worthy of the papers which they accompany. It is a tremendously worthwhile project to make Hardy's papers available as a collection, for this will be a source of stimulation even to those of us who are already familiar with parts of his work, and mathematicians everywhere owe a great debt to the London Mathematical Society and its editorial committee. PHILIP HEYWOOD

HARDY, G. H.. *Collected Papers*, including joint papers with J. E. Littlewood and others. Vol. IV. Edited by a Committee appointed by the London Mathematical Society. (Clarendon Press; Oxford University Press, 1969), 722 pp, 120s.

For anyone to attempt a critical review of a volume of Hardy's papers would be very difficult; for one of his former pupils to do so would be impertinent. All I can do is to state briefly what this volume contains.

There are 54 papers, of which only five are joint, written between 1902 and 1946; and they are divided into two sections.

The first section of 42 papers is on Special Functions. § I(*a*) contains 12 papers on the zeros and asymptotic properties of certain special integral functions. At the beginning of the century, the theory of integral functions was an active field of research. Hardy's objective was to illustrate the general theory and to try to find new results by the consideration of particular examples. He starts by considering the zeros of large modulus of such functions as $\sin x - x$ and $\frac{1}{\Gamma(x+1)} - c$. He then goes on to integral functions defined by Taylor Series. An interesting example is

$$\sum_{0}^{\infty} \frac{z^{n^3}}{(n^3)!}$$

When $|z| = N^3$ where N is large, this function is dominated by its Nth term, from which it follows that there are N^3 zeros in $|z| < N^3$. In the annulus

$$N^3 < |z| < (N+1)^3,$$

the function is dominated by the sum of its Nth and $(N+1)$th terms. And the $3N^2+3N+1$ zeros in the annulus are given approximately by equating to zero the sum of those terms. They lie approximately at the vertices of a regular polygon midway between the bounding circles. The result is generalised by replacing n^3 by a function $\phi(n)$ whose increase is regular and sufficiently rapid.

§ I(*b*), consisting of five papers, deals with the singularities on its circle of convergence $|z| = 1$ of a function defixed by a Taylor series $\Sigma a_n z^n$ where

$$a_n = \int_0^1 \left(\log \frac{1}{u}\right)^{\alpha-1} (1-u)^{\beta-1} u^{\gamma-1+n} \phi(u) du.$$

The discussion involves replacing this integral by an integral round a loop in the complex u-plane. Various examples, such as the hypergeometric series, are worked out in detail. He also discusses functions of two variables; the problem is to determine the behaviour of a series $\Sigma a_{mn} c_{mn} x^m y^n$ from a knowledge of the behaviour of a " base series " $\Sigma c_{mn} x^m y^n$ of simple type.

The three papers of § I(*c*) are essentially a supplement to Hardy's 1910 Cambridge Tract *Orders of Infinity* and applications of the Infinitärcalcül of du Bois-Reymond to oscillating Dirichlet integrals.

§ I(*d*) is a miscellaneous collection of 21 papers on Special Functions, the Gamma and Zeta Functions and their generalisations, Airy's Integral, the Modular Functions, Weierstrass's Non-Differentiable Function, the Laguerre Polynomials, and some formulae of Ramanujan.

§ II contains 12 papers, written between 1910 and 1946, on widely scattered topics in the Theory of Functions.

§ I was edited by E. M. Wright, § 2 by M. L. Cartwright, who give valuable comments, corrections and references to more recent work.

E. T. COPSON

CLEGG, JOHN C., *Calculus of Variations* (University Mathematical Texts Series, Oliver and Boyd, 1968), ix + 190 pp., 21s. (Paper bound), 27s. 6d. (cloth bound).

In a modern undergraduate course on mathematics calculus of variations is often disposed of in a few lectures, where most of the emphasis is on physical applications. Less often a much fuller course, with care taken to put the subject on a more rigorous basis, may be given. For students in both these types of courses, the book under review should prove an excellent text. For a comparatively short book a surprisingly large number of topics is covered: to mention a few, corner conditions, fields of extremals and the sufficiency conditions in terms of Weierstrass's E-functions. In the last chapter there are short discussions of integrals involving higher derivatives than the first and of the extrema of double integrals. There are many illustrative examples worked out in full detail and physical applications are given a prominent place. A minor criticism is that more examples for the student to tackle might have been provided.

R. P. GILLESPIE

POLYA, G., *Patterns of Plausible Inference* (Second Edition, Princeton University Press, 1968), x + 225 pp, 72s.

In a review of the second edition of so celebrated a book, there is obviously no need either for a synopsis of the contents of the main body of the text or for a detailing of the features which will continue to provide entertainment, instruction, food for thought, and inspiration for thought to readers of all sorts of mathematical tastes and backgrounds.

The first edition was published in 1954. New in this edition is an appendix consisting of (i) a 10-page article " Heuristic Reasoning in the Theory of Numbers " (reprinted from the *American Mathematical Monthly*, 1959), and (ii) 23 pages of additional comments and problems. This material is designed as a supplement to the whole of Polya's two-volume work " Mathematics and Plausible Reasoning " (of which " Patterns of Plausible Inference " is volume II). Solutions are given for the 33 new problems.

T. A. WHITELAW