

CORRIGENDUM TO THE PAPER “NILPOTENCY OF DERIVATIONS”

BY

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An error in [1] has been kindly pointed out to the present authors by Professor Warren Dicks. The hypothesis $\partial^{2n-1}R_p \neq (0)$ should be added in both Lemma 5 and the first part of Lemma 6. Such restoration will secure the main theorem if R is either torsion free or of characteristic a prime p . However, for general semiprime ring R some change is necessary.

In the proof of Lemma 7, the last two sentences should be replaced by:

Assuming to the contrary that $\partial^{2n-1}R_p \neq (0)$, by Lemma 5(i) we obtain $\partial^{2n-1}R(\partial^{2n-1}R_p)S = (0)$ since $S = \sum R_q$ and $R_pR_q = (0)$ for $p \neq q$. This by the semiprimeness of R implies $\partial^{2n-1}R\partial^{2n-1}R_p = (0)$ and hence, by Lemma 6, $\partial^{2n-1}R_p = (0)$, a contradiction.

In the proof of the main theorem, lines 13–15 on p. 345 should be replaced by the following argument:

Let $\mathcal{S} = \{(s, t) | c \in R_p \cap [R(\partial^{2n-1}R)R] \text{ such that } c \neq 0, \partial c = 0, (\partial^s R)c = c(\partial^t R) = (0)\}$. Partially order \mathcal{S} by $(s, t) < (s', t')$ iff $s \leq s'$ and $t \leq t'$. Let (s_0, t_0) be a minimal one in \mathcal{S} and $0 \neq c_0 \in R_p \cap [R(\partial^{2n-1}R)R]$, $\partial c_0 = 0$, $(\partial^{s_0} R)c_0 = c_0(\partial^{t_0} R) = (0)$. Let $k = \sum_{i=0}^M \beta_i p^i$ be the nilpotency of ∂ on R_p where $0 \leq \beta_i < p$, $\beta_M \neq 0$ are integers (β_i 's must be not all even), and let $m = \sum_{i=0}^M [\beta_i/2] p^i$. Using the technique in the proof of Lemma 4, we have $n < k \leq 2n - 1$, $m < t_0, s_0$. Let j be the largest index with β_j odd. Then $hp^j \leq k \leq (h + 1)p^j$, where $h = \sum_{i=j}^M \beta_i p^{i-j}$ and, moreover, $\delta^{h+1}R_p = (0)$ where $\delta = \partial^{p^j}$ is a derivation of R_p . But we already know that the nilpotency of a derivation of R_p must be odd. So $\partial^{hp^j}R_p = \delta^h R_p = (0)$. Hence $k = hp^j$ and $\beta_i = 0$ for $i < j$. Now we claim $k - m < s_0$. If not, for any $x \in R$, $\delta^{(h+1)/2} x c_0 = \partial^{k-m} x c_0 = 0$. Using again the technique in the proof of Lemma 4, we obtain $\partial^k x y c_0 = \delta^h x y c_0 = 0$ for all $x, y \in R$. Since $k < 2n - 1$ and $c_0 \in R(\partial^{2n-1}R)R$, $c_0 = 0$, a contradiction. Thus $k - m < s_0$. That $\partial^k (\partial^{s_0 - (k-m+1)} x c_0 \partial^{t_0 - (m+1)} y) = 0$ yields $\partial^{s_0-1} x c_0 \partial^{t_0-1} y = 0$ for all $x, y \in R$, since

$$\binom{k}{m} \equiv \prod_{i=0}^M \binom{\beta_i}{[\frac{\beta_i}{2}]} \not\equiv 0 \pmod{p}.$$

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Let $y_0 \in R$ be such that $c_1 = c_0 \partial^{t_0-1} y_0 \neq 0$. Then $c_1 \in R_p \cap [R(\partial^{2n-1}R)R] \partial^{s_0-1} R c_1 = 0$, $c_1 \partial^{t_0} R = c_0 \partial^{t_0} ((\partial^{t_0-1} y_0)R) = (0)$ and hence $(s_0 - 1, t_0) \in \mathcal{S}$, again a contradiction.

REFERENCES

1. L. O. Chung and Jiang Luh, *Nilpotency of derivations*, *Canad. Math. Bull.* **26** (1983), pp. 341–346.

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