INTERPRETATION OF SOURCE COUNTS AND REDSHIFT DATA IN EVOLUTIONARY
UNIVERSES

J. V. Wall, T. J. Pearson and M. S. Longair
Mullard Radio Astronomy Observatory,
Cavendish Laboratory, Cambridge, England

Conventional interpretation of the N(S) relation requires cosmic
evolution of the radio source population. Investigators agree on the
general features of this evolution: it must be confined to the most
luminous sources, and must be strong, the numbers of such sources at
redshifts of 1 to 4 exceeding the present numbers by a factor $\gtrsim 10^3$.
There is no consensus as to whether density or luminosity evolution
prevails (or both), whether a cutoff in redshift is necessary, or
whether the source populations found in high-frequency surveys follow
even the general evolutionary picture deduced for the low-frequency
survey population. It is therefore hardly surprising that the physical
basis of the evolution, the ultimate goal of N(S) interpretation,
remains largely "in the realm of imaginative speculation" (P. A. G.
Scheuer).

Recently the observational basis for source count interpretation
has been greatly strengthened. There now exist N(S) relations at
frequencies of 408, 1410, 2700 and 5000 MHz which all reach to $\sim 10^5$
sources sr^{-1}, and for which the statistical definition at S < 1 Jy is
greatly improved over that available for previous investigations.
Furthermore, at high flux densities at all frequencies, identifications
and redshifts approach completeness and therefore luminosity distribu-
tions may be defined with greater certainty. We have undertaken a
numerical investigation of the implications of this much-improved data
base for cosmic histories of the different radio source populations.
The early results are reported here.

1. A SCHEME FOR SOURCE COUNT ANALYSIS

We wish to determine $\rho(P,z,type)$, the generalized luminosity function,
which describes the spatial density of radio sources of luminosity P
and given type at all redshifts z. The function may be factored as
$\rho = F(P,z,type) \cdot \rho_0(P,type)$; F is the evolution function and ρ_0 is the
local luminosity function.

We have developed a new numerical scheme which guarantees that
these models will at least be consistent with luminosity distributions
defined at high flux densities. Consider a single type of source, and
suppose that for these sources a complete luminosity distribution n(P)
for S > S_0 is available. For a given world model and evolution function
F(P,z), the local luminosity function may be obtained directly:

$$\rho_0(P)dP \;=\; n(P)dP \,/\, \int_0^{z(S_0)} F(P,z)\; dV(z)$$

where $z(S_0)$ is the redshift at which a source of luminosity P has an
observed flux density S_0. The count for this type of source is then
computed in the usual manner:

$$N(>S) \;=\; \int_0^\infty dP \; \int_0^{z(S)} F.\rho_0.dV$$

A complete luminosity distribution, statistically adequate in size,
therefore makes it possible to proceed directly to a predicted count
for each postulated evolution function F. The requisite input guesses
for our approach are thus (i) a world model and (ii) an evolution
function, while input data supplied are (iii) a luminosity distribution
n(P,S_0) and (iv) an observed source count N(S); the procedure is to
use (i), (ii) and (iii) to calculate the model count directly as above,
and to compare it statistically via a χ^2-test with (iv), the observed
N(S) for this type of object. At the outset we have used simple
analytic forms of evolution functions, whose free parameters are
allowed to vary widely. The evolution models which survive the χ^2-test
may be examined in more detail by comparison with observational data at
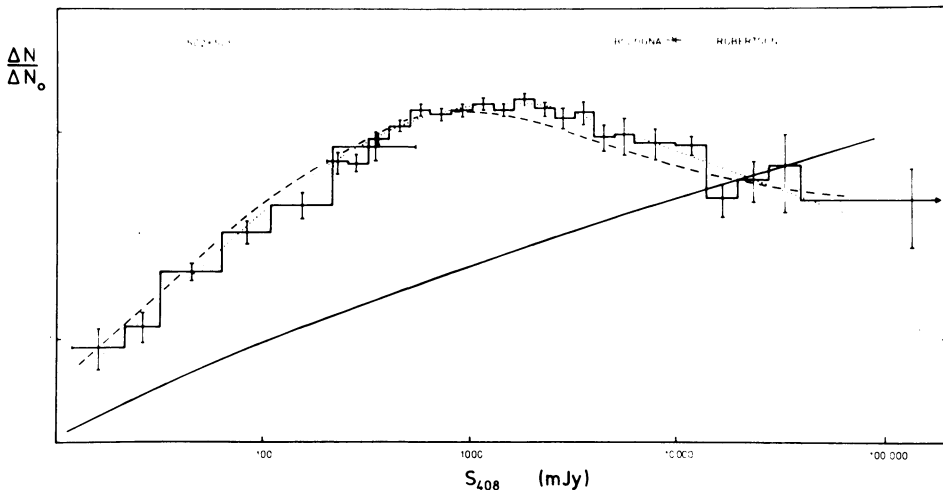lower flux densities.

Figure 1.   The 408 MHz source count.   Solid curve: model 1;
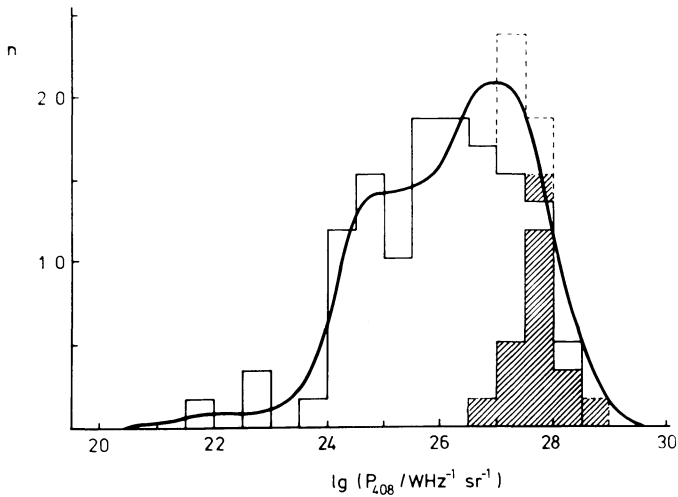dashed: model 2a (optimized);   dotted: model 5 (optimized).

Figure 2. The 408 MHz luminosity distribution, $S_{408} > 10$ Jy, 5.86 sr. Hatched area: QSOs; dotted area: no redshift estimates available; solid curve derived by smoothing the unbinned data.

2. THE 408 MHz SOURCE COUNT

The evolutionary requirements of the 408 MHz count (Fig. 1) are particularly amenable to exploration by the present method. Some 90 % of all sources found at this frequency are of the extended, steep-spectrum type, so that a valid first approximation is to assume that the count consists of only these sources, with a spectral index $\alpha = 0.75$ ($S \propto \nu^{-\alpha}$). Moreover, recent identifications and redshift determinations yield the well-defined luminosity distribution of Fig. 2, the details of which will be described elsewhere.

The guesses of our initial investigation are (i) world models of Friedmann geometries with $\Omega = 0$ and 1, $H_0 = 50$ km s^{-1} Mpc^{-1}, and (ii) the following forms for the evolution function F.

1. $F(P,z) = 1$ (non-evolutionary, source-conserving)

2. $F(P,z) = \chi_1(P) + \phi\chi_2(P)$, with $\phi = \exp\{M(1-t/t_0)\}$, and

$\chi_1 = (P_c/P)^m/\{1+(P_c/P)^m\}$, $\chi_2 = 1/\{1+(P_c/P)^m\}$;

2a: no redshift cutoff; 2b: $F = 0$ if $z > z_c$.

3.   As for 2b, but with $\phi(P,z) = (1+z)^\beta$ for $z < z_c$.

4. $F(P,z) = \exp\{M(P)(1-t/t_0)\}$, with

$M(P) = 0$ for $P < P_1$, $M(P) = M_{max}$ for $P > P_2$, and

$M(P) = M_{max}(\lg P - \lg P_1)/(\lg P_2 - \lg P_1)$ for $P_1 < P < P_2$;

4a: no redshift cutoff; 4b: F = 0 if z > z_c.

5.   As for 2a, but with $\lg P_c$ = algz + b.

The first three are conventional models from the literature. All
fail to produce satisfactory models for the 408 MHz source count for
all possible variations of the parameters M, m, z_c, β, and P_c. Model 1
(no evolution) produces a count which differs from that observed to an
extreme degree of statistical significance, as is evident in Fig. 1.
Model 2a has attractively few parameters (M, P_c, m), but after optim-
izing these, is rejected at the 99.9% level of significance. The
difficulty (Fig. 1) is in obtaining the rapid turnover towards the
faint flux densities when enough evolution is supplied to fit the
steeply-rising count of bright flux densities; and when this turnover
is achieved, falloff is too rapid. Imposing a redshift cutoff (2b)
aggravates this behaviour, and model 3 is even worse in this respect
because the density enhancement increases rapidly up to the redshift
cutoff. The basic problem is that these models produce too many
sources at the low-luminosity end of the evolving component. Models 4
and 5 were invented to avoid the difficulty. Model 4 was derived by
examining how the evolution function depends on radio luminosity; the
value of $<V/V_m>$ as a function of luminosity for sources with S_{408} > 5
Jy was used to define M(P), and in this sense there are no free
parameters in 4a, and only one (z_c) in 4b. However, we succeeded in
finding acceptable models of type 4 only in the Ω = 1 geometry. Model
5 has the transition luminosity between evolving and non-evolving
components as a function of redshift, thereby reducing the contribution
to the count of weak, high-redshift sources. It produces the best fit
to the source count, and bears considerable resemblance to luminosity
evolution. The dependence of luminosity function on epoch given by
models 4 and 5 is shown in Fig. 3, and the optimum parameters are as
follows:

   Model 4a: $\lg P_1$ (W Hz^{-1} sr^{-1}) = 26.0, $\lg P_2$ = 27.1, M_{max} = 11.5
        4b: $\lg P_1$ = 25.0, $\lg P_2$ = 27.3, M_{max} = 11.0, z_c = 3.5
        5:   M = 9.8, a = 3.14, b = 26.8, m =1.06.

These models are very different in character, as exemplified by
the redshift distribution which each predicts at low flux densities
(Fig. 4); the identification content of such a sample clearly provides
a powerful discriminant. For example, the identifications of Richter
(1975) for the 5C3 survey suggest to us that $\gtrsim$ 35% of all sources with
S_{408} > 10 mJy have redshifts less than 0.6. The proportions of sources
with z < 0.6 are 26% and 42% for models 4a and 4b, and 28% for model 5,
and it is therefore not clear that models 4a and 5 remain tenable. It
is clear that identifications and redshifts for even a small sample of
sources at this level would provide powerful constraints to the
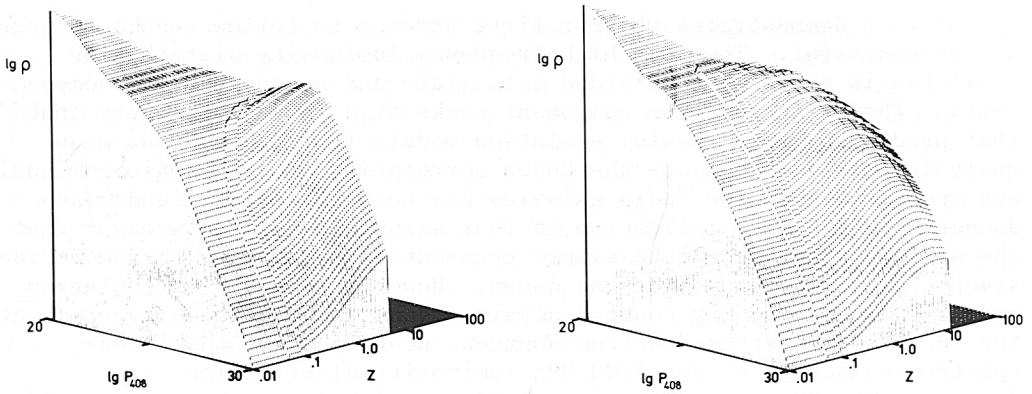permitted forms of evolution.

Figure 3. Dependence of luminosity function on redshift in model
4b (left) and model 5 (right).

3. THE 2700 MHz SOURCE COUNT

In surveys at frequencies above 408 MHz, the proportion of compact
objects with 'flat' radio spectra becomes significant and increases as
survey frequency increases. Consider for simplicity that two classes
of source are represented in the 2700 MHz count: 'flat-spectrum' and
'steep-spectrum' sources. A first attempt to derive evolution models
for these two populations might proceed as follows. (1) Assume that the
steep-spectrum objects are the same as those in the 408 MHz count.
(2) Translate a successful model for the 408 MHz count to 2700 MHz,
obtaining the appropriate normalization either from the spectral index
distribution at some S_{2700} or from the 'steep-spectrum' component of
the 2700 MHz count (Fig. 5 of Wall, this volume). (3) Subtract this from
the total 2700 MHz count to obtain a complete count for the flat-
spectrum sources. (4) Analyse this with the procedure of section 1 to
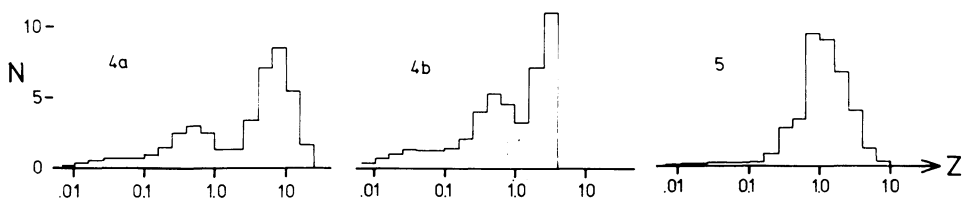set limits on the evolution required for these sources.

Figure 4. Predicted distributions of redshifts for sources with
$10 < S_{408} < 50$ mJy; models 4a, 4b and 5.

Fig. 5 demonstrates why our first attempt to follow such a procedure was unsuccessful. The best high-frequency luminosity distribution available is shown here, divided into flat- and steep-spectrum components. The steep-spectrum component peaks at a lower luminosity than that predicted by successful evolution models from the 408 MHz count analysis. This is because the known correlation between radio luminosity and spectral index for radio galaxies has been neglected (Condon and Jauncey 1974). The implication is that assumption (1) is wrong - that the 408 MHz count cannot be simply translated to 2700 MHz to remove the steep-spectrum component of the count. However, successful evolution models from the 408 MHz count analysis may be used to <u>calculate</u> a count for the 2700 MHz steep-spectrum sources, provided that the steep-spectrum component of the 2700 MHz luminosity distribution is well defined. The 28 objects of Fig. 5, 12 of which do not have measured redshifts, are clearly inadequate in this regard. Moreover, the flat-spectrum component of the luminosity function contains only 20 objects, and exploration of the evolution function for flat-spectrum sources on the basis of such a sample is somewhat optimistic.

There is nevertheless some indication that the cosmic history of the flat-spectrum population differs considerably from that of the steep-spectrum population. There are differences amongst the source counts themselves (Figures 4 and 5 of Wall, this volume). These may be due to differences in luminosity functions rather than evolution functions, but there is further indication that this is not so. C. J. Masson has measured magnitudes to $\pm 0^m.3$ for a complete sample of 55 QSOs in unobscured regions of the $\pm 4^\circ$ declination zone of the Parkes 2700 MHz survey. Redshifts for many of these were kindly communicated to us by R. Lynds and D. Wills, and the $<V/V_m>$ for the sample (Masson and Wall, in preparation) is 0.58 ± 0.03, significantly less than the values of 0.65 to 0.7 obtained for objects from low-frequency surveys (Schmidt, this volume). The result suggests that flat-spectrum
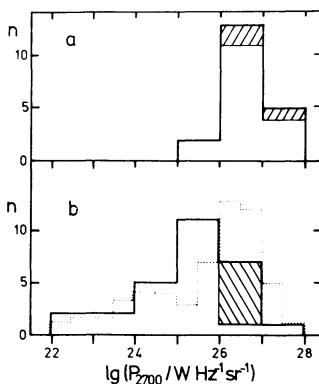

Figure 5. The 2700 MHz luminosity distribution (a) for flat-spectrum sources, (b) for steep-spectrum sources. Hatched areas: no redshift estimates available; dotted histogram: prediction from 408 MHz results.

sources show relatively low $<V/V_m>$, and indeed a Spearman rank test on the 55 objects showed a correlation between V/V_m and high-frequency spectral index which is significant at the 95% confidence level. Dividing the sample at $\alpha_{HF} = 0.5$ yields a steep-spectrum subsample of 14 objects with $<V/V_m> = 0.70 \pm 0.05$ whose properties appear to be identical with the (steep-spectrum) 3CR and 4C samples studied by Schmidt and by Lynds and Wills, and a flat-spectrum subsample of 41 objects with $<V/V_m> = 0.54 \pm 0.04$. All 41 of these QSOs have structures dominated by compact components less than 0".1 arc in extent (Bentley et al. 1976). These results do not require a complete lack of evolution for compact QSOs, but they do imply different cosmic histories for QSOs of different radio structures.

4. CONCLUSIONS

Some authors have argued that satisfactory evolution functions are trivial to obtain, firstly because of a wide range of permissible assumptions - geometry, local luminosity function, and of course the form of the evolution itself, and secondly because of highly permissive data - poorly defined source counts, luminosity distributions, background temperature requirements. In the face of modern data this point of view is hard to sustain, and the definition of satisfactory models becomes distinctly non-trivial. Progress is therefore possible on investigating the form of the evolution function for different source populations, and in particular on providing constraints on this evolution. From our examination of the 408 MHz counts we conclude that all conventional models of evolution for steep-spectrum sources are unsatisfactory, and that identifications and redshifts of sources at the flux density level of the 5C surveys provide powerful constraints on models which do result in correct source counts. From our preliminary 2700 MHz results we conclude that existing luminosity distributions permit only the crudest of analyses, but the difference in evolution for compact and and extended QSOs suggested by the $<V/V_m>$ test provides considerable incentive to obtain the necessary observations for this investigation.

Bentley, M., Haves, P., Spencer, R.E. and Stannard, D.: 1976, Monthly Notices Roy. Astron. Soc. 176, 275.
Condon, J.J. and Jauncey, D.L.: 1974, Astron. J. 79, 1220.
Richter, G.A.: 1975, Astron. Nachr. 296, 65.

DISCUSSION

Murdoch: Does the luminosity evolution force a sharp modification to the density law in the vicinity of z=3?

Wall: We have considered basically density evolution; of our successful models, type 1 works (in $q_o = 0.5$ geometry) both with (1a) and without

(lb) redshift cut-offs, while type 2 imposes an effective redshift cut-off by progressively narrowing the range of luminosities allowed to evolve.

Murdoch: Robertson (Sydney) is doing similar work, but instead of parametrized models he uses an array of numbers for $\delta(z)$ which are modified until convergence to the source counts is obtained. The conclusions are similar: $(1+z)^\beta$ must be modified, and in particular it is forced to drop sharply at $z \sim 3$.

Wall: Robertson's approach is one that we are very interested in trying. The difficulty comes in communicating the answers; if you have to invent a parameterized model to describe the resultant array, you might as well have started with this model in the first place. It has always been known that $(1+z)^\beta$ models require a redshift cut-off.

Grueff: You stressed the necessity of optical identifications at the 5C flux density level. Did you compare the predictions of your most successful models with the data already available at ~ 1 Jy? The 1 Jy point in the log N - log S is a rather interesting one, since it is there that we observe the largest source excess with respect to Euclidean prediction.

Wall: I confess not; our look at the identification data has been preliminary, and we went for the longest baseline in flux density, namely to what is known of the 5C identifications. Our luminosity distribution is "defined" at 10 Jy; 1 Jy is not very far away considering the breadth of the luminosity function.

E.M. Burbidge: I am still puzzled as to how you estimate redshifts for radiogalaxies which do not have measured redshifts.

Wall: We do what we can from estimated magnitudes and the m-z plot for radio galaxies. But I want to emphasize that extreme assumptions about these objects do not affect the structure of the luminosity distribution markedly, and in particular do not change the conclusions of the analysis.

Roeder: Why did your (1/2) model not work in the empty universe, $q_o = 0$?

Wall: The difficulty is at the faint (5C) end of the 408 MHz count, and is the same as the one which destroys the "conventional" models of the literature. The model in question just fits the counts with $q_o = 0.5$ geometry, because the relatively small volumes, ΔV, corresponding to the Δz's assist in converging the faint source counts gradually; these ΔV's are much <u>larger</u> for $q_o = 0$, and the faint counts take on quite the wrong shape.

Ekers: We know from the bivariate luminosity function analysis presented by Perola that there is a variation of optical luminosity for the lower radio power ranges, so the estimation of distance for these galaxies could be affected. Perhaps you should consider including the bivariate

luminosity function in your analysis so that you can correctly include the galaxies with no redshift determination.

Wall: I agree in principle.   However, from several considerations, it is clear that the missing-z objects have high radio powers, off the top of the well-defined part of Perola's bivariate function, and before this refinement becomes necessary, the missing redshifts will probably be available.