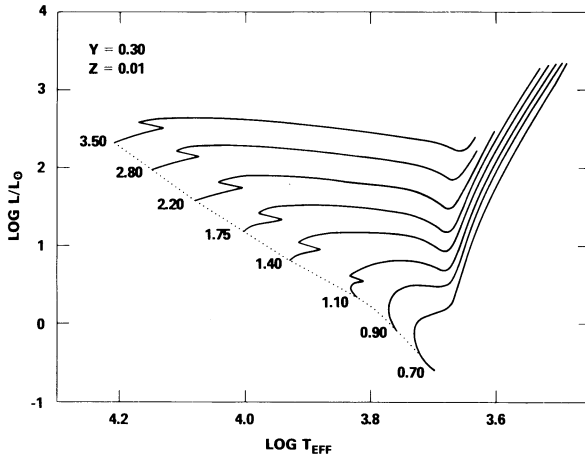of helium burning.

Fig. 2. Main-sequence-turnoff and red-giant tracks for (Y, Z) =
(0.30, 0.01). The location of the ZAMS is indicated by the
dotted line. Each track is labeled by the mass in solar units.
The tracks for $0.70 \leq M \leq 2.20$ $M_\odot$ have been evolved to the onset
of helium burning.

a given track to larger values of log T_{eff} and Log L.

     The dependence of the turnoff tracks on Z is illustrated in
Fig. 3. Simoda and Iben (1968) have demonstrated that an increase
in Z will reduce log T_{eff} and log L at the turnoff point. Such
an effect is evident in Fig. 3 and is due primarily to the effects
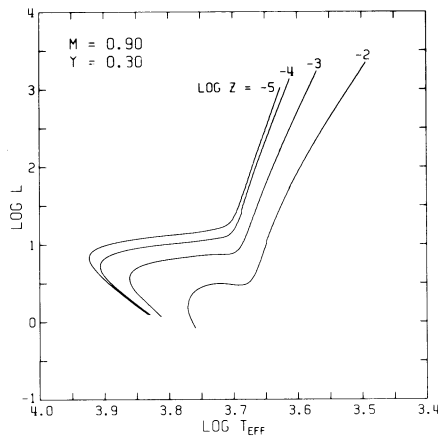

Fig. 3. Main-sequence-turnoff and red-giant tracks for (M, Y) =
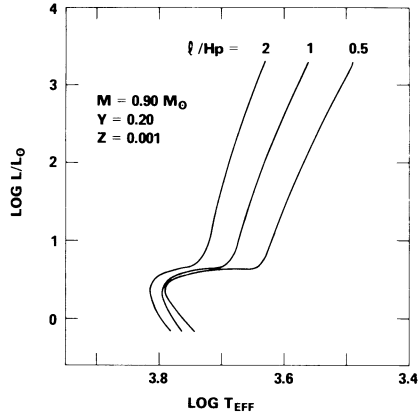(0.90, 0.30). Each track is labeled by the value of log Z.

Fig. 4. Main-sequence-turnoff and red-giant tracks for (M, Y, Z) = $(0.90, 0.20, 0.001)$. Each track is labeled by the value of the mixing-length ratio $\alpha = \ell/H_p$.

of the CNO elements on the hydrogen-burning rate and the opacity (Renzini 1977). Any variations in the CNO abundance relative to the Fe abundance could therefore significantly influence the turnoff characteristics.

Since the lower mass sequences have a convective envelope, their tracks are affected by the value adopted for the ratio α of the mixing length ℓ to the pressure-scale height H_p (Sweigart 1978). All of the present sequences use $\alpha = 1$ except for the 2 cases plotted in Fig. 4. In this figure log T_{eff} at the ZAMS and along the red-giant branch is approximately proportional to log α. Not surprisingly, log L is rather insensitive to the value of α. Around the turnoff point the dependence of log T_{eff} on log α is quite nonlinear, since the envelope convection has almost dis-appeared in the $\alpha = 0.5$ and 1 cases while it is still present in the $\alpha = 2$ case. Sequences with a deeper convective envelope would be expected to show a smoother variation of log T_{eff} at turnoff with log α. The caution which must be exercised when ages are obtained by fitting isochrones to cluster HR diagrams is indic-ated in Fig. 4. If the fit is based essentially on the turnoff color and if the correct α has not been used, the derived ages can be rather uncertain, a point emphasized by Iben and Rood (1970).

3. RED-GIANT SEQUENCES

Some of the red-giant tracks have already been displayed in Figs. 1 to 4. From these tracks and the others listed in Table I the following conclusions can be drawn (Rood 1972; Sweigart and
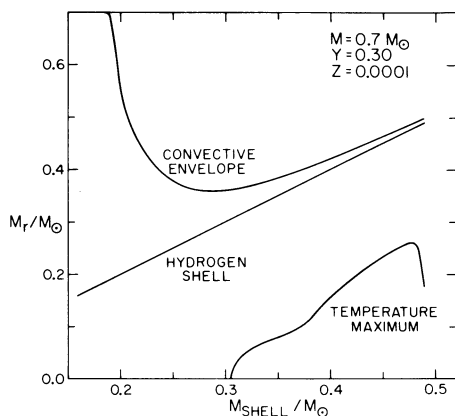
Fig. 5. Variations in the locations of the convective envelope, the hydrogen shell and the core-temperature maximum along a red-giant sequence for $(M, Y, Z) = (0.70, 0.30, 0.0001)$. The evolution between the end of the main-sequence phase and the onset of the helium-core flash is shown.

Gross 1978; Sweigart 1978). First, there is a blueward shift of the red giant branch with increasing Y, M or α or decreasing Z. Second, an increase in Y or a decrease in Z lowers the surface luminosity at the tip of the red-giant branch and reduces the maximum stellar mass at which the helium-core flash still takes place.

How the edge of the convective envelope and the point of maximum temperature within the degenerate helium core vary along a typical red giant sequence is illustrated in Fig. 5. The abscissa gives the location of the center of the hydrogen shell, defined as the point where the hydrogen abundance is one half of its envelope value. The location of the hydrogen shell is thus represented by a straight line. The temperature inversion produced by neutrino emission becomes progressively more extensive as the mass within the core grows. Shortly before the helium-core flash the temperature maximum suddenly shifts inward. The flash point can be quite off center, especially for small values of Y and M.

Of particular interest is the behavior of the convective envelope. Initially the convective envelope moves inward in mass until deepest penetration is reached on the lower subgiant branch. At that time the convective envelope extends into the region that was partially depleted of hydrogen during the main-sequence phase. A discontinuity in the hydrogen abundance is therefore created at the point of deepest penetration. The convective envelope then retreats outward due to the approach of the hydrogen shell. When
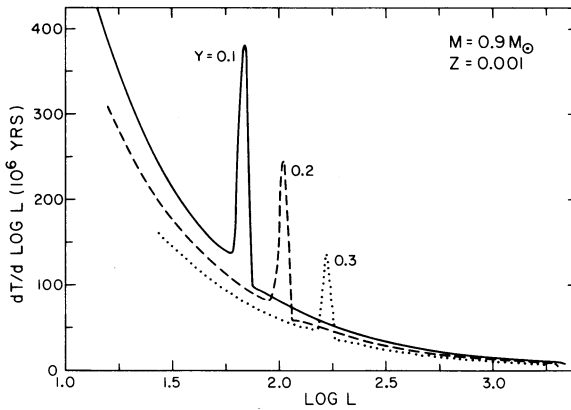
Fig. 6. Theoretical red-giant luminosity functions for (M, Z) = (0.90, 0.001) and Y = 0.10, 0.20 and 0.30. The ordinate gives the time T spent per unit interval in log L in units of 10^6 yrs.

the hydrogen shell later encounters this discontinuity, a readjustment of the interior structure takes place, leading to a hesitation in the rate of evolution up the giant branch and often to a temporary drop in the surface luminosity (Rood 1972). The result can be a rather pronounced peak in the luminosity function as is demonstrated by Figs. 6 and 7 (Sweigart 1978). In general the prominence of this peak is increased and the luminosity at which it occurs is lowered by smaller values of Y and larger values of Z. Such composition changes tend to produce a deeper
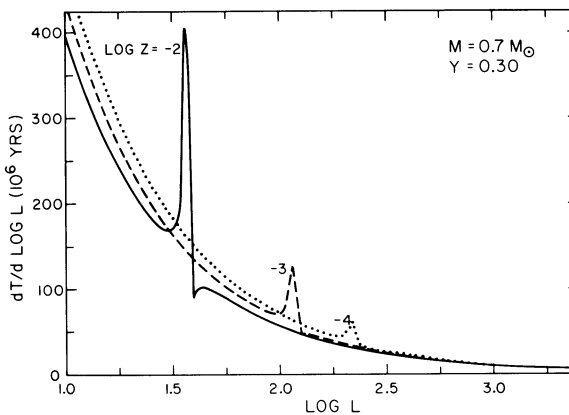

Fig. 7. Theoretical red-giant luminosity functions for (M, Y) = (0.70, 0.30) and log Z = -2, -3 and -4. The ordinate gives the time T spent per unit interval in log L in units of 10^6 yrs.

penetration of the convective envelope on the subgiant branch and thus a more pronounced hydrogen discontinuity.

A major reason for constructing red-giant sequences is to determine the core mass M_c at the time of the helium-core flash. The precise value of M_c strongly affects the morphology of the horizontal-branch tracks as well as the horizontal-branch life-times. The present calculations give

$$M_c/M_\odot = 0.474 - 0.24(Y - 0.30) - 0.007 (\log Z + 3)$$
$$- 0.038(M - 0.80) \tag{1}$$

in the neighborhood of the case $(M, Y, Z) = (0.80, 0.30, 0.001)$. Eqn. (1) assumes the neutrino emission rates of Beaudet et al. (1967). If neutrino emission is omitted, M_c decreases by 0.03 $M_\odot$, while the additional neutrino emission due to neutral current interactions raises M_c by 0.007 $M_\odot$ (Sweigart and Gross 1978).

4. HORIZONTAL-BRANCH SEQUENCES

The interior structure and observable properties of hori-zontal-branch models are substantially altered by convective overshooting and semiconvection within the helium core (Schwarz-schild 1970). During the initial horizontal-branch evolution the convective core propagates outward in mass because of convective overshooting. This growth in the convective core is driven fun-damentally by the increase in the opacity and hence the radiative gradient caused by the gradual depletion of the central helium abundance (Castellani et al. 1971a, b). Tentative estimates for the rate of propagation suggest that convective neutrality will, to a good approximation, be maintained at the convective-core edge (Renzini 1977). When the central helium abundance falls below about 0.7, a semiconvective zone develops in the layers surrounding the convective core. The subsequent changes in the composition distribution are shown in Fig. 1 of Sweigart and Gross (1974). The effects of semiconvection on the evolution have been examined in a series of papers (Demarque and Mengel 1972; Sweigart and Demarque 1972; Sweigart and Gross 1974, 1976). The results show that semiconvection can almost double the hori-zontal-branch lifetime and track length when compared with cal-culations in which neither convective overshooting nor semicon-vection is permitted.

Some typical horizontal-branch tracks computed with semiconvec-tion (Sweigart and Gross 1976) are presented in Fig. 8. These tracks extend from the ZAHB to the initial helium-shell burning stage and are similar to those obtained by Gingold (1976). The 0.50 $M_\odot$ track evolves vertically in the HR diagram and would not be expected to return to the asymptotic branch. The 0.54 $M_\odot$ track
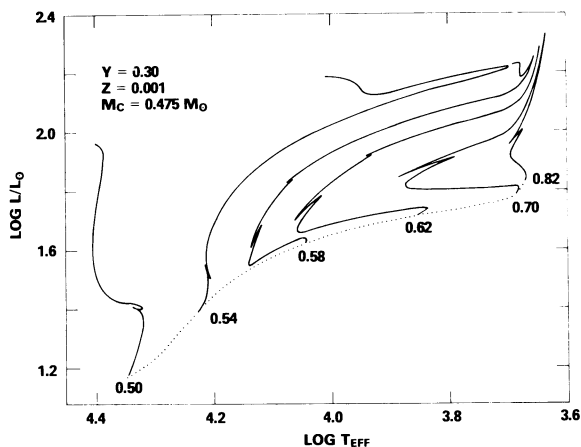
Fig. 8. Horizontal-branch and post-horizontal-branch tracks for $(Y, Z, M_c) = (0.30, 0.001, 0.475)$. The location of the ZAHB is indicated by the dotted line. Each track is labeled by the mass in solar units.

makes an excursion into the region of the BL Her stars in a manner similar to the 0.51 $M_\odot$ track of Sweigart et al. (1974). At higher masses the tracks all evolve back to the asymptotic branch. The spike visible at the end of the main horizontal-branch phase of each track is due to a rapid growth in the extent of the convective core and appears to be caused by an instability in the composition distribution within the core (Sweigart and Demarque 1973). The same effect has been reported by Gingold (1976) and Taam et al. (1976).

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DISCUSSION

COX: Can you tell me what your best Y value is for the RR
Lyrae stars?

SWEIGART: The value of Y can be estimated from the ratio R of
the number of horizontal-branch stars to the number of red giant
stars, since the corresponding ratio of theoretical lifetimes is
a sensitive function of Y. The original number counts gave a value
of 1 for R, implying a Y value of about 0.3. This Y value was
determined from a theoretical calibration in which the effects of
semiconvection were neglected. It is now well-known that semi-
convection can substantially increase the horizontal-branch life-
time. The resulting change in the calibration between R and Y
reduces the Y value for R = 1 to 0.12. Renzini (1977) has recent-
ly re-examined the number counts in more detail and has found that
a significant number of faint horizontal-branch stars were omitted
in the original counts. According to Renzini (1977), the average
value of R for a number of globular clusters is about 1.6. For
this R value one obtains Y∼0.22, when the calibration including
semiconvection is used. This value for the helium abundance rep-
resents the best estimate currently available from the R method.

COX: You know, of course, that pulsation theory gives a value
of about 0.3. Do you have any comment?

SWEIGART: All I can say is that the value based upon the number
counts is about 0.20 - 0.25. I do not think that 0.3 is possible.
However, a detailed calibration of the R method using the latest
horizontal-branch and red giant sequences has not yet been made,
and thus the value I have given is a preliminary one.

ROOD: In reply to Cox's suggestion that RR Lyrae theory requires
Y∼0.28 to 0.30, I think most of the indications, at least from the
linear theory, are that Y∼0.20 to 0.25. Iben and Tuggle's blue
edge calculations are consistent with this.

CASTELLANI: What is the dependence of "RG clumps" on the assum-
ed value of the mixing length?

SWEIGART: The peak in the red-giant luminosity function does
depend somewhat on the assumed value for the ratio, α, of the
mixing length to the pressure-scale height. An increase in α
reduces the size of the peak and raises the luminosity L at which
it occurs. Although detailed luminosity functions have not yet
been computed for $\alpha \neq 1$, this dependence appears to be mild. For
example, in the case $(M, Y, Z) = (0.90, 0.20, 0.001)$, a change
in α from 1 to 2 increases log L at the peak by only 0.02.

CASTELLANI: I have one more comment on the problem of the RR
Lyrae stars raised by Dr. Cox. I know that in decreasing the
helium content the width of the instability strip is expected to
decrease. Nevertheless I wish to emphasize that, if one assumes
a constant width of the strip (i.e. the observed one) and looks
at the fitting of the observed period-frequency histograms then
available pulsational theories, when connected with evolutionary
prescriptions, clearly indicate a helium content on the order of
0.23 - 0.25. On the contrary, no agreement can be found with the
assumption that $Y = 0.30$ for the large majority of galactic
globular clusters.

FLOWER: I have one comment to make concerning the proper choice
of the value of the mixing length. The evolutionary tracks (Mengel,
et. al., 1978) used to construct isochrones were computed using a
value of 1.0 for the mixing length. Since, for the present
generation of evolutionary codes, a value closer to 1.5 or 1.6
seems to be necessary to (1) reproduce the surface temperature of
the present day Sun; (2) reproduce the location of the red giant
branch on the HR diagram for old galactic clusters; and (3)
reproduce the location of red giants on the HR diagram in young
populous clusters in the LMC, the isochrones must be corrected
to reflect these "calibrations" of the mixing length.

SWEIGART: I agree that a value closer to 1.5 gives a better
fit with observations. It is particularly important to be
aware of the dependence of the sequences on the mixing length
when using the present results.