

## TIDAL EFFECTS ON GLOBULAR CLUSTERS

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It was von Hoerner (1957) who pointed out that the tidal field of the Galaxy imposed a boundary condition on the size of globular clusters, a fact that had observational consequences when the first deep star counts became available (King 1962). More recently, though, dispute (cf Innanen, et al. 1983) has arisen over the identification of the observed limiting radius from star counts with that predicted by simple dynamical theory (King 1962, 1966; Szebehely 1967). Straightforward application of the classical three body problem seemed to imply that no stable stellar orbits would be found beyond the collinear Lagrangian points. Thus, to first order, the observed limiting radius would be just the radius of the inner Lagrangian point  $L_2$  from the cluster center. For clusters in eccentric orbits, the tidal radius would be set at perigalacticon, where the tidal force is strongest. Numerical models of stellar orbits about a cluster in a realistic galactic potential by several authors (Jefferys 1976; Keenan and Innanen 1975; Keenan 1981) showed that stable orbits can exist with apocluster distances greater than  $r_{L_2}$ , due to the presence of non algebraic integrals of the motion that constrained the stars to move in a restricted region about the cluster rather than escaping.

I undertook a study of the effects of the tidal field on globular clusters in order to quantify the above (Seitzer 1983). I followed 1000 test particles orbiting in the outer half of a cluster (represented by a Plummer potential) for 10 cluster orbits about the galaxy. Escape was arbitrarily defined as when a particle went beyond 3 times the expected tidal radius as computed at apogalacticon. Models were computed for different cluster orbital eccentricities  $e = 0.0, 0.3, 0.5, \text{ and } 0.7$  to check for the dependence of observed tidal radius on eccentricity. The conclusions are summarized below:

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1. The observed limiting radius after ten orbits of the cluster about the galaxy was very close to the predicted inner Lagrangian point. A very few stars had apocluster radii outside this limit. This means that the observed tidal radius can be used to estimate the eccentricity of the cluster's orbit about the galaxy, or (to a much lower accuracy) the mass of the galaxy inside the cluster's orbit (Peterson 1974).
2. Most of the stars escaped in the first eight cluster orbits. One model ( $e = 0.5$ ) was followed for 50 cluster orbits: very few stars escaped after 10 orbits. This suggests that one has to be very careful about interpreting the observed limiting radius of distant systems such as dwarf spheroidals, who probably have completed fewer than 10 orbits about the galaxy since their formation. Conditions today are probably an unknown mixture of initial conditions and tidal effects, and hence tidal radii of these distant systems can not be used as probes of the galactic potential.
3. Something like 50 cluster orbits will be necessary for a state of tidal relaxation. Thus, only the inner clusters will have their outer parts tidally relaxed, the outer halo systems will still reflect (to some level) their initial conditions.
4. There was a slight preference for stars with retrograde orbits about the cluster (compared to the direction of the cluster's orbital angular momentum vector) to be more stable than stars in a direct orbit. However, the effect is not going to be large enough to set up a net retrograde rotation of the outer half of a cluster. In Omega Centauri, the one cluster where the kinematics has been followed beyond half the tidal radius (Freeman et. al. 1984), no rotation is observed.

It must be emphasized that my calculations were not a true N body simulation of a self gravitating system, and therefore can be applied only to very centrally condensed systems such as globular clusters where tidal effects will be negligible in the center. Dwarf galaxies will have to be followed with true N body codes.

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## DISCUSSION

OSTRIKER: There is a possible explanation for the correlation you pointed out between radius and central relaxation time based on work done at Princeton several years ago. As galactocentric radius decreases, the effects of tidal shocks increase and evolution of the core to larger densities and shorter relaxation times becomes more rapid. This is due to depopulation of orbits at intermediate energies by tidal phenomena which speeds diffusion from the center to these intermediate regions.

WHITE: I'd like to comment on two points. I think that for many clusters it may be important to include two-body relaxation effects in a discussion of limiting radii since their core relaxation times are short enough that significant repopulation of a depleted halo may occur within one or two orbits about the galaxy. I'd also like to reemphasize that it may take a number of orbits to establish a tidal cut-off in binding energy. I have carried out some self-consistent simulations of the Milky Way - Draco system. These simulations show that in a single eccentric encounter, tides do not impose a cut-off in binding energy - many weakly bound stars survive while a number of much more strongly bound stars are lost. Whether a star is lost or not clearly depends on its orbital phase and orientation as well as on its binding energy. This suggests that it may be extremely risky to use the tidal radii of dwarf spheroidals to estimate their mass. If I try to apply this technique to my simulations of Draco I find that the present profile of the model shows no evident limiting radius and that attempts to derive such a radius using standard observational techniques can lead to overestimates of the mass by factors as large as 500. I urge the N-body people to run more self consistent simulations of loosely bound systems.

SHAPIRO: Do I gather, then, that estimates of dwarf spheroidal galaxy masses from measurements of their tidal cut-off radii are completely invalid?

SEITZER: I do not put much faith in these, due to both the observational errors and the fact that these clusters may not have reached a tidally relaxed state.

COHN: Concerning your final figure, which relaxation time was plotted - the core relaxation time or the half-mass relaxation time?

SEITZER: The central relaxation time.

TREMAINE: If the cluster is on a circular orbit the tidal radius is located at the Lagrange points. In the elliptic restricted three-body problem the Lagrange points move closer or further from the cluster as it moves closer or further from the galactic center, keeping at a fixed fraction of the distance from the galactic center. This suggests that the tidal radius may be a fixed fraction of the present distance rather than the pericenter distance. At what phase in the cluster orbit did you measure the tidal radius and would you get the same answer if you measured it at a different phase of the orbit?

SEITZER: I measured the density profiles at apogalacticon, and found that the observed limiting radius is not too far from the calculated radius at perigalacticon for tightly bound systems. For loosely bound system (dwarf galaxies), this may not be true.

KING: I would like to emphasize your point that many of the "observed" limiting radii are rather weak. When Peterson and I determined them, we began with the cases where the models were a good fit and the observations were good too. But then we went on and estimated a limiting radius for any cluster where we thought it could be done at all. These should surely be redone from better observational material that can be obtained today.

SEMENZATO: 1) What is the agreement of your results with those of Keenan's numerical experiments? 2) I have seen reports of evidence of observations of retrograde rotation in M13 and  $\omega$  Cen. Can you explain this rotation with only the slight surplus of retrograde orbit stars that you get from your numerical simulations?

SEITZER: 1) My models produced nearly the same results as those of Keenan. However, Keenan would say that because there are unstable orbits at  $r < r_t$ , the observed limiting radius will be less than the collinear Lagrangian radius. But the King models do not have all possible orbits in phase space at  $r < r_t$ . Considerable volumes of phase space are not inhabited in a spherically symmetric King model at  $r < r_t$ : the observed cut-off is very gradual with continuous derivatives. 2) Rotation in M13 and  $\omega$  Cen is almost certainly due to initial conditions, since tidal forces do not penetrate very far into the cluster.

FREEMAN: Seitzer, White and van Albada have all pointed out that the tidal radius of a satellite or globular cluster takes several galactic orbits to be established. There is at least one situation where this is not so. The young globular clusters in the LMC have ages that are only a fraction of their orbital time around the LMC. However, the masses that one estimates for these clusters from their tidal radii are in excellent agreement with the masses estimated from their measured luminosity functions and from a recent direct measurement of velocity dispersion (for one somewhat older cluster). The tidal estimates work because these clusters have nearly circular orbits: the tidal radii are probably associated more with the clusters' *formation* than with the subsequent dynamical evolution. If we knew that the dwarf spheroidal satellites of our galaxy were also born in low eccentricity orbits, then their tidally estimated masses could also be reliable, despite the low number of galactic orbits that they have made.

SEITZER: Agreed. A knowledge of initial conditions when clusters and dwarf galaxies formed is essential to understand their current tidal state.