to the point where it could be applied to some ring theoretic problems. The book ends with three appendices; the first concerns the representation of a commutative ring as a ring of functions and the second and third develop some of the theory of group rings. Appendix 3, on semiprime group rings, was written by I. G. Connell.

This should be an excellent textbook for a course in ring theory. There is a large selection of problems on all levels of difficulty.

A few typographical errors were noted. The only confusing one occurs on page 29, line 2: Replace $1 = rx$ by $1 - rx$. The author has also communicated the following correction: "Proposition 2 on page 110 and Colollary 1 on page 111 require the additional assumption that R_R be finite dimensional. The proof of the former is incomplete; it remains to be shown that, for any non-zero divisor r of R, $qr = 0$ implies $q = 0$. This is an easy consequence of the fact that rR is large, which is proved as follows, using an argument by Lesieur and Croisot (1959): Suppose $rR \cap sR = 0$ for some s in R, then $\Sigma_{i \geq 0} r^i sR$ is seen to be a direct sum, and the finite dimensionality of R_R leads to $s = 0$."

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Ring and radicals, by N. J. Divinsky. University of Toronto. $7.50.

A review of a book on ring theory should start with a short summary of the history of the subject, and we take this opportunity to correct some of the historical remarks made by the author.

The general theory of associative rings has its roots in the theory of finite dimensional algebras as developed by Dickson and Wedderburn, and the abstract ideal theory introduced by Emmy Noether. Probably, the cornerstone of general ring theory was laid in Artin's famous paper of 1927, where the structure of rings with both chain conditions was given: Students of E. Noether have tried to extend this result, and the structure of rings with only the minimum condition was given almost simultaneously by Hopkin (1938) and Levitzki (1939). (Levitzki's paper had been sent earlier to Artin, but due to the pre-war conditions in Europe, the first copy was lost, and when this was found out it was published hastily some months after Hopkin's publication.)

The structure theory of rings emphasizes the radical N of a ring R, by which the structure problem can be split into three parts: characterization of semi-simple rings R/N; the structure of radical rings N, and finally the patching together of N and R/N (where cohomology may be useful). The various characterizations of the radicals, especially Perlis' (1942), led to the basic result of Jacobson (1945) on the structure of ring without finiteness conditions, which was a turning point in the ring theory. In the same attempt of avoiding chain conditions other types of radicals were found: the upper and lower nil radical by Baer (1943); the locally nilpotent radical by Levitzki (1943) and the Brown-McCoy radical (1947). These developments revealed that there are two ways to attack radical theory: one, the 'going up' method, searches for a maximal ideal N with a given property; and the other approach, the 'going down', looks for a maximal homomorphic image of R with a given property whose kernel is the radical N. Both approaches were carried out in the general theory almost simultaneously. The first approach being presented by Amitsur* (1952, 1954), and the second by Kurosh (1953). The methods are complementary, and the useful radicals for a general structure theory are those which can be attacked from both directions. Since then the general theory of radicals has been developed by Andrunakievitch, Divinsky, and others, and is now in a 'categorical algebra' phase of expansion.

This is as far as avoiding chain conditions goes. Recently a surprising push has been given to associative ring theory by going back to the ascending chain condition, where all is based on trying to extend the pre-classical fact that a commutative integral domain can be embedded in a field. This push was given in Goldie's paper (1958), on the embedding of prime rings with ascending chain conditions in simple rings with the minimum conditions, and its generalization to semi-prime rings. We take this opportunity to point out that many of these results are already found in another form in the papers of R.E. Johnson in 1951-1954, which unfortunately have not influenced the community of algebraists as Goldie's paper did.

These are the stages of the theory of rings, which the book of Divinsky covers. Namely, the developments in the general theory of radicals, rings with minimum conditions, rings of quotients of rings with maximum conditions. The literature in Algebra (in the languages known to the reviewer) lack a book in ring theory which, on one hand, is not too elementary (like McCoy's Theory of Rings), and on the other hand is not too sophisticated and encyclopaedic (like Jacobson s Structure of Rings). A book which would be intended for the student who has already made his first steps in modern algebra, but who is not yet an expert in rings - was missing. The present book seems to fill this gap, and even covers an area (radical theory) which is missing in Jacobson's book - though in an approach which is somewhat one-sided, but which is justified by the taste of the author. This book is intended for students who are acquainted with fundamental notions of modern algebra (like those appearing in Maclane-Birkhoff's book) and are used to homomorphisms and ideals. It is written clearly, and proofs are easily read, though the author, in some cases, has preferred longer computational proofs to the more abstract approach.

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* First published for rings in a notice in Proc. Am. Math. Soc., v. 57 (1951), p.118.

The book contains the following chapters:

(1) The general theory of radicals, where this theory is presented following Kurosh;

(2) Rings with descending chain conditions. The structure theory of the rings is presented in the classical way, which has its roots in Wedderburn's methods and which is somewhat outdated (but still good);

(3) Rings with ascending chain condition, which starts with Hopkin's result that rings with descending chain with a unit satisfy also the ascending chain condition. Next, there is a detailed discussion of the nilproperty of rings - its relation to the maximum condition and the various radicals defined by this property, namely the lower radical of Baer, the upper radical, and finally Levitzki-Nagata result that the intersection of the prime ideals is the lower radical. However, the main object of this chapter is the presentation of Goldie's theory on the ring of quotients or ring with ascending chain condition. It is here that the contribution of R. E. Johnson and the other approach of using injectives might have been mentioned. The next three chapters are devoted to particular radicals.

(4) The Jacobson radical; (5) the Brown-McCoy radical; (6) the Levitzki radical. The chapter on the Jacobson radical covers also Jacobson's structure theory of rings without finiteness assumptions. The other two radicals are defined, and their basic properties are covered.

Neither of these last two radicals is satisfactory from the point of view of structure theory, since while the Brown-McCoy radical M has an interesting characterization by the structure of R/M - the characterization of M is not satisfactory, or useful; on the other hand, the Levitzki radical L does not have a good description for its quotient ring R/L. To the disappointment of the reviewer these observations have not been pointed out in the text. The last chapter, (7) The eight radicals and recent results, is the high point of the book. Here the general theory of radical is applied to reveal the relations between the various radicals. Methods of constructing radicals are given and some recent results of Andrunakievitch are presented.

One has to introduce a word of caution to the readers who are interested in radicals. There are infinitely many ways to define radicals, but the importance of the new radicals lies in their usefulness. Thus, Jacobson's, Baer's and Levitzki's radicals have proved their importance, while for the others we still await their applications. On the other hand, the general theory of radical is interesting due to the fact that it reveals the role which homomorphisms and subdirect sums play in the category of rings, and this is the reason for its extension to arbitrary categories.

A drawback of this book is the uniform treatment given to each of the radicals without any indication to their relative usefulness. This criticism probably reflects the reviewer's taste in ring theory which does not always coincide with the author's. We also leave to the reader s judgment the numerous historical and emotional remarks of the author, like those on pages 39, 51, 63 and 75.

In spite of these differences of opinion, we do recommend this book. The general outline is satisfactory, and it is unique, as far as known to the reviewer, in presenting the general radical theory. Undoubtedly, it can be used in a course in ring theory for advanced students though no exercises are included. The book contains many simple and complicated examples which help to clarify the abstract theory.

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Mathematics for the physical sciences, by L. Schwartz. Hermann, Editeurs des Sciences et des Arts, Paris, and Addison-Wesley Publishing Company, Reading, Mass., 1966. 358 pages. $14.00.

The title of this book may be somewhat misleading in that it is neither a treatise on the methods of applied mathematics, nor a compendium of mathematical methods that should be known, or may be useful, to physicists. Rather, the emphasis is almost entirely on distribution theory and its application to physics. The book succeeds well in this, although it appears to be written more for the mathematician who may wish to learn something of the role of distribution theory in physics than for the physicist.

The first chapter is a review of various results on series and the Lebesque integral. Discussions of multiple integration and convergence of sequences of functions are included. The second chapter is concerned with elementary properties of distributions: their definition, examples, differentiation of distributions, and topology in distribution spaces. The convolution of distributions is defined in the third chapter and applied to the operational calculus, Volterra's integral equation (where the kernel is a function of the difference of its arguments) and electric circuit theory. The next three chapters are concerned with Fourier series, the Fourier Transform and the Laplace transform; the discussion is again strongly distribution theoretic. The chapter on Fourier series includes also the definition of Hilbert space and L^2 and a statement, and explanation of the significance, of the Riesz-Fischer theorem. The application of the Laplace transform to the operational calculus is also discussed. In Chapter 7 the preceding material is applied to the wave and heat-flow equations. The discussion of the wave equation is quite extensive, but that of the heat-flow equation is rather cursory; the latter, however, is discussed as an example in other parts of the book. The last two chapters are devoted to the gamma and Bessel functions.

Exercises are included at the end of each chapter. These exercises are both interesting and challenging.