POYNTING-ROBERTSON EFFECT AND COLLISIONS IN THE INTERPLANETARY DUST CLOUD

Jan Trulsen and Arild Wikan
Institute of Mathematical and Physical Sciences
University of Tromsø, Tromsø, Norway

ABSTRACT

A simulation model has been developed to study the results of the Poynting-Robertson (PR) effect and collisions on the dynamical evolution of an interplanetary dust cloud. Fragmentational and accretional effects are neglected. With a mean free collision time of the order of the PR lifetime collisional effects become of importance. As the individual grains still spiral inwards collisions act to make the mean eccentricity and inclination of the grain orbits both decrease at comparable rates, giving rise to an expanding fan shaped dust cloud.

INTRODUCTION

Due to the solar radiation the mean orbital elements of a dust grain evolve as given by Wyatt and Whipple (1950):

$$\dot{a} = -K \frac{2 + 3e^2}{a(1-e^2)^{3/2}}$$

$$\dot{e} = -K \frac{5e}{2a^2(1-e^2)^{\frac{1}{2}}} \tag{1}$$

$$\dot{i} = 0 \ .$$

K is a constant determined by the radiated power and the size and mass of the grain. Thus, the grains spiral inwards, their orbits are getting more circular, but their orbital planes remain unchanged. Representing each dust grain by a dot in an e-i diagram, the PR effect therefore leads to the evolution shown in figure 1 b,c.

Collisions between grains will also give rise to dynamical effects (Trulsen 1976). Collisions will keep eccentricities and inclinations Rayleigh distributed:

I. Halliday and B. A. McIntosh (eds.), Solid Particles in the Solar System, 299–302.

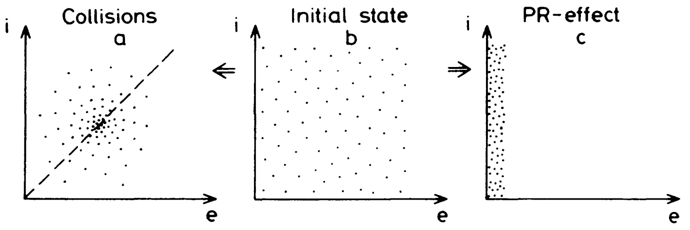

Fig. 1. PR and collisional evolutions.

$$
\begin{aligned}
f_e(e) &\sim e \exp(-E\, e^2) \\
f_i(i) &\sim i \exp(-I\, i^2) \,.
\end{aligned}
\tag{2}
$$

The establishment of these Rayleigh distributions from arbitrary initial distributions is a rapid process, taking of the order of one mean free collision time. On a slightly longer time scale collisions will also adjust these two Rayleigh distributions to each other such that the mean eccentricity and mean inclination (in radians) remain approximately equal. Returning to the e-i diagram collisions lead to the development shown in figure 1 b,a.

SIMULATION MODEL

As seen from figure 1 the PR effect and collisions taken alone lead to different results. To study their combined effect a numerical simulation model was developed by Trulsen and Wikan (1979). Some important aspects of this model are:

- A number of identical simulation particles (typically 1000) are followed in an annular region limited by a_{min} and a_{max}.
- The PR effect is included by integrating the orbital element equations (1) between successive collisions. As particles spiral inwards and past the inner boundary new particles are continuously added at the outer boundary to make a steady state situation possible.
- Collision events are prescribed through a stochastic procedure. Care has been taken to ensure that the collision rate varies in accordance with particle density and velocity distribution within the cloud. Fragmentational and accretional processes are neglected but the degree of inelasticity can be prescribed.

Each simulation is carried on until a steady state population is reached. We are interested in the form of this steady state as a function of:

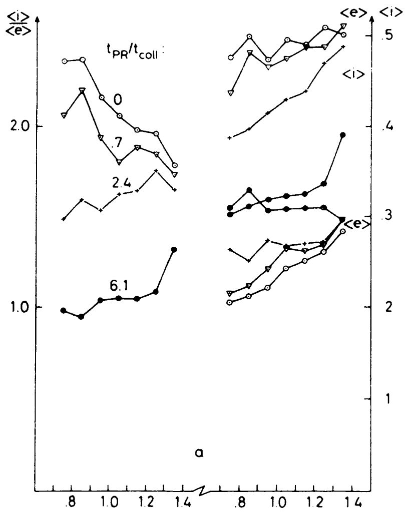

Fig. 2. Simulation results

i) the distributions of eccentricity and inclination of particles added at the outer boundary,

ii) the degree of inelasticity, and

iii) the ratio of the PR lifetime t_{PR} to the mean free collision time t_{coll}.

The latter turns out to be the most important parameter (Trulsen and Wikan 1979) so we restrict subsequent discussion to this case.

RESULTS

In figure 2 the results of 4 simulations for different t_{PR}/t_{coll}-ratios are given. Shown are the mean inclination, the mean eccentricity and the ratio of these two quantities as functions of semi-major axis.

$t_{PR}/t_{coll} = 0$ corresponds to the classical PR effect with constant mean inclination, decreasing mean eccentricity and an increasing ratio of these quantities with decreasing a. The "bumpiness" of the result is an indication of the statistical fluctuations associated with the simulation method.

With increasing t_{PR}/t_{coll}-ratio the mean inclination starts to decrease, the mean eccentricity does not decrease as fast any more and with $t_{PR}/t_{coll} \gtrsim 2$ even the ratio of these two quantities starts to decrease as we move inwards from the source region. This means that we have moved over into the collision dominated regime of figure 1a. The eccentricities and inclinations will again be Rayleigh distributed but the width of these distributions are now functions of the semi-major axis.

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Our main conclusion is, therefore, that with a mean free collision time of the order of or shorter than the PR lifetime, orbital effects due to collisions will be important. Collisions will determine the e-i distribution in the cloud, increase the PR spiralling-in rate by maintaining an increased mean eccentricity and work to maintain equal mean values of eccentricity and inclination of grain orbits.

With this t_{PR}/t_{coll} condition satisfied we can also predict the particle density distribution in the cloud to be expected. Starting from Rayleigh distributed eccentricities and inclinations the particle density as a function of radial distance r and distance z above the symmetric plane can be approximated by

$$n(r,z) \sim \frac{I(r)}{r} \exp(-I(r) \frac{z^2}{r^2}) \ . \quad (3)$$

I(r) depends on the variation of the mean orbital inclination with solar distance and will normally be an increasing function with decreasing r. We suggest that Zodiacal light data should be fitted to this expanding fan model.

REFERENCES

Trulsen, J.: 1976, in H. Elsässer and H. Fechtig (eds.) "Interpanetary Dust and Zodiacal Light", Springer-Verlag.
Trulsen, J. and Wikan A.: 1979, Institute Report, University of Tromsø.
Wyatt, W.P. and Whipple, F.L.: 1950, Astrophys. J. 14, p. 134.

DISCUSSION (Edited replies by Trulsen)

Reply to *Singer:* The angular momentum will be determined by the particle source prescribed. The result can be guessed by remembering that collisions work to equalize eccentricity and inclination, while the Poynting-Robertson effect decreases eccentricity.
Reply to *Leinert:* Result (3) is derived assuming no particle source in the region studied. I(r) is determined by the variation of the mean inclination. A contracting fan-shaped form of the outer part of the dust cloud, as indicated by zodiacal light observations, can be produced if particles are injected mainly into orbits of high eccentricity and low inclination.
Reply to *Grün:* Including a size spectrum and fragmentation may give some different results; but, on the basis of my experience with different number-conserving collision models, I do not expect drastic changes.
Reply to *Lamy:* Starting from arbitrary initial distributions, eccentricities and inclinations will be Rayleigh distributed after about one collision per particle. Steady state is reached after one or two PR lifetimes.

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