Sourav Tarafder, ***Non-Classical Set Theories and Logics Associated With Them***, University of Calcutta, India, 2017. Supervised by Mihir Kr. Chakraborty and Benedikt Löwe. MSC: 03E70, 03B53, 03E40, 03B50. Keywords: paraconsistent logic, set theory, algebra-valued models, ordinal numbers.

Abstract

The theory of *algebra-valued models of set theory* was initiated in the 1960s by Dana Scott, Robert M. Solovay, and Petr Vopěnka. They took a model of set theory V and a Boolean algebra $\mathbb{B}$ to construct a new algebra-valued model of set theory $V^{\mathbb{B}}$. If the algebra is a Boolean algebra, this model will be a model of classical set theory ZFC.

If the algebra used is not a Boolean algebra, then the resulting model can be a model of nonclassical set theory. This was first done by [1] with Heyting algebras to construct models of intuitionistic set theory and later by Takeuti, Titani, Kozawa, and Ozawa for various lattices to obtain models of quantum set theories

In this thesis, we generalise this approach to *deductive reasonable implication algebras* and show that $V^{\mathbb{A}}$ becomes an algebra-valued model of the some or all of the axioms of the *negation-free fragment* of ZFC (cf. also [2]).

We also study a particular example of such an algebra, the three-valued matrix PS_3 which gives a semantics of the paraconsistent logic $\mathbb{L}\mathrm{PS}_3$ (i.e., $\mathbb{L}\mathrm{PS}_3$ is sound and complete with respect to PS_3), and show that $\mathbb{L}\mathrm{PS}_3$ is a maximal paraconsistent logic relative to classical logic (cf. also [4]).

Combining these two results, we obtain an algebra-valued model V^{PS_3} which is a model of paraconsistent set theory considerably different from other paraconsistent set theories that have been proposed. In particular the axiom scheme of comprehension remains invalid in this model.

We study the properties of the set theory validated in V^{PS_3}. Its paraconsistency is closely related to the fact that the set theory violates Leibniz's law of indiscernibility of identicals, i.e., being equal does not enforce that all properties are shared. We study the representation of natural numbers and ordinal numbers and prove that analogues of mathematical induction and Cantor's theorem are valid in V^{PS_3} (cf. also [3]).

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