

# Formation of terrestrial planets

Eiichiro Kokubo

National Astronomical Observatory of Japan,  
2-21-1, Osawa, Mitaka, Tokyo 181-8588 Japan  
email: [kokubo.eiichiro@nao.ac.jp](mailto:kokubo.eiichiro@nao.ac.jp)

**Abstract.** In the standard formation scenario of planetary systems, planets form from a protoplanetary disk that consists of gas and dust. The scenario can be divided into three stages: (1) formation of planetesimals from dust, (2) formation of protoplanets from planetesimals, and (3) formation of planets from protoplanets. In stage (1), planetesimals form from dust through coagulation of dust grains and/or some instability of a dust layer. Planetesimals grow by mutual collisions to protoplanets or planetary embryos through runaway and oligarchic growth in stage (2). The final stage (3) of terrestrial planet formation is giant impacts among protoplanets while sweeping residual planetesimals. In the present paper, we review the elementary processes of terrestrial planet formation and discuss the extension of the standard scenario.

**Keywords.** solar system: formation, planets and satellites: formation

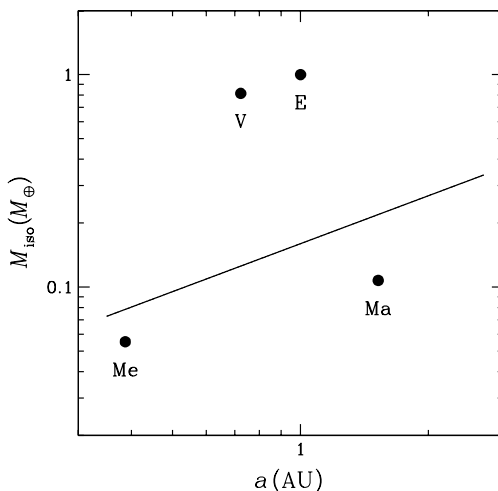
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## 1. Introduction

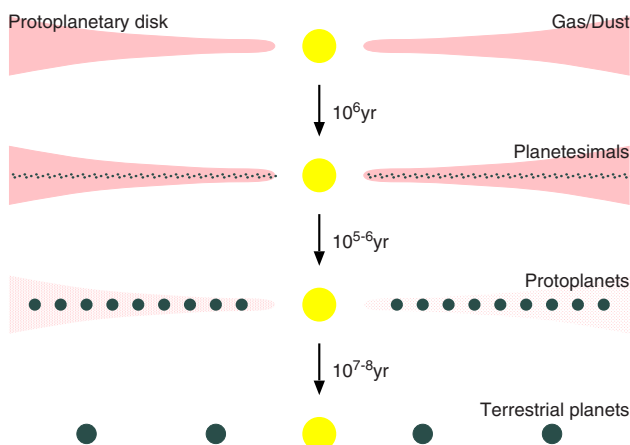
Needless to say, the formation of terrestrial planets is one of the most important problems in modern planetary astrophysics. At the beginning we overview the important properties of the terrestrial planets in the solar system when we consider their origin. The terrestrial planets have a bimodal mass distribution, namely, large planets with  $\sim M_{\oplus}$  (Venus and Earth) and small planets with  $\sim 0.1M_{\oplus}$  (Mercury and Mars), where  $M_{\oplus}$  is the Earth mass (Fig. 1). Their orbits have the semimajor axis of  $\simeq 0.4$ -1.5 au in which the large planets reside inside and the small ones outside. The large planets have smaller eccentricities and inclinations ( $e, i \sim 0.01$ ) than the small ones. The obliquity of the planets ranges from  $0^{\circ}$  to about  $180^{\circ}$  and the spin periods are from  $\sim 1$  d to  $\sim 100$  d. The effective spin period for the Earth-Moon system is  $\simeq 4$  hr. The core (iron) fraction of the planets ranges from 20% to 70%. Note that all these properties affect the habitability of planets.

In the standard scenario for solar system formation, planets form from a protosolar or protoplanetary disk that consists of gas and dust (e.g., [Hayashi et al. 1985](#), [Raymond et al. 2014](#)). The formation scenario can be divided into three stages: (1) formation of planetesimals from dust, (2) formation of protoplanets or planetary embryos from planetesimals, and (3) formation of planets from protoplanets (Fig. 2). In stage (1), planetesimals form from dust through coagulation of dust grains and/or some instability of a dust layer. Planetesimals are small building blocks of solid planets. Planetesimals grow by mutual collisions to protoplanets through runaway and oligarchic growth in stage (2). The final stage (3) depends on the type of planets. The final stage of terrestrial planet formation is giant impacts among protoplanets while sweeping residual planetesimals. This scenario gives a general framework of terrestrial planet formation though it still has many problems.

In the present paper, we review the basic elementary processes of terrestrial planet formation, showing some recent simulations (e.g., [Kokubo & Ida 2012](#)). We also discuss the extension of the standard scenario and the origin of the diversity of terrestrial planets.



**Figure 1.** Mass of the terrestrial planets, Mercury, Venus, Earth, and Mars against the semimajor axis. The solid line shows the isolation mass of protoplanets for the standard protoplanetary disk model.



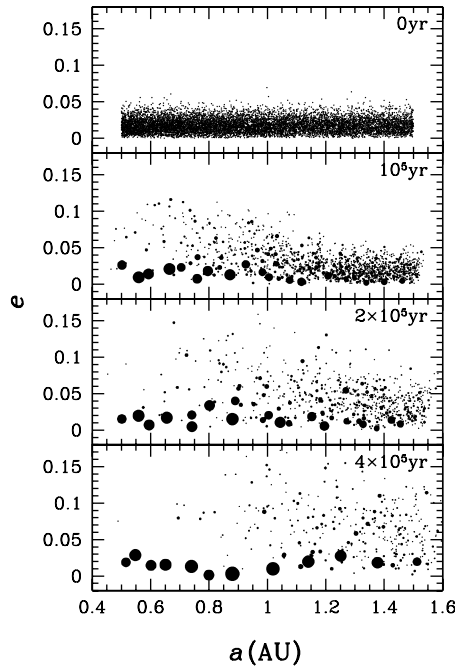
**Figure 2.** Schematic illustration of the standard scenario of terrestrial planet formation.

## 2. Planetesimal Formation

The formation of planetesimals from dust is one of the most important unsolved problems in planet formation theory. Since planetesimals are building blocks of planets, understanding their formation is a key to understand the structure and evolution of protoplanetary systems.

There are two classical models, the pairwise coagulation of (compact) dust grains (e.g., [Weidenschilling & Cuzzi 1993](#)) and the (dynamical) gravitational instability of a dust layer (e.g., [Goldreich & Ward 1973](#)). The coagulation model has serious difficulties known as fragmentation, bouncing and radial drift barriers, while the protoplanetary gas disk seems too turbulent for the gravitational instability model to operate (for a review, see e.g., [Chiang & Youdin 2010](#)).

A few new models that can possibly overcome these difficulties have been proposed recently. In the streaming instability model, due to clogging of dust grains through radial drift, dust clumps develop to form planetesimals (e.g., [Youdin & Goodman 2005](#)). In the



**Figure 3.** Snapshots of a planetesimal system on the semimajor axis-eccentricity plane at  $t = 0, 10^5, 2 \times 10^5, 2 \times 10^5,$  and  $4 \times 10^5$  years (Kokubo & Ida 2002). The size of circles is proportional to the radius of planetesimals.

secular gravitational instability model, the gravitational instability of a dust layer driven by gas drag can take place on the longer timescale than the classical dynamical instability (e.g., Michikoshi *et al.* 2012). The porous dust model assumes a pairwise coagulation of low density dust aggregates. In this model, porous dust aggregates with fractal dimension of about 2 grow by pairwise coagulation. They can grow beyond the three barriers because of their low-density (e.g., Kataoka *et al.* 2013). As the porous aggregates grow their layer becomes gravitationally unstable to fragment to form planetesimals (e.g., Michikoshi & Kokubo 2016).

The planetesimal formation process depends on the condition of the protoplanetary disk. The high-resolution observation of the early stage of protoplanetary disks is necessary to understand the realistic formation process.

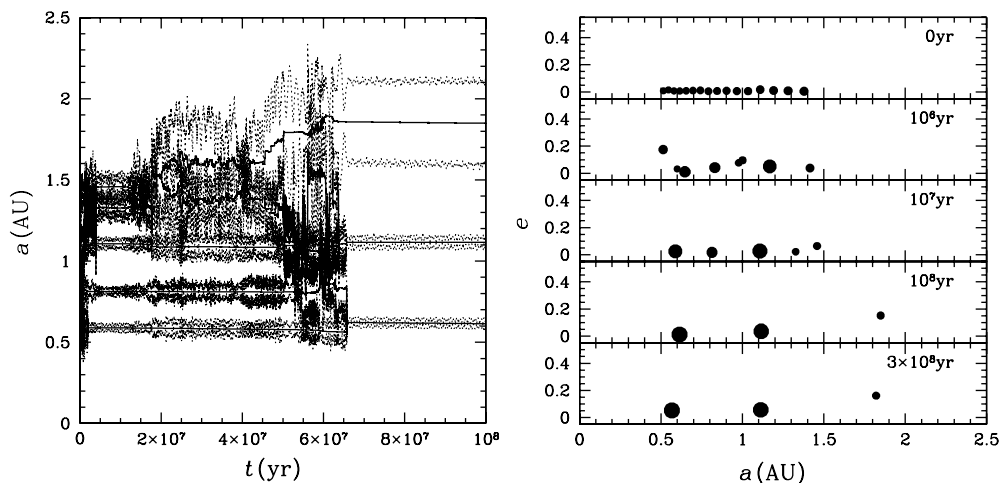
### 3. Protoplanet Formation

The formation of protoplanets from planetesimals is relatively well understood, compared with planetesimal formation (Fig. 3). For the details of the basic dynamical models, see Kokubo & Ida (2012).

We assume that the relative growth rate of planetesimals with mass  $M$  by collisions among themselves  $(1/M)(dM/dt)$  is proportional to  $M^p$ . There are two modes of planetesimal growth, namely orderly ( $p < 0$ ) and runaway ( $p > 0$ ) (e.g., Kokubo & Ida 1996). In the orderly growth planetesimals tend to grow equally, while larger planetesimals grow faster than smaller ones in the runaway growth.

In the course of planetesimal growth  $(1/M)(dM/dt)$  can be approximated as

$$\frac{1}{M} \frac{dM}{dt} \propto M^{1/3} v_{\text{ran}}^{-2}, \quad (3.1)$$



**Figure 4.** Evolution of a protoplanet system on the time-semimajor axis plane (left) and the semimajor axis-eccentricity plane at  $t = 0, 10^6, 10^7, 10^8,$  and  $3 \times 10^8$  years (right) (Kokubo *et al.* 2006). The dashed curves present apocenter and pericenter distances (left) and the size of circles are proportional to the radius of planets (right).

where  $v_{\text{ran}}$  is the random velocity of planetesimals, which is a deviation velocity from a non-inclined circular orbit approximately given by  $(e^2 + i^2)^{1/2}v_K$ , where  $e$  and  $i$  are the eccentricity and inclination of planetesimals and  $v_K$  is the Kepler circular velocity. Thus it is the random velocity of planetesimals that controls the growth mode and timescale. There exists the equilibrium random velocity at which the effects of gravitational scattering among planetesimals and drag from the disk gas balance. At least in the initial stage of planetesimal growth, the growth mode is runaway, where the random velocity is independent of the mass of a growing planetesimal. In this case  $(1/M)(dM/dt) \propto M^{1/3}$ , which shows the runaway growth. We call a runaway-growing planetesimal as a protoplanet.

Once the mass of a protoplanet exceeds about a hundred times the mass of surrounding planetesimals, the runaway growth slows down to shift to the oligarchic growth where the random velocity of planetesimals becomes proportional to the Hill radius of the protoplanet ( $r_H \propto M^{1/3}$ ). In this case  $(1/M)(dM/dt) \propto M^{-1/3}$ , which means the orderly growth. Among protoplanets orbital repulsion keeps their orbital separation  $b \simeq 10r_H$ . In summary, on the oligarchic growth stage, protoplanets dominantly grow in the orderly way with certain orbital separations (e.g., Kokubo & Ida 1998). Given the orbital separation  $b$  of adjacent pairs we can estimate the isolation or final mass of protoplanets formed by the oligarchic growth. For the standard disk with the planetesimal surface density  $\Sigma_d = 10(a/1 \text{ au})^{-3/2} \text{ g cm}^{-2}$ , we obtain

$$M_{\text{iso}} \simeq 2\pi ab\Sigma_d = 0.16 \left( \frac{b}{10r_H} \right)^{3/2} \left( \frac{a}{1 \text{ au}} \right)^{3/4} M_{\oplus} \quad (3.2)$$

(Kokubo & Ida 2002). The mass of protoplanets around the terrestrial planet region is on the order of  $0.1 M_{\oplus}$ , in other words, about Mercury to Mars mass (Fig. 1).

#### 4. Giant Impacts

In order to complete the large terrestrial planets such as Venus and Earth, further accretion among protoplanets is necessary. While protoplanets are in the disk gas,

their orbits are stable since their eccentricities are damped by the gravitational drag from gas (e.g., Tanaka & Ward 2004). However, from the observations, the lifetime of disk gas is estimated to  $\lesssim 10^7$  years. After the disk gas dispersal, protoplanets gradually build up their eccentricities by gravitational interaction. It is empirically known from the numerical experiments that for an equal-mass protoplanet system with equal orbital separations in the mutual Hill radius the instability timescale of the systems is given as

$$\log t_{\text{inst}} \simeq c_1(b/r_H) + c_2, \quad (4.3)$$

where  $c_1$  and  $c_2$  are the functions of  $e$  and  $i$  (e.g., Chambers *et al.* 1996; Yoshinaga *et al.* 1999). In this timescale close encounters between protoplanets occur and the giant impact stage begins, where protoplanets collide with one another.

The giant impacts continue until the system becomes orbitally stable where all orbits are nearly circular and widely separated. In other words, the mass and orbital properties of planets are determined by the system stability. The timescale of this stage is on the order of  $\sim 10^8$  years (e.g., Chambers & Wetherill 1998). A typical outcome for the standard disk is two Earth-sized planets and one or two leftover protoplanets (Fig. 4) (e.g., Kokubo *et al.* 2006). Their eccentricities and inclinations are  $\simeq 0.1$  that are higher than those of Venus and Earth. Therefore some damping is necessary after formation, which may be dynamical friction from residual planetesimals (collisional debris) and/or disk gas.

The spin parameters are also determined on this stage (Kokubo & Genda 2010). The typical spin angular velocity is about 70% of the critical spin angular velocity for rotational breakup

$$\omega_{\text{cr}} = 3.3 \left( \frac{\rho}{3 \text{ gcm}^{-3}} \right)^{1/2} \text{ hr}^{-1}, \quad (4.4)$$

where  $\rho$  is the internal density of protoplanets, and is independent of the planet mass. The obliquity distribution obeys an isotropic distribution

$$nd\varepsilon = \frac{1}{2} \sin \varepsilon d\varepsilon, \quad (4.5)$$

which means that the typical obliquity is around  $90^\circ$ . This isotropic distribution is an outcome of isotropic collisions of protoplanets due to much larger scale-height of the protoplanet system than the physical size of planets.

## 5. Extension of the Standard Scenario

The basic dynamical model of terrestrial planet formation gives us a general framework. However we may need to modify it when we apply it to individual systems.

For simplicity, the standard scenario assumes a continuous power-law disk except for the snow-line, in-situ formation, and km-sized planetesimals as building blocks. Of course, these assumptions may not be always justified in a realistic system. Now many authors are extending the standard scenario by including the effects of an evolving disk, radial migration of planet-building materials, and larger planetesimals and small pebbles. Here we focus on the localization of terrestrial planets in the solar system and introduce two models to explain it.

In the solar system, terrestrial planets reside in the limited radial range  $0.4 \text{ au} \lesssim a \lesssim 1.5 \text{ au}$  and the two planets, Mercury and Mars, near the inner and outer edges are smaller than those in the middle, Venus and Earth. Explaining this property of the mass distribution is often referred to as the small Mars problem. Hansen (2009) showed

that if planets are accreted from a narrow ring of protoplanets and planetesimals with  $0.7 \text{ au} \lesssim a \lesssim 1.0 \text{ au}$ , the mass distribution of the planets is easily reproduced. Then the question is how to form such a narrow ring. Here the so called grand-tack model comes in (Walsh *et al.* 2011). In the model first Jupiter migrates inward to cut off the planetesimal disk around 1 au and then tacks to migrate outward with Saturn. This model succeeds in forming a narrow ring, depletion of the asteroid belt, and water delivery to the Earth. Note that the grand-tack model does not explain the inner cut-off of the planetesimal disk.

Another model is the disk bump model. The evolution of a protoplanetary disk with viscous accretion and disk winds naturally leads to formation of a global radial pressure bump in the disk (Suzuki *et al.* 2016). Around the pressure bump planetesimals accumulate due to gas drag, which forms a narrow planetesimal ring. In the evolved gas disk planetesimals and protoplanets barely migrate radially due to an almost flat surface density distribution of gas. The accretion from such a disk can reproduce the localization of terrestrial planets (Ogihara *et al.* 2018).

## 6. Summary

We have reviewed the standard dynamical model of terrestrial planet formation in which rocky dust grains grow through planetesimals and protoplanets to terrestrial planets on the timescale of  $10^8$  years. At the present after the discovery of diverse terrestrial planets in exoplanetary systems, the formation model is being modified and extended. This extension includes a discrete distribution of planetesimals, formation with migration, collisional disruption, and pebbles as building blocks. We are aiming to construct a general comprehensive formation model that explains not only the solar system terrestrial planets but also the diverse terrestrial planets in exoplanetary systems.

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**Discussion**

LINSKY: Please comment on the role of resonance in determining the localization of planets. TRAPPIST-1 has 7 planets all in resonances.

KOKUBO: Capturing into resonances is possible when planets migrate in a gas disk. Since the giant impact stage starts after disk gas dispersal, planets formed by giant impacts are not in resonances.

MAROV: Did you try to figure out in the collisional processes assembly/repulsion ratios depending on size particles specifically beyond the size barrier, and how it depends on the assumed fractal dimension?

KOKUBO: We investigated the accretionary conditions of planetesimals and protoplanets and used them in the N-body simulations.