AN ANALYTICAL STUDY OF THE STRUCTURE OF TWO-DIMENSIONAL MAGNETOSTATIC EQUILIBRIA IN THE PRESENCE OF GRAVITY

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1 - INTRODUCTION

In order to understand how prominences are supported in the solar corona, we have recently developed several 2D models describing x-invariant equilibrium configurations of a plasma occupying a half-space $(z > 0)$ and submitted to a magnetic field then of the form

$$\underline{B}(y,z) = B_x(y,z)\,\hat{x} + \nabla A(y,z) \times \hat{x}, \qquad (1)$$

and to a vertical gravitational field

$$\underline{g} = -g\hat{z} \qquad (g > 0). \qquad (2)$$

We report here some results concerning three of these models, which are based on different assumptions about the way matter is spatially distributed. The details of the calculations and references to the current literature on the subject may be found in Amari (1988), and in Amari and Aly (1988a,b,c)

2 - MODEL 1: EQUILIBRIUM OF A MASSIVE FILAMENT IN A CONSTANT-α FORCE-FREE FIELD

2.1 - Assumptions

i) the system is confined between the two planes $\{y = \pm L/2\}$ (or equivalently, is assumed to be y-periodic) ;

ii) all the plasma is condensed into a massive filament (density of mass λ per unit of length), located along the line $\{-\infty < x < +\infty \,;\, y = 0 \,;\, z = h\}$, in which an electric current of given intensity I $(I > 0)$ is flowing either parallelly $(\epsilon = +1)$ or antiparallelly $(\epsilon = -1)$ to the x-axis ;

iii) the magnetic field outside the filament is a constant-α force-free field: then $B_x = \alpha A$ (α given) and A obeys the equation

$$-\Delta A = \alpha^2 A + (4\pi\epsilon I/c)\,\delta(y)\,\delta(z-h) \qquad (3)$$

iv) $A(y,z)$ satisfies the boundary conditions ($B_0 > 0$ is a constant)

$$A(y,0) = B_0 L \cos(\pi y/L) \qquad \text{for } -L/2 < y < +L/2 \qquad (4a)$$

$$A(\pm L/2,\, z) = 0 \quad ; \quad \lim_{z\to\infty} A(y,z) = 0 \qquad (4b)$$

i.e. the normal component of $\underline{B}$ is fixed on the boundary of the domain occupied by the plasma. Conditions (4) can be satisfied only if $0 \leq |\alpha| < \pi/L$;

v) the filament is in equilibrium (the gravity force being balanced by the Lorentz force) and then h is determined by solving

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$$4\pi I^2 \sum_{p=o}^{\infty} e^{2\gamma_{2p+1}h} - \epsilon I B_o L^2 c\gamma_1 e^{\gamma_1 h} = \lambda g L c^2 \qquad (5)$$

where $\gamma_n^2 = n^2\pi^2/L^2 - \alpha^2$.

2.2 - Results

i) Equ. (5) admits one and only one solution for α ($0 \leq |\alpha| < \pi/L$) and I ($0 < I < \infty$) fixed (when $\epsilon = -1$, one needs to assume $\lambda \neq 0$) ; when I (or $|\alpha|$) increases, our model does not exhibit any non-equilibrium phenomenon of the Kuperus-Van Trend (1981) type, with a loss of equilibrium of the filament and its eruption outwards. It should be noted, however, that such an effect is recovered if a more complex expression for A(y,o) is assumed (Démoulin and Priest, 1988) ;

ii) h is a strictly increasing function of I - with $\lim_{I\to\infty} h = \infty$ - and a strictly decreasing function of λ ;

iii) for $\epsilon = +1$, h is a strictly increasing function of $|\alpha|$ - with $\lim_{|\alpha|\to\pi/L} h = \infty$ - ; for $\epsilon = -1$, h increases with $|\alpha|$ if λ is smaller than some critical value $\lambda_c(|\alpha|,I)$, while it decreases with $|\alpha|$ if $\lambda > \lambda_c(|\alpha|,I)$;

iv) the topology of the projection of the field lines onto the plane {x = 0} is as shown on Figure 1.

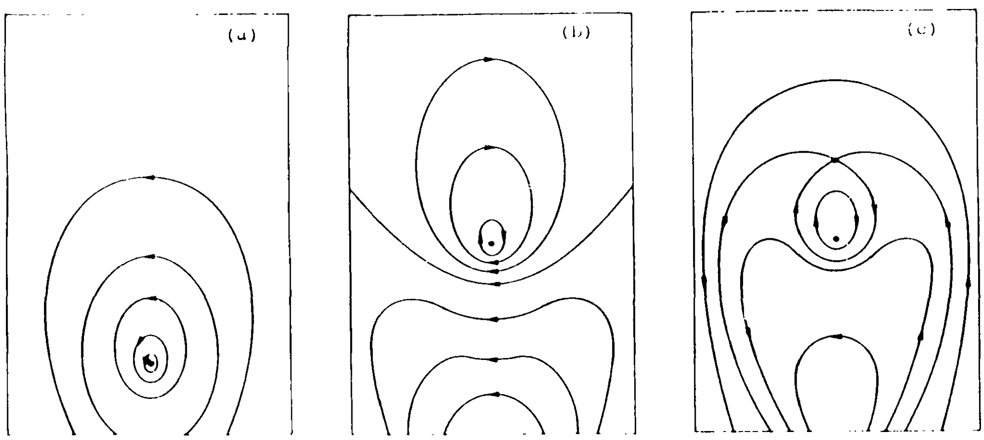

Figure 1: Topology of the lines in {x = 0}
a) $\epsilon = +1$;
b) $\epsilon = -1$, $\lambda < \lambda_c'(|\alpha|,I)$, with λ_c' a computable critical value;
c) $\epsilon = -1$, $\lambda > \lambda_c'(|\alpha|,I)$.

3 - MODEL 2: EQUILIBRIUM OF A MASSIVE CURRENT SHEET IN A CONSTANT-α FORCE-FREE FIELD

3.1 - Assumptions

i) the system is confined between the two planes {y = ±L/2} ;

ii) all the plasma is condensed into a massive current sheet (density σ per unit of surface), located along the strip $\{-\infty < x < +\infty$; $y = 0$; $a < z < b\}$, in which an electric surface current of density $\underset{\sim}{j} = (6\epsilon I/h^3)(b-z)$

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$(z-a)\hat{x}$ is flowing ($\epsilon = \pm 1$; $I > 0$; $h = b-a$) ;

iii) the magnetic field outside the sheet is a constant-α force-free field : $B_x = \alpha A$ and A obeys ($X^+ = \max(X,0)$):

$$-\Delta A = \alpha^2 A + (24\pi\epsilon I/h^3 c)(z-a)^+(b-z)^+\delta(y) \qquad (6)$$

iv) A satisfies the boundary conditions (4) ;

v) the sheet is in equilibrium, i.e.

$$2\pi\sigma g = (B_y B_z)(0^+, z) \qquad \text{for } a < z < b \qquad (7)$$

In this model, actually, Equ. (7) is used to compute a value of σ consistent with equilibrium when $\underset{\sim}{B}$ corresponding to $(a,b,\epsilon I,\alpha)$ has been determined by solving (6) ; of course, one must have $\sigma \geqslant 0$ and this condition determines the existence or non-existence of an equilibrium associated with given values of these parameters.

3.2 - Results

Equation (6) may be solved analytically for A. Reporting the values of the resulting magnetic field into (7), one then finds that:

i) for $\epsilon = +1$, the sheet cannot be in equilibrium ;

ii) for $\epsilon = -1$, the sheet can be in equilibrium only if $I < I_c(a,b,|\alpha|)$, where I_c is a computable critical value for I ; a consequence of this result is the existence of a maximal mass $m_c(a,b,|\alpha|)$ which can be supported by the field when a, b, and $|\alpha|$ are fixed.

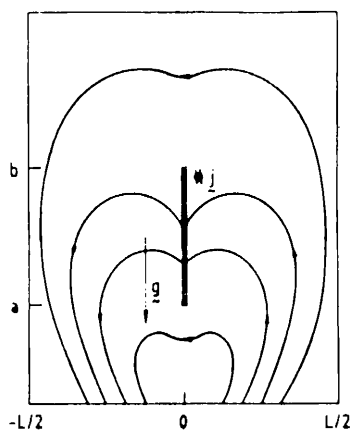

Figure 2: See text.

We have also studied the topology of the field lines in the plane $\{x = 0\}$ and shown that it is of the type shown on Figure 2 ("Kippenhahn-Schlüter" type topology) if $I < I_c'(a,b,|\alpha|) < I_c(a,b,|\alpha|)$, while it becomes more complex if $I_c' < I < I_c$, in which case neutral points (where $\nabla A = 0$) do appear in the configuration.

4 - MODEL 3: EQUILIBRIUM OF AN ISOTHERMAL PLASMA IN $\{z > 0\}$

4.1 - Assumptions

i) the magnetic field is shearless: $B_x = 0$ (the case $B_x \neq 0$ is currently under study) ;

ii) A (and then B_z) is fixed on the boundary $\{z = 0\}$: $A(y,o) = g(y)$, where g is a given non-negative function tending to zero at infinity ;

iii) the plasma is isothermal: $T = T_o$ = given constant ; then, at

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