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Extending the reach of Lagrangian analysis in turbulence

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The hypothesis in the classical Kolmogorov picture of turbulence with perhaps the most far-reaching consequences is that of universality, the notion that small-scale turbulence dynamics are independent of the way the turbulence was generated. The assumption of universality can be evaluated by comparing measurements taken in many kinds of flows. However, up to now the range of flows that can be used to study universality from a Lagrangian viewpoint has been highly constrained, because large-scale Eulerian inhomogeneity manifests as Lagrangian non-stationarity. The recent work of Viggiano *et al.* (*J. Fluid Mech.*, vol. 918, 2021, A25) significantly extends this range by showing how the dynamics along Lagrangian trajectories can be continuously renormalised using local Eulerian scales, at least in flows whose development is self-similar. They demonstrate their results on a turbulent jet, a classical flow that is well studied from the Eulerian perspective, though not in a Lagrangian sense. Their work provides an exciting roadmap for expanding the scope of Lagrangian analysis of turbulent flows.

Key words: turbulent flows, jets, turbulent mixing

1. Introduction

Turbulence remains one of the most challenging problems in physics and engineering, largely owing to its vast complexity. Grasping the full multi-scale structure of its violently fluctuating but not fully random dynamical properties is daunting. Thus, it has proved useful to examine turbulence from as many perspectives as possible. Such perspectives can take many forms. We can take a statistical approach to turbulence in the spirit of Kolmogorov (Frisch 1996), for example, or instead consider it as the outcome of the interaction of dynamically generated coherent structures (Hussain 1986). We can measure the spatial structure of turbulence in an Eulerian approach, or instead consider the spatiotemporal dynamics of fluid elements from a Lagrangian perspective. Finally, we can also study turbulence generated in different ways and in different geometries, to try to tease out common features that are independent of the driving forces or boundary conditions. This last aspect is often underappreciated, but its importance for understanding turbulence should not be discounted; before we can agree on what turbulence is, we need to be sure we are all studying the same phenomenon.

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Comparing measurements taken in different kinds of turbulent flows is necessary for assessing the hypothesis of universality, namely the notion (as introduced by Kolmogorov 1941) that, at least at small scales and at high Reynolds numbers, the statistical properties of all turbulent flows are the same. Although it does not have the same clear practical utility as the other Kolmogorov hypotheses and cannot directly be used to predict scaling laws, the assumption of universality has far-reaching consequences. Any hope of developing a single theory or model of turbulence ultimately rests on universality. Like the vast majority of empirical work in turbulence, universality has primarily been investigated from an Eulerian perspective. Comparisons of various turbulence properties such as the Kolmogorov constant in the energy spectrum (Sreenivasan 1995) or the scaling exponents of the Eulerian velocity structure functions (Arnèodo *et al.* 1996) across many different kinds of turbulent flows largely appear to support universality, at least within measurement and statistical uncertainty. More subtle investigations, however, have suggested that universality is not perfect, and that instead, the structure of the large scales may indeed affect the small-scale turbulence properties (Blum *et al.* 2011).

We might expect to see stronger signatures of the possible breakdown of universality if we take a Lagrangian viewpoint and consider the properties of turbulence along the trajectories of fluid elements. In doing so, we gain information not just on the structure of the flow field but also on how fluid elements sample it; this additional facet builds in dynamics in an essential way. It is reasonable to conjecture that distinctions in the dynamical process of energy passing from large to small scales in different flows may be more evident by explicitly incorporating such dynamics into our thinking. As a practical matter, however, taking a Lagrangian approach introduces its own challenges, and to date the range of flows that have been investigated from a Lagrangian standpoint has been highly restricted. Compendia of Lagrangian statistics are limited to a few types of low-mean-flow experimental set-ups and direct numerical simulations of homogeneous isotropic turbulence in periodic boxes (Arnèodo *et al.* 2008).

One of the primary reasons for these limitations is the difficulty in dealing with inhomogeneity or spatial development of the flow field, both of which necessarily imply a lack of statistical stationarity along trajectories. As a fluid element moves downstream in a turbulent jet, for example, it samples a flow for which the scales are ever-changing owing simply to the development of the large scales and not to the turbulence dynamics. This may seem an insurmountable problem. But now, drawing on old but largely untested ideas from Batchelor (1957), Viggiano *et al.* (2021) has given us a roadmap for how we might extend Lagrangian analyses to a large new class of turbulent flows: those that are inhomogeneous but self-similar, including jets, wakes, mixing layers and so forth. And along the way, Viggiano *et al.* (2021) also give us some of the first precise measurements of several key Lagrangian turbulence quantities in such flows.

2. Overview

Batchelor (1957) specifically considered how one might modify Taylor's celebrated theory of turbulent diffusion (Taylor 1922) to handle free shear flows. Taylor linked the displacement of fluid elements leaving a point source to the Lagrangian two-point correlation function (or equivalently, and perhaps more saliently from the standpoint of small-scale turbulence dynamics, the second-order Lagrangian structure function). Both the short- and long-time limiting behaviour of turbulent diffusion from this point source can then be captured knowing only the velocity variance and the Lagrangian correlation time, with no need for any further details of the turbulence. Taylor's theory, however, requires that the Lagrangian dynamics be stationary; if the properties of the velocity field change along trajectories, then his arguments do not hold. Stationarity in turn requires statistical homogeneity, so that as fluid elements move away from their source, they keep sampling realisations of the same velocity field. Unfortunately, despite the ubiquity of homogeneous flows in academic studies of turbulence, a vast range of realistic flows (including many where one might want to use Taylor's theory) are not homogeneous.

Batchelor's insight was to conjecture that the problems introduced by inhomogeneity are not necessarily insurmountable, as long as the flow field that the fluid elements experience as they move along their trajectories changes in a known and predictable way. In that case, velocity and time can be continually renormalised along the trajectories so that the (modified) random process representing the fluid-element velocity becomes stationary. Batchelor (1957) reasoned that turbulent free shear flows such as jets, wakes and mixing layers are ideal cases where this renormalisation process can be accomplished, for two reasons. First, such flows typically have a significant mean velocity, meaning that fluid elements are dominantly advected downstream in a predictable way. And second, they develop self-similarly (Pope 2000), so that the turbulent fluctuations at any downstream location can be mapped on to those at any other location by a known rescaling. Appealingly, one only needs to know the Eulerian properties of the flow, such as the local velocity variance and correlation time, which are well characterised, to implement this rescaling.

Elegant though these ideas are, they have been subjected to very little direct testing (as is unfortunately common for Lagrangian theories, given the historical challenges associated with Lagrangian measurements (Yeung 2002; Toschi & Bodenschatz 2009)), and therefore, as discussed above, the variety of flows used for Lagrangian studies remains highly circumscribed. To address this lack of validation, Viggiano *et al.* (2021) made measurements using Lagrangian particle tracking in a turbulent round jet so that they could directly evaluate how well Batchelor's proposed renormalisation performed in a real setting. At the same time, they generalised his work: whereas Batchelor (1957) assumed the known self-similar development of a round jet, Viggiano *et al.* (2021) simply used the directly measured Eulerian properties. Although this renormalisation was not perfect in the near field of the jet, it worked very well in the far field, where both Lagrangian structure functions and correlation functions computed for tracer particles moving through small spatial regions at different downstream locations collapsed after velocity and time were renormalised.

Viggiano *et al.* (2021) were not only able to test Batchelor's ideas, but given the success of the renormalisation procedure, they were also able to add measurements in this inhomogeneous flow environment to the compendium of Lagrangian turbulence knowledge, just as data from turbulent jets have long been used in Eulerian studies (Sreenivasan 1995; Arnèodo *et al.* 1996). In particular, they measured C_0, the Lagrangian structure function scaling constant, which is a key parameter in models of turbulent transport (Sawford 1991). Although they found C_0 values of the same order of magnitude as in previous studies, the range of reported values is still too large to rule out a systematic dependence on the large-scale features of the flow (and therefore a lack of universality). Viggiano *et al.* (2021) also explored the Lagrangian acceleration in the jet, though their measurements were affected by the finite size of their tracer particles.

3. Future

The results of Viggiano *et al.* (2021) give us a framework for how to incorporate a whole new class of flows into what we can analyse from a Lagrangian vantage point.

This represents an important step forward; without comparing the properties of turbulence generated in different ways, there is no way to assess our foundational assumptions of turbulent universality. And because Lagrangian analysis incorporates the multipoint, multiscale dynamics of turbulence in an essential way, there is reason to believe that violations of universality may be more apparent in a Lagrangian context, further underscoring the value of this work. Additional measurements of Lagrangian quantities such as C_0 and the acceleration variance scaling constant a_0 in a variety of flows are certainly needed. The values reported by Viggiano *et al.* (2021) are consistent with, though different in detail, from what has been found in other, more homogeneous flows. It remains to be seen how much of the discrepancy can be ascribed to measurement challenges and how much is due to differences in the flows themselves.

More broadly, the results of Viggiano *et al.* (2021) also represent another promising step in the longstanding quest to link the Eulerian and Lagrangian descriptions of turbulence (Borgas 1993; Chevillard *et al.* 2003; Biferale *et al.* 2004). Doing so has proved to be very challenging; yet, if we actually understood turbulence, it is something we ought to be able to do. Viggiano *et al.* (2021) have now shown us how to rescale the Lagrangian evolution with Eulerian values in self-similarly developing flows; it will be interesting to try similar ideas in more generally inhomogeneous cases.

Declaration of interests

The author reports no conflict of interest.

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