Instability, Chaos and Predictability
in Celestial Mechanics and
Stellar Dynamics

ISBN 1-56072-054-9

UNSTABLE TRIPLE SYSTEMS – ESCAPE FROM INITIAL CONFIGURATIONS

J.P. Anosova
St. Petersburg State University, Russia

ABSTRACT

The statistical material obtained at the Leningrad observatory amounts to about $3{,}10^4$ tripe systems with negative total energy (Anosova and Orlov, 1985; Anosova 1986). From this material the basic qualitative results have been summarized.

1) In the majority of cases (∿95%), the dynamical evolution was completed by escape. Escape always arises from a close triple approach of the bodies, and the moment of escape has been assumed to coincide with the minimum triangular perimeter.

The most effective triple approaches are those in which a temporary binary is first formed, whereupon the ejected body returns for a favourable interaction with the two binary components. This is a fly-by interaction.During the triple approach, the trajectory of the escaping body is often nearly rectilinear and the three-body configuration takes the form of an isosceles triangle in the equal-mass case. Synchronization of motions also has an effect on triple approaches of "fly-by" - type.Thus, if a single component passes through the centre of mass of the system just as the two other bodies are approaching, its velocity is decreased and escape does not occur, whereas escape is promoted if the binary components are expanding at the moment of closest approach. Escape is more probable when this passage is closer. In those triple approaches which are separated by a sequence of exchanges, escape from an equal-mass system seldom takes place.

2) The life-time tends to increase with increasing angular momentum, especially by the appearance of a small proportion

(∿ 5%) of hierarchical triple configurations which are stable over long times. Triple approaches are absent in such systems, and the motion can be described by the superposition of two perturbed Keplerian orbits during intervals $t > 10000$ crossing times. Application of stability criteria (Szebehely and Zare, 1977; Harrington 1972; Goldbev 1967) to these systems confirms this result.

A convenient method for generating initial configurations is achieved in the following way (Agekian and Anosova, 1967, 1968). The components A and B of the triple system shown in Figure 1 are placed at the points (-0.5, 0) and (0.5, 0) in the Cartesian coordinate system ξ, η. A circle of unit radius centred on (-0.5, 0) is then drawn. The initial position of the third component C are distributed with uniform probability within the positive quadrant ($\eta \geq 0$, $\eta \geq 0$). In this way, all possible configurations of the triple system with equal masses are sampled. If the components have different masses, it is also necessary to take permutations of the positions A, B, and C, thereby obtaining six initial states each time.

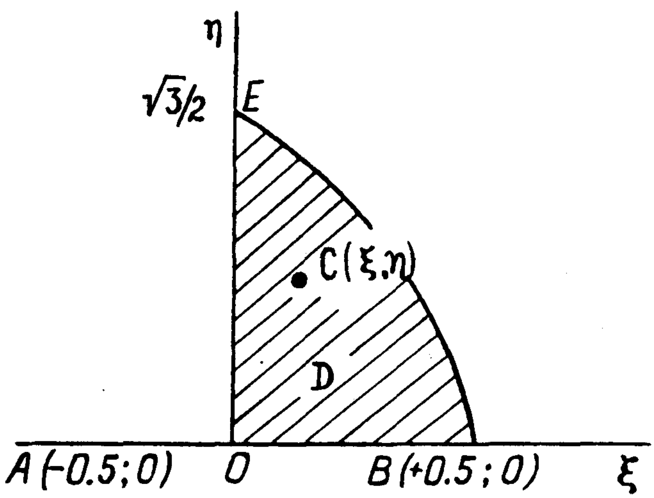

Figure 1

3) Small changes of initial conditions can lead in some cases to widely different outcome. It has been shown that the region D of all possible configurations breaks up into a number of small islands s, inside which the escape time T is a continuous function of the initial coordinates. At the boundaries of s the function $T(\xi, \eta)$ has a discontinuity such that $T(\xi, \eta) \to \infty$. During a crossing of the boundary, the number of triple approaches $n(\xi, \eta)$ may change by an arbitrary value.

Moreover, the islands s may consist of subregions s^x inside which the function $n(\xi,\eta)$ is constant. When crossing the boundary of s^x, the function $T(\xi,\eta)$ changes continuously and $n(\xi,\eta)$ changes by 1. Figure 2 presents the function $T(\xi,\eta)$ for the rotating triple systems with equal-mass components. The initial positions of component C are indicated and the numbers shown correspond to the time of escape. This figure shows that the average escape time increases for a increasing degree of hierarchical structure, as measured by the initial configuration parameter $q = \rho/r_{min}$ where r_{min} is the distance between the closest components, and ρ is the distance from the distant body to the centre of mass of the two other bodies. The region D^x in Fig. 2 is the region of the initial configurations of the stable hierarchical rotating triple systems.

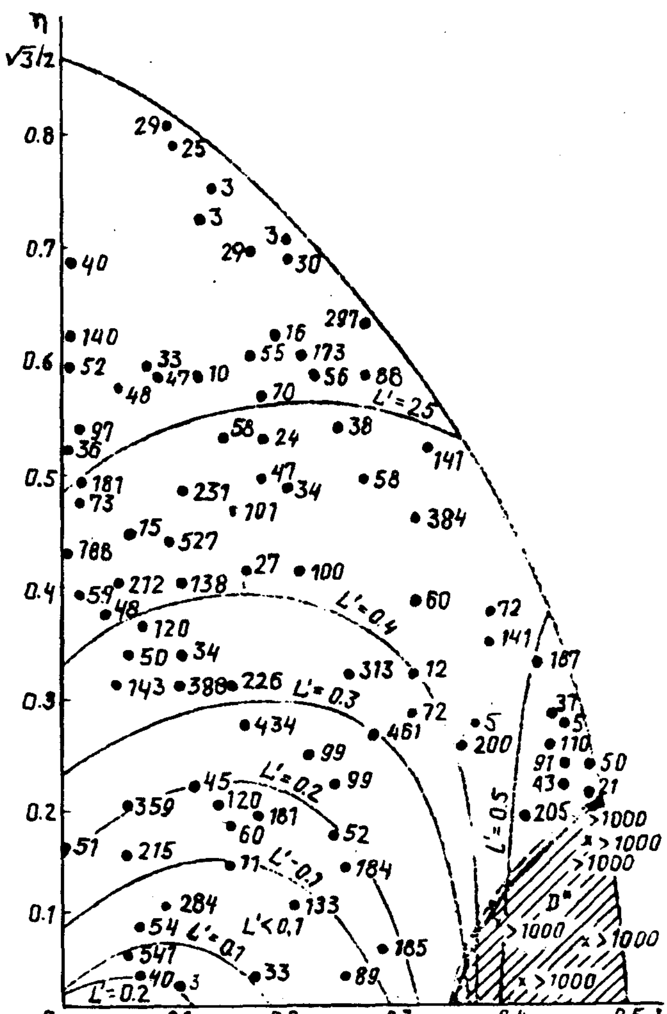

Figure 2

https://doi.org/10.1017/S0252921100066161 Published online by Cambridge University Press

4) The Figure 3 shows the subregions D_s (with s = 1,2,3,4, 5) inside the region D of all possible initial configurations which lead to escape after the first triple approach of the bodies. These regions D_s can be characterized as follows:

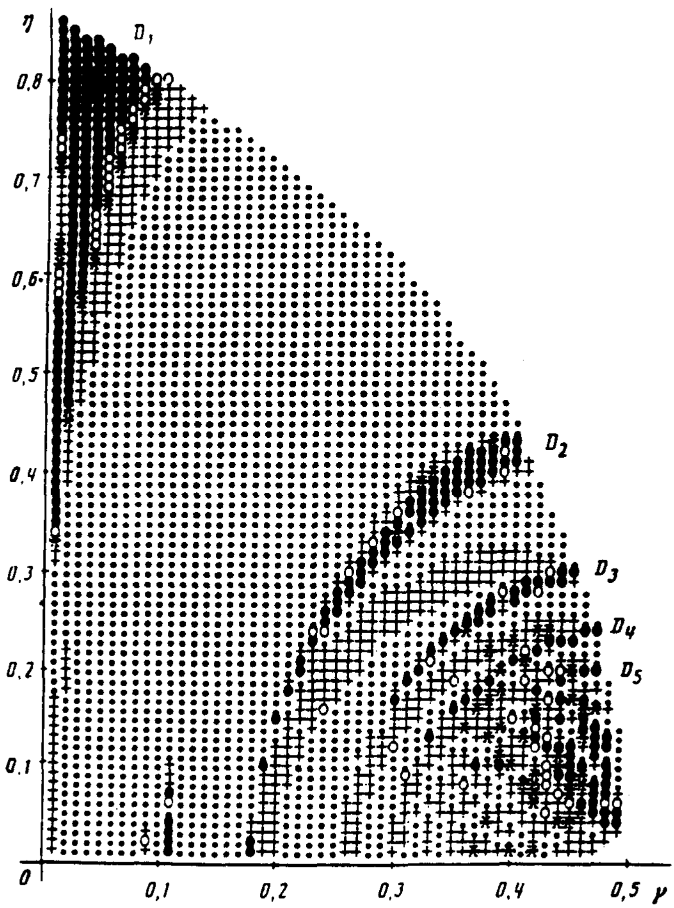

Figure 3

(1) The "axis" of the regions D_s are the contours

$$x = r_{AC}/r_{BC} = U_{BC}/U_{AC} = \text{const.},$$

for which ratio of interaction forces (and the ratio of relative potential energies U_{AC} and U_{BC}) from the two first bodies A and B to the third body C is equal.

https://doi.org/10.1017/S0252921100066161 Published online by Cambridge University Press

(2) On the contours of the regions D_s the values that are approximately equal to an integer (1, 5,10,16,23).

(3) The boundaries of the regions D_s correspond to very distant ejections resulting from the first triple approach.

(4) In the regions D_s with increasing value of x, the number n^x of double approaches of the bodies B and C in a temporary binary before the triple approach also increases; for x = I, n^x = 0, and for x = 2, n^x = 1, etc. Thus, the time of escape increases for larger values of x.

(5) The regions D_s have a complicated structure; they are almost symmetrical relative to the contours but different bodies escape from each side; always a component A escapes from the right side, a component B from the left, and a component C escapes from a narrow strip along the contours.

(6) On escape of components A or B, the centre of mass of the other two bodies is only approached once, whereas component C escapes after two approaches (one weak, one strong) to the temporary binary AB.

(7) The orbits of the bodies during a triple approach is determined by the initial configuration. Thus, for hierarchical systems near an isosceles triangle with a base r_{AC} or r_{BC}, the only passage of component A or B takes place through the centre of mass of the other two bodies. On the other hand, for configurations near an isosceles triangle with a base r_{AB} greater than r_{AC} (or r_{BC}), two passages take place, one slow and the other a rapid fly-by.

(8) The basis result of this study is that a determining parameter for the course of the force dynamical evolution in these triple systems is given by the value of the ratio $x = r_{AC}/r_{BC} = U_{BC}/U_{AC}$.

REFERENCES

[1] Agekian, T.A. and Anosova, J.P. (1967), Astron. Zh.44, 1261, Soviet Astron.2, 1006.

[2] Agekian, T.A. and Anosova, J.P. (1968), Astrofizika 4,31.

[3] Anosova, J.P. and Orlov, V.V. (1985), Trans. Astron.Obs. Leningr. Univ.40, 65.

[4] Anosova, J.P. (1986), Astrophys. Space Sci.,124,217-