Abstract

Intuitionistic logic formalises the foundational ideas of L.E.J. Brouwer's mathematical programme of *intuitionism*. It is one of the earliest non-classical logics, and the difference between classical and intuitionistic logic may be interpreted to lie in *the law of the excluded middle*, which asserts that either a proposition is true or its negation is true. This principle is deemed unacceptable from the constructive point of view, in whose understanding the law means that there is an effective procedure to determine the truth of all propositions. This understanding of the distinction between the two logics supports the view that negation plays a vital role in the formulation of intuitionistic logic.

Nonetheless, the formalisation of negation in intuitionistic logic has not been universally accepted, and many alternative accounts of negation have been proposed. Some seek to weaken or strengthen the negation, and others actively supporting negative inferences that are impossible with it.

This thesis follows this tradition and investigates various aspects of negation in intuitionistic logic. Firstly, we look at a problem proposed by H. Ishihara, which asks how effectively one can conserve the deducibility of classical theorems into intuitionistic logic, by assuming atomic classes of non-constructive principles. The classes given in this section improve a previous class given by K. Ishii in two respects: (a) instead of a single class for the law of the excluded middle, two classes are given in terms of weaker principles, allowing a finer analysis and (b) the conservation now extends to a subsystem of intuitionistic logic called *Glivenko's logic*. This section also discusses the extension of Ishihara's problem to minimal logic.

Secondly, we study the relationship between two frameworks for weak constructive negation, the approach of D. Vakarelov on one hand and the framework of *subminimal negation* by A. Colacito, D. de Jongh, and A. L. Vargas on the other hand. We capture a version of Vakarelov's logic with the semantics of the latter framework, and clarify the relationship between the two semantics. This also provides proof-theoretic insights, which results in the formulation of a cut-free sequent calculus for the aforementioned system.

Thirdly, we investigate the ways to unify the formalisations of some logics with contra-intuitionistic inferences. The enquiry concerns paraconsistent logics by R. Sylvan and A. B. Gordienko, as well as the logic of co-negation by G. Priest and of empirical negation by M. De and H. Omori. We take Sylvan's system as basic, and formulate the frame conditions of the defining axioms of the other systems. The conditions are then used to obtain cut-free labelled sequent calculi for the systems.

Finally, we consider L. Humberstone's actuality operator for intuitionistic logic, which can be seen as the dualisation of a contra-intuitionistic negation. A compete axiomatisation of intuitionistic logic with actuality operator is given, and comparisons are made for some related operators.

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We prove various results connected together by the common thread of computability theory.

First, we investigate a new notion of algorithmic dimension, the inescapable dimension, which lies between the effective Hausdorff and packing dimensions. We also study its generalizations, obtaining an embedding of the Turing degrees into notions of dimension.

We then investigate a new notion of computability theoretic immunity that arose in the course of the previous study, that of a set of natural numbers with no co-enumerable subsets.

We demonstrate how this notion of Π_1^0-immunity is connected to other immunity notions, and construct Π_1^0-immune reals throughout the high/low and Ershov hierarchies. We also study those degrees that cannot compute or cannot co-enumerate a Π_1^0-immune set.

Finally, we discuss a recently discovered truth-table reduction for transforming a Kolmogorov–Loveland random input into a Martin-Löf random output by exploiting the fact that at least one half of such a KL-random is itself ML-random. We show that there is no better algorithm relying on this fact, in the sense that there is no positive, linear, or bounded truth-table reduction which does this. We also generalize these results to the problem of outputting randomness from infinitely many inputs, only some of which are random.

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XINHE WU, ***Boolean-Valued Models and Their Applications***, Massachusetts Institute of Technology, Cambridge, MA, USA. 2022. Supervised by Vann McGee. MSC: 03C90, 03E40, 03G05. Key words and phrases: Boolean-valued models, indeterminacy, set theory with urelements.

Abstract

Boolean-valued models generalize classical two-valued models by allowing arbitrary complete Boolean algebras as value ranges. The goal of my dissertation is to study Boolean-valued models and explore their philosophical and mathematical applications.

In Chapter 1, I build a robust theory of first-order Boolean-valued models that parallels the existing theory of two-valued models. I develop essential model-theoretic notions like "Boolean-valuation," "diagram," and "elementary diagram," and prove a series of theorems on Boolean-valued models, including the (strengthened) Soundness and Completeness Theorem, the Löwenheim–Skolem Theorems, the Elementary Chain Theorem, and many more.

Chapter 2 gives an example of a philosophical application of Boolean-valued models. I apply Boolean-valued models to the language of mereology to model indeterminacy in the parthood relation. I argue that Boolean-valued semantics is the best degree-theoretic semantics for the language of mereology. In particular, it trumps the well-known alternative—fuzzy-valued semantics. I also show that, contrary to what many have argued, indeterminacy in parthood entails neither indeterminacy in existence nor indeterminacy in identity, though being compatible with both.

Chapter 3 (joint work with Bokai Yao) gives an example of a mathematical application of Boolean-valued models. Scott and Solovay famously used Boolean-valued models on set theory to obtain relative consistency results. In Chapter 3, I investigate two ways of extending the Scott–Solovay construction to set theory with urelements. I argue that the standard way of extending the construction faces a serious problem, and offer a new way that is free from the problem.

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Abstract

Logic is traditionally considered to be a purely syntactic discipline, at least in principle. However, prof. David Isles has shown that this ideal is not yet met in traditional logic. Semantic residue is present in the assumption that the domain of a variable should be fixed