Why do we study the history of mathematics? The late Ivor Grattan-Guinness identified the crucial point, summarised here by Fraser and Schroter:

> [There is] a disjunction between *heritage* (our tracking of a particular concept's journey along the "royal road" from the past to the present) and *history* (our attempt to explain why a certain mathematical development happened). The "heritage approach" evaluates past mathematics in the light of recent theories, looking for similarities that reveal the gradual unveiling of a mathematical concept. Conversely, "history" instinctively looks for differences and discontinuities.

Guicciardini and his team ask key questions and offer some answers. Although there may be a slight risk of emulating the centipede which, asked which foot was being moved, became unable to move at all, anyone writing, or seriously interested in, the history of mathematics should read this important book.

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Abstract algebra, a comprehensive introduction by John W. Lawrence and Frank A. Zorzitto, pp. 619, £64.99 (hard), ISBN 978-1-10883-665-4, Cambridge University Press (2021)

This book is aimed at "senior undergraduate students" and "those more gifted in mathematics", as well as "beginning graduate students who need a refresher". It assumes knowledge of injections and surjections, elementary matrix properties and linear transformations, but it starts with a "refresher" chapter on basic number theory, up to Fermat's Little Theorem. There are then two chapters on groups; the first goes as far as quotient groups and external and internal products, the second covers Cauchy's and Sylow's Theorems and chains of solvable groups. Chapter 4 covers rings, including maximal ideals; Chapter 5, on primes and unique factorisation, is largely concerned with Noetherian domains, and Chapters 6 and 7 are on Galois theory, from algebraic field extensions to the insolvability of the general quintic, with a mention of the inverse Galois problem. The last two chapters cover principal ideal domains and division algorithms, ending with an extensive treatment of Gröbner bases. An appendix discusses infinite sets, including Zorn's lemma, cardinality and the algebraic closure of a field. The subtitle 'comprehensive introduction' is indeed accurate.

The treatment is concise and rigorous but approachable in style, with plenty of helpful advice and motivation. For example, from the introduction to the section on Cauchy's and Sylow's theorems:

> If m is the power of a single prime and m divides the order of the group, subgroups of order m will exist, and quite a bit can be said about them. That is what the upcoming results are about. It takes quite a bit of slogging to work through the ensuing ideas, but the reward will be a more profound understanding of finite groups.

Another characteristic of the authors is to write proofs formally in content but not in style, often using constructions such as "Well, …": "What are the conjugates of τ? Well, the group relations yield $\sigma\tau\sigma^{-1} = \sigma^2\tau$." This approach is very user-friendly, although readers need to get used to the fact that when the authors say that something 'seems' to be the case, they are asserting it, and not indicating that it is a false impression. There are plenty of examples given, although perhaps not always soon enough after the definitions – for instance, kernels and automorphisms.

The introduction to groups uses mainly permutations, with cycle notation, although there are several geometric examples. An unusual feature is that, at the end of the second chapter on groups, twenty pages are devoted to the group theory involved in breaking the Enigma codes, centred around "Rejewski factorisation" of a permutation σ of $2n$ letters into a pair of permutations $\alpha\beta$ of products of disjoint transpositions. Other topics given more space than one might expect in a first course include semi-direct products and Buchberger's algorithm.

I took the opportunity to work in detail through two of the chapters. I found the explanations clear and the material well arranged to help understanding. For example, group actions, much used in later sections, are introduced carefully, with a clear progression from a homomorphism from one group to the permutation group of a set, to action on cosets, before orbits and stabilisers are defined. There is then a substantial section devoted to what is first called "the Cauchy-Frobenius-Burnside formula", but then just "Burnside's formula", and a whole exercise of combinatorial examples. (Each main section of each chapter ends with a substantial exercise; answers are not provided.) The authors are based at the University of Waterloo, Ontario; their experience, and care for the needs, of learners are admirably apparent, and they avoid the pitfalls of some other recent texts. For instance, they spell out the need to show that the product of quotient groups is well defined, before proving it. Another example of careful exposition comes in the proof that all 3-cycles are mutually conjugate in A_n (for $n \geqslant 5$): "It is not enough to say that, since all 3-cycles have identical orbit structure $[3,\ 1,\ 1,\ \dots\ ,1]$, then they are mutually conjugate by Proposition 2.46. The conjugation must happen inside A_n." I found few typos.

Students for whom concepts such as quotient groups are hard may need a slower initiation. Otherwise this is an admirable treatment of its subject, perhaps the best since Fraleigh [1]. For those who can cope with the quantity of material, this is an excellent book for learning, and it can be recommended very highly.

References

1. John B. Fraleigh, *A first course in abstract algebra*, Addison-Wesley Publishing Company, Inc, 1966.

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Series and products in the development of mathematics (second edition) by Ranjan Roy, Vol. 1 pp. 776, £69.99 (paperback), ISBN 978-1-10870-945-3; Vol. 2 pp. 476, £45.99 (paperback), ISBN 978-1-10870-937-8, Cambridge University Press (2021)

As the late Ranjan Roy (1947-2020), already well-known as a co-author of *Special Functions* [1], writes in his preface, the book is essentially an updated edition of his *Sources in the Development of Mathematics* (2011). However, in addition to the material from the first edition, 250 pages of new material have been added, making it a gigantic two-volume work of over 1200 pages. Series and products have played a fundamental role in the development of both pure and applied mathematics. But the title gives only an inadequate idea of its scope. It is