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## APPENDIX 4. SUMMARY OF WORK DONE IN U.S.S.R. ON COSMOLOGY AND RELATED TOPICS OF GENERAL RELATIVITY

(prepared by A. L. Zelmanov)

A. Z. Petrov (1) showed that in the case when Einstein tensor is proportional to the metrical tensor the geometry of the four-dimensional space (space-time) is almost entirely determined by the totality of geodetics: if two such spaces allow a one-to-one correspondence between the geodetics, then either they are spaces of constant curvature or their metrical tensors differ only by a constant factor of proportionality.

Using his earlier results concerning the existence of three different types of space-time every type of which includes, in particular, its own kind of an empty world, A. Z. Petrov (2) proposed a principle for an invariant-group formulation of the boundary conditions at infinity giving to every type of space-time an appropriate kind of boundary conditions.

Analysing from the geometrical point of view the situation, which leads to the appearance of the time singularity in the solutions of gravitational equations in a so-called synchronous (semi-geodetical) system of co-ordinates, E. M. Lifshitz, V. V. Sudakov and I. M. Khalatnikov (3) concluded that the general case of an arbitrary distribution of matter and gravitational field leads to the absence of such a singularity.

Accepting the usual interpretation of the energy-momentum tensor and rejecting the assumptions of homogeneity and isotropy, a theorem concerning the existence of solutions of Einstein field equations with any (arbitrary) behaviour of the volume of the co-moving space was proved by A. L. Zelmanov (4). In particular the following consequences were drawn: the possibility of a combination of an expansion of the co-moving space in one region with its contraction in another one and also the possibility, due to absolute rotation or the pressure gradient, of the passage of the elements of the co-moving space through a regular finite (non-zero) minimum of volume (of the scale-factor as well) instead of a singularity.

The possibility of such a minimum (due to the pressure gradient) simultaneous in the whole space, in the case of some spherically symmetrical mass distributions was then shown by I. D. Novikov (5). In the general case of spherical symmetry the existence of the regions of essential non-stationarity was also shown. In these regions it is impossible to eliminate the dependence of the angular part of the line element on the time co-ordinate, but it is possible to exclude its dependence on the radial co-ordinate (the space inside the singular Schwarzschild sphere is such a region in vacuum). The results are applied to the Metagalaxy. The properties of metrics inside the singular spheres in the cases of Schwarzschild and Nordström fields, as well as the known problem of relativity of finiteness or infiniteness in these cases were considered (6).

The possibility of the removal of the singularity of the Schwarzschild sphere was pointed out by Y. A. Rylov (7). An opposite point of view was supported by V. Unt (8). I. D. Novikov and L. M. Ozernoi (9) considered the propagation of light in Lemaître system of reference, free falling in the spherically symmetrical gravitational field outside the masses. The results are applied to the computation of the appearance of an exploding mass with initial size smaller than the gravitational radius.

A. Z. Petrov (10) showed that the well known Birkhoff theorem (stating that each spherically symmetrical gravitational field in empty space should be static) is true only under some additional conditions. The author came to the conclusion that these conditions exclude solutions of the type of shock waves and presented a general solution of the problem on the spherically symmetrical field in vacuum.

Still earlier V. Unt (11) showed that it is impossible to prove Birkhoff theorem without violation of well known Lichnerowicz conditions. The same author pointed out also, that the gravitational field inside a spherical hole in the expanding universe cannot be static if only Lichnerowicz admissible co-ordinates are used (12).

Y. B. Zeldovich (13) and, independently, I. D. Novikov (14), considered spherically symmetrical models of a semi-closed world, which includes more than a half of a non-stationary spherical Friedman world with its surrounding region and which is connected with the outer space through a narrow neck. These models show, in particular, that an essential decrease of the gravitational mass of a sphere is possible by adding a spherically symmetrical layer of matter because of the increased gravitational mass-defect. The questions concerning the evolution of a semi-closed world and the possibility for the exchange of the information between it and the outer space are also discussed.

Exact axially symmetrical static solutions of Einstein equations were considered by A. Koppel (15).

B. Pontecorvo and Y. Smorodinsky (16) considered the possibility that the density of

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neutrinos and antineutrinos is comparable to, or greater than, the density of hydrogen. It is suggested that the present day data do not contradict hypotheses about the separation of the matter from anti-matter as a result of fluctuations in a charge-symmetric universe in the past when density of neutrinos and anti-neutrinos exceed the nucleon density by many orders magnitude.

Y. B. Zeldovich and Y. A. Smorodinsky (**17**) estimated the upper limit for the present density of all kinds of matter in the homogeneous isotropic universe, including neutrinos and gravitons, as $2 \cdot 10^{-28}$ g cm$^{-3}$ (if the reciprocal of Hubble parameter is $10^{10}$ years) since a greater density leads to a value less than $4 \cdot 10^{9}$ years for the age of the universe, and this is inconsistent with the age of the Earth and of radioactive elements. This gives an upper limit for the density of neutrinos and gravitons.

Y. B. Zeldovich (**18**) showed that in the limiting case of ultra-high density the pressure of matter can be equal to its energy density instead of one third of this magnitude, as it is usually supposed. As a consequence on the earliest stage of the expansion of the homogeneous isotropic universe (when the density was much greater than the nuclear one) the scale-factor increased as the cube (instead of the square) root from the time of expansion (**19**). According to the hypothesis of the same author (**20**), the matter in the prestellar stage was cold and was composed of protons, electrons and neutrinos. When the density was high, all these particles were in the state of degenerated Fermi-gases. The presence of neutrinos forbids the capture of an electron by a proton and the generation of a neutron and neutrino. During the expansion this matter turned into hydrogen and the first generation of stars was composed of it. The hypothesis by Y. B. Zeldovich (**21**) about the origin of inhomogeneities in the expanding universe was not confirmed in the course of the further computations of the author.

A. G. Doroshkevich and Y. B. Zeldovich (**22**) considered in non-relativistic approximation the gravitational development of perturbations of density and velocity in a homogeneous medium, undergoing homogeneous isotropic expansion. Limiting cases of low pressure and long-wave perturbations were considered.

N. A. Dmitriev and Y. B. Zeldovich (**23**) concluded that the law of variation (decrease) of momentum of a particle moving in the homogeneous isotropic expanding universe, becomes invalid when the local inhomogeneities of density are taken into account. A differential equation for the kinetic energy of the random motion, taking into account local inhomogeneities, is proposed.

According to Y. B. Zeldovich (**24**), the usual conclusions about the angular size and apparent luminosity of distant objects in the homogeneous isotropic universe are true only in case when the mean density of matter in the cone of rays connecting the object with the observer does not differ from the mean density of matter in the universe. If in this cone masses are not present, the angular diameter of the object does not pass through the minimum.

M. F. Shirokov and I. Z. Fisher (**25**) deduce cosmological equations for homogeneous isotropic models by space-time averaging of the equations for models with local inhomogeneities bearing in mind the possibility of the elimination of the singularities by such an account of local inhomogeneities.

A criticism on the theory of the creation of matter and of variation of the universal constants can be found in the papers (**26**) by Y. B. Zeldovich. This criticism is based, in particular, on the account of the influence of local inhomogeneities.

A version of the theory of variation of the universal constants based on Dirac cosmological principle is proposed by K. P. Staniukovich (**27**) According to this version, the total mass of all particles is conserved, the gravitational constant and the number of particles (nucleons) increase with time, while the Planck constant as well as the charges and the masses of the particles decrease. The decrease of the masses of elementary particles is treated as a result

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of emission of gravitational waves. The right hand parts of Einstein gravitational equations are modified (generalized). A generalization of Friedman models relating to the variation of the gravitational constant and to the modified gravitational equations is considered (**28**).

Identifying the bright emissional band $\lambda = 5170$ Å in the optical spectrum of the radio-source 3C286 with the resonance line of ionized magnesium ($\lambda = 2798$ Å), I. S. Shklovsky (**29**) obtains $1 + Z = 1{\cdot}848$, which is confirmed by the possibility of identifying the very faint band near $\lambda = 4390$ Å with the resonance triplet of ionized iron (2383 Å, 2374 Å, 2344 Å). The absence of lines in the optical spectrum of 3C196 obtained by Schmidt is interpreted as the result of a great redshift, supposedly $1 + Z > 2{\cdot}25$. Accepting the Einstein-de Sitter model and the Hubble parameter equal to 100 km sec$^{-1}$ Mpc$^{-1}$, the luminosity distances of the two objects are estimated respectively as $2{\cdot}9 \times 10^3$ Mpc and more than $4 \times 10^3$ Mpc.

Continuing his investigations Y. P. Pskovsky (**30**) considered the character of the motions in the Hypergalaxy (Supergalaxy) using the redshifts of 370 galaxies. The principal values of the strain-velocity tensor were found. It is concluded that the Hypergalaxy has no unified axis of rotation and the galaxies move around the central condensation (in Virgo cluster) on disorderly oriented orbits.

An attempt is made by A. V. Zasov (**31**) to obtain a more precise dependence between the luminosity distances to clusters of galaxies and field galaxies on the one hand and the redshifts on the other. Altogether 25 clusters of galaxies and also 277 field galaxies were considered. The Hubble parameter is found to be $175 \pm 10$ km sec$^{-1}$ Mpc$^{-1}$. The second order term in the dependence between the redshift and the corrected photovisual magnitude of the tenth brightest galaxy in a cluster is found to be $-2{\cdot}2 \pm 1{\cdot}0$ (that corresponds to $q = +3 \pm 1$).

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## APPENDIX 4 (SUPPL.). SUPPLEMENT TO THE SUMMARY OF WORK DONE IN THE U.S.S.R. ON COSMOLOGY AND RELATED TOPICS OF GENERAL RELATIVITY

(prepared by A. L. Zelmanov)

G. M. Idlis (**32**) considered the principle of causality as the base of cosmology and proposed cosmological consequences from this principle.

A. J. Kipper (**33**) proposed a solution of the gravitational paradox problem in newtonian theory by introducing a generalized integral (called by him 'integral with an actually infinite volume of integration') for the potential, and in connection with this solution, introduced two systems (scales) for space and time, measuring called gravitational and atomic systems.

The behaviour of a gravitating homogeneous ellipsoidal distribution of dust matter with a linear velocity field, including homogeneous deformation (not isotropic in general) and rotation was considered in non-relativistic theory by Y. B. Zeldovich (**34**). The infinite homogeneous distribution of matter was treated as a limiting case of an ellipsoidal homogeneous distribution. A comparison between the non-relativistic treatment of the problem and the relativistic one was given.

Using free-falling non-rotating systems of reference both in non-relativistic gravitational theory and in general relativity, H. Keres (**35**) proposed a method for the comparison of fields described by these theories.

A refinement of Birkhoff's theorem was proposed by A. P. Ryabushko (**36**) according to which any spherically symmetrical field in the empty space, satisfying Einstein equations, is equivalent to Schwarzschild field or to a part of it, if the continuity of coordinates transformations and one-to-one correspondence are not required.

The works (**5**) and (**14**) by I. D. Novikov were continued by their author in (**37**). In particular, the propagation of light and the motion of particles in the regions of essential non-stationarity were considered and a conclusion about the possibility of an arbitrary behaviour of the volume of a medium element on an infinite interval of time was drawn.

G. M. Idlis (**38**) regarded the Metagalaxy as a quasi-autonomic cosmical system, which can be approximately represented by a closed homogeneous isotropic model with the Hubble parameter equal to 75 km sec$^{-1}$ Mpc$^{-1}$ and with the prevalence of the radiation over the matter,

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