FILTRATION THEORY FOR OIL-WELL DRILLING FLUIDS

By

D. T. OAKES Lion Oil Company, El Dorado, Arkansas

ABSTRACT

In the oil industry it has been customary to describe water loss or filtration from bentonite drilling fluids by adaptation of Darcy's law to the filtration process. Thus, the rate of filtration varies directly with the differential pressure and permeability and inversely with the filtrate viscosity and the thickness of the deposited filter cake. Since the thickness of the filter cake is variable, however, it is necessary that the rate of deposition of the filter cake be related to the volume of filtrate. In establishing this relation certain workers have made questionable allowance for the water adsorbed on the bentonite particles with the result that the filtration equation is not valid in certain instances (Larsen, 1938; Rogers, 1953, p.268).

Other workers have indicated the significance of the adsorbed phase in their presentation of experimental techniques for relating the filter cake and the filtrate (Williams and Can· non, 1938; von Englehardt, 1954).

The filtration relationship may be derived analytically by the simultaneous use of Darcy's law and mass and volumetric balances on the filtration process. Such a derivation is here presented. The validity of the filtration relationship presented is experimentally confirmed using three groups of unlike systems-Wyoming bentonite suspensions, low-yield clay suspensions, and sodium carboxymethylcellulose suspensions-as well as four sets of clay filtration data from the literature.

INTRODUCTION

The reported studies of bentonite-water systems have, in general, been limited to the study of the relationships between a few of the properties or characteristics, particularly as they were influenced by ionic contamination. Few studies have been restricted to simple systems wherein properties have been measured as a function of the concentration. Accordingly, a study was instituted to determine all feasibly measured, fundamental properties and characteristics as a function of the single parameter, concentration of a particular bentonite.

Certain parts of this study, those dealing with the filtration of oil-well drilling fluids, are here presented. Detailed results were presented as a dissertation at The Pennsylvania State University. The aid of those faculty members who contributed to this study is gratefully acknowledged.

THEORY

General

Consider a cylindrical sample of a dispersion such as that contained in a simple, cylindrical filter press. Let the cross-sectional area of filtration be *A,* the initial depth of dispersion on the filter be L_i in centimeters, and the initial density of the sample be d_i in grams per cubic centimeter. If a volume, V (cubic

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centimeters) , of the dispersing medium is removed by filtration, the sample in the filter press decreases in mass and volume. During this process a filter cake of thickness, *h,* is accumulated.

Expressing a volumetric balance on the system from initial time to the end of the filtration:

Initial Volume $=$ Filtrate Volume + Filter-cake Volume + Volume Change $AL_i = V + Ah + \Delta V$

The volume change, ΔV , represents any slight decrease in the volume of water adsorbed on the solid particles as the concentration increases from the initial value to that of the filter cake.

Assuming a filter cake that is uniform throughout, the volume change is expressed:

$$
\Delta V = AhB = Ah[(V_b)_i - (V_b)_f]c
$$
 Equation 1

where $h =$ thickness of filter cake, cm

 $c =$ grams water per milliliter of filter cake

 V_b = specific volume of adsorbed water, cubic cm per gram

Subscripts, and $_f$ refer to the initial dispersion and filter cake, respectively.</sub> The volumetric balance is then expressed:

$$
AL_i = V + Ah(1 + B)
$$
 Equation 2

Using the same terms the mass balance may be expressed:

Initial Mass $=$ Mass Removed $+$ Mass Remaining

$$
AL_i d_i = Vd + Ahd_f
$$
 Equation 3

where d_f and d are the density of the filter cake and filtrate, respectively. Eliminating *ALi* and solving for *Ah/V:*

$$
\frac{Ah}{V} = \frac{d_i - d}{(d_f - d_i - Bd_i)} = \frac{d_i - d}{d_f - d_i (1 + B)}
$$
 Equation 4

During the filtration process the hydrated particles are brought into relatively close proximity. As they approach distances of about 100 A or less, laminar particles such as bentonite particles tend to orient so that the crystals are parallel. (See discussion to MacEwan, 1955, p. 84). Similar orientation has also been observed in the study of sedimentation volumes. Hauser (1939, p. 212) states,

If we allow particles of an anisodimensional suspension to settle out of their own accord, they will tend to pack themselves in such a way that a sediment of minimum free energy results. This means they line up with their large surfaces lying opposite to each other. The unchanged solvated layers or lyospheres will thereby act like a lubricant, permitting the particles to slide over each other until they have found their proper location. If, with increasing sedimentation, the solvated layers of the first settled particles are distorted, because of pressure exerted upon them, this can end only in making the packing somewhat denser.

Thus, the geometrical arrangement is such that water is not retained by simple, mechanical occlusion but rather is retained as an adsorbed phase, or lyosphere, in the terminology of Hauser. This adsorbed water is capable of resisting the filtration pressure which tends to extrude free water from between the laminar particles. The quantities of adsorbed water which remain will vary

somewhat depending on the rigorousness of the separation process. This is quite analogous to the distortion of the solvated layers referred to by Hauser.

It is reasonable to assume, therefore, that a bentonite filter cake contains water only in the form of adsorbed water. Referring to Equation 4, *Ah/V* represents the volume of filter cake per unit volume of filtrate, which is equal to the volume of solids plus volume of adsorbed water per unit volume of free water. Denoting this ratio by C' , Equation 4 becomes:

$$
\frac{Ah}{V} = \frac{d_i - d}{d_f - d_i (1 + B)} = \frac{\text{Volume Solids} + \text{Volume Adsorbed Water}}{\text{Volume Free Water}} = C'
$$
\nEquation 5

$$
h = \frac{C'V}{A}
$$
 Equation 5(a)

Substituting Equation $5(a)$ into the differential expression of Darcy's Law then gives:

$$
\frac{dV}{dt} = \frac{KAP}{uh} = \frac{KA^2P}{uCV}
$$
 Equation 6

For the usual conditions of filtration at constant temperature and pressure, permeability, *K,* and C' can be shown to be independent of time or time-dependent as follows: At a fixed value of water loss, \hat{V} , the flow rate dV/dt is plotted against the reciprocal of C' . If the curve is linear, either the permeability and C' are known to be independent of time, or each contains a time-dependent factor which varies identically. In either case, the linearity indicates that Equation 6 may be integrated.

$$
VdV = \frac{KA^2P}{uC'} dt
$$

$$
\frac{V^2}{2} = \frac{KA^2P}{uC'} t + \text{Constant}
$$

At zero time the volume of filtrate, V , is equal to zero so that constant of integration is equal to zero.

$$
V = \left[\frac{2KA^2Pt}{u}\right]^{1/2} \left[C'\right]^{-1/2}
$$
 Equation 7

where
$$
C' = \frac{d_i - d}{d_f - d_i (1 + B)}
$$
 Equation 5
\n
$$
B = c[(V_b)_i - (V_b)_f]
$$

and where

The term *B* represents the change in volume of the water adsorbed on the solid particles as the particles pass from the dispersion to the filter cake. It can be shown that even in hydrophilic colloids little or no change in volume occurs

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when a particle passes from the bulk dispersion to the filter cake, so that the value of *B* will approach zero. Equation 5 then reduces to:

$$
C' = \frac{d_i - d}{d_f - d_i} = \frac{\text{Volume Solids} + \text{Volume Adsorbed Water}}{\text{Volume Free Water}}
$$
 Equation 8

If this expression for *C'* is utilized, Equation 7 becomes:

$$
V = \left[\frac{2KA^2Pt}{u}\right]^{1/2} \left[\frac{d_f - d_i}{d_i - d}\right]^{1/2}
$$
 Equation 9

Thus, for a given time, pressure, and temperature the water loss should be in· versely proportional to the square root of C' and the plot should pass through the origin.

It is interesting to note that the approximation presented in Equation 10 (based on concentrations) is essentially equivalent to that presented by Will· iams and Cannon (1938) which is here cited:

A weighed sample of the mud under consideration is taken to dryness, and weight and vol· ume of residue determined. A weighed portion of the filter cake from the same mud is taken to dryness, the residue weighed, and appropriate corrections for the volume of water retained in the cake during filtration applied to the residue first determined. *v* is then taken as the volume of solid in mud, plus volume of water retained in the cake, divided by the vol· ume of water in mud, less that retained in cake.

Utilization of the Filtration Equation

The density of the filter cake, d_f , is usually not measured but may be estimated as follows. Since C' is usually shown to be independent of time and *di* and d are constants, it follows that d_f is likewise independent of time. Thus, even though the curve does not pass through the origin, a plot of *V* versus the reciprocal of the square root of C' should be linear. When the correct value of d_t is assumed, the straight line should pass through the origin.

In the event that density data are not available, the value of C' may be approximated. Assuming that the sum of the volumes of the constituents is equal to the total volume, the densities may be expressed:

$$
d_i = \frac{100.0 + C_i}{V_w(100) + VcC_i}
$$
 where V_w = specific volume of water, cc/g,
\n
$$
d_f = \frac{100.0 + C_f}{V_w(100) + VcC_f}
$$
 V_c = specific volume of clay, cc/g,
\n
$$
C_f
$$
 = filter cake concentration, grams,
\n
$$
C_i
$$
 = initial concentration, grams.

Substituting those density terms into Equation 8 and simplifying:

$$
C' = \frac{d_i - d}{d_f - d_i} = \frac{(100Vw + VcC_f) C_i}{100Vw (C_f - C_i)}
$$
 Equation 10

The concentration of the filter cake, C_f , may be estimated in the same manner as was described above for estimating density of the filter cake.

MATERIAL, APPARATUS, AND PROCEDURES

Preparation of Materials

The colloidal dispersions of bentonite in water which were used in this investigation were prepared from commercial Wyoming bentonite in quantities of each concentration sufficient for all measurements described below_ These samples were agitated vigorously with electric stirrers for approximately six hours and then placed in stoppered, glass containers. Before any measurements were made on these samples, they were allowed to hydrate at approximately 30°C for at least six months during which time they were periodically agitated.

Dispersions used in the supplemental filtration tests (sodium carboxymethylcellulose dispersions and Magcobar Xact clay dispersions) were prepared in a comparable manner but filtration tests were made approximately three weeks after mixing.

Apparatus and Procedures

Concentration.-Exact concentration of clay samples was determined by drying a weighed sample to constant weight at 130 to 135°C. All weighings were made to within 0.0002 grams on the analytical balance using calibrated weights. Concentrations are expressed in grams of solids per 100 grams of water. Unless specified to the contrary, concentrations will be simplified to the expression of concentration in grams, with the implication that the material was dispersed in 100 grams of water_

Density.-The densities of the various samples of clay dispersions and filter cakes were determined by one of the two methods described below depending on the consistency of the particular sample.

The densities of those samples which were sufficiently fluid to be placed in an accurately calibrated pycnometer (50 ml Leach specific-gravity bottle) were determined using the standard procedure.

Certain samples, namely, the higher concentrations of Xact clay and all filter cakes, could not be placed in the pycnometer without inclusion of air bubbles. Fortunately, these samples were of such a consistency that they could be formed into lumps on one end of a platinum support wire of known weight. Great care was taken in the formation of these lumps so that no air was occluded.

These lumps were weighed first in air and then in benzene of known temperature. The volumes of the lumps were calculated using Archimedes' principle of buoyancy. Presumably because of the slight mutual solubility of benzene and water, no difficulty was experienced in displacing air from the surfaces of the clay lumps.

The density of the dry bentonite was determined using the standard pycnometer procedure for measurement of the density of solids. Approximately 20 grams of dry bentonite was placed in a pycnometer and dried at 130 to 135°C for a period of five days after which it was allowed to cool to room temperature in a desiccator and was weighed with stopper and cap in place. The stopper and cap were removed and the pycnometer was partially filled with pure ben-

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zene. The resulting unstable suspension was agitated carefully for five hours to allow complete wetting of the bentonite by the benzene. The bentonite was allowed to settle until the upper layers of benzene were completely free of bentonite, after which additional benzene was added. The stopper was inserted and the pycnometer and its contents were heated to 30°C in the water bath. Excess benzene was removed from the overflow capillary which was sealed with its cap. The pycnometer was washed, dried, and weighed on the analytical balance. As the weights of the pycnometer and the bentonite were known, the weight of benzene and its volume were calculated. The difference in volume between the benzene and the pycnometer was taken as the volume of the bentonite. The density was calculated from this volume and the previously determined weight.

Water loss.—Water loss determinations were made using a standard, wallbuilding tester thermostated in an air bath at 30° C (\pm 1.0°). A constant filtration pressure of 100 psig was maintained throughout all water loss determinations. Water loss data were taken at short intervals of time up to approximately 50 minutes. Filtrate volumes were plotted against the square root of time and the standard adjustment to zero time was made (Rogers, 1953, p. 270). For simplicity, unless otherwise stated, the 30-minute water loss at 30.0°C and 100 psig, corrected to zero time, will be referred to as the water loss.

EXPERIMENTAL VERIFICATION OF FILTRATION THEORY

Verification by Bentonite-Water Systems

The analysis of the filtration process requires the use of density-concentration data for the determination of the *effective concentration,* C', as indicated by Equations 8 and 9. These data for the bentonite-water systems are presented in tabular form in Table 1, since a very large graph is necessary to show pertinent details. While not here presented, several features of the density-concentration curve are worthy of mention:

a) A significant change in the slope occurs at approximately 35 grams of bentonite per 100 grams of water. This is indicative of a change in the physical mechanism of the density-concentration relationship.

b) Actual densities are significantly greater than densities calculated assuming volumetric additivity of the two components, bentonite and water. The greater densities are attributed to interaction between the solid and liquid, which increases the density of the adsorbed liquid.

The water loss as a function of concentration is also presented in Table 1. These data may be shown to agree with the filtration relationship presented above (Eq. 9) in the following manner: Using assumed d_t values from 1.120 to 1.200 and the density data presented in Table 1, values of $(C')^{-1/2}$ were calculated. Equation 9 indicates that if the values of water loss are plotted against (C') ^{-1/2} the curve should be a straight line that passes through the origin when the correct value of d_f is assumed. This relationship is presented in Figure 1. When the density of the filter cake is equal to 1.200 the curve passes through the origin, and all curves are linear within experimental limits. The value of

Concentration (grams)	Density $({\rm grams/ml})$	C'	1/C'	$(C')^{-1/2}$	Water Loss (m _l)	dV/dt (ml/sec)	Water Loss (Calculated)
0.000	0.99564	0	Inf.	Inf.	Inf.		Inf.
0.992	1.00155	.0298	33.579	5.79	35.9	2.220	36.2
1.827	1.00730	.0605	16.527	4.06	25.1	1.150	25.4
3.770	1.01950	.1322	7.565	2.75	17.7	0.536	17.2
5.660	1.03145	.2124	4.707	2.17	13.5	.309	13.5
8.136	1.04594	.3265	3.063	1.75	10.8	.198	10.9
9.534	1.05452	.4047	2.471	1.57	9.8	.164	9.8
11.855	1.06870	.5858	1.707	1.31	8.5	0.118	8.4
12.910	1.07500	.6349	1.580	1.25	----		
33.395	1.200						
47.59	1.25719						
76.48	1.37194				Density of dry bentonite $= 2.5196$ grams/ml at 30°C		
90.88	1.42410						
109.69	1.48944						

TABLE 1.-DENSITY AND WATER-LOSS DATA, WYOMING BENTONITE (AQUAGEL), (30°C)

filter-cake density was confirmed experimentally and found to be 1.205 $(\pm.005)$ grams/cc.

To confirm the validity of the assumptions made in the derivation of Equation 9, values of dV/dt were determined (at 10 minutes) and plotted against the reciprocal of C' as shown in Figure 2. Values of C' were based on d_f of 1.200. The curve is linear and passes through the origin. This indicates that

FIGURE 1.-Estimation of filter-cake density, Wyoming bentonite

. FIGURE 2.-Validation of integrated filtration equation by use of differential equation.

the permeability and C' of Equation 6 are not dependent upon time, so that Equation 6 may be integrated to give a valid Equation 9.

Returning to Figure 1, it is found that the slope of the curve that passes through the origin is equal to 6.24 for the particular filtration conditions (100 psig, 30°C, and 30 minutes).

$$
V = \left[\frac{2KA^2Pt}{u}\right]^{1/2} (C')^{-1/2} = \text{slope} (C')^{-1/2} = 6.24 (C')^{-1/2} \qquad \text{Equation 11}
$$

Using Equation 11 the water loss versus concentration curve was determined; it is presented in Figure 3 along with experimental points for particular concentrations.

Verification by a Low-Yield Clay (Magcobar Xact Clay)

The density-concentration data for the Xact clay are presented in Table 2. The concentration range presented is greater than the range for the bentonite dispersion presented. The relation between water loss and the reciprocal of the square root of C' is presented in Figure 4, and it is observed to be linear except at the highest concentrations. In this range it is observed that a significant deviation occurs and the theory is no longer valid. This deviation is presumed to be the result of the high gel-strength of these samples which prevents a sharp transition between the filter cake and the un filtered dispersion. Further, the lack of a sharp transition between the filter cake and the dispersion pre-

FIGURE 3.-Comparison of experimental and calculated water loss, Wyoming bentonite. (Solid curve from Equation 11. Points from experimental data.)

eluded the accurate, experimental determination of the density of the filter cake. Accordingly, an approximate filter-cake concentration was assumed and the density was determined by extrapolation of the density-concentration curve. The values of C' calculated in this manner are given in Table 2. Presumably the assumption of a lower value of d_f would bring the linear part of the curve through the origin. In view of the fact that the density could not be confirmed, this was not done.

Thus, the mechanism of filtration on which the equations are based does

Concentration (grams)	Density (g/ml)	1/C'	$(C')^{-1/2}$	Water Loss (ml)
0.0	0.996	Inf.	Inf.	Inf.
27.5	1.152	2.02	1.42	36.4
35.8	1.192	1.39	1.18	29.9
52.0	1.243	.889	.943	20.9
56.1	1.262	.752	.867	18.0
62.1	1.290	.582	.763	15.1
69.2	1.323	.421	.649	12.3
78.7	1.366	.254	.504	11.3
86.4	1.398	.153	.391	10.9
100.0	1.459 (est.)	0	$\bf{0}$	Filter Cake

TABLE 2.-DENSITY AND WATER·LOSS DATA, MAGCOBAR XACT CLAY

FIGURE 4.-Water loss versus concentration function, Magcobar Xact clay.

not adequately represent the mechanism of filtration at concentrations that exhibit a very high gel strength. From a general standpoint, this defines the upper limit of the applicability of this approach to filtration, the lower limit being the ability of the material to form a continuous filter cake. The concentrations having this very high gel strength (which may be compared to the consistency of modeling clay) are far removed from the concentration ranges normally used in drilling fluids so the upper limit does not impose any practical limitation.

Verification by Sodium Carboxymethylcellulose

The density of the sodium carboxymethylcellulose dispersions could not be measured because of the persistence of bubbles within the samples. These bubbles could not be removed entirely even by simultaneous heating and evacuation

Concentration (grams)	1/C'	$(C')^{-1/2}$	Water Loss (m _l)
0.0	Inf.	Inf.	Inf.
1.0	11.95	3.46	16.85
2.0	5.43	2.33	10.45
	2.17	1.48	5.70
$\frac{4.0}{6.0}$	1.09	1.04	3.75
8.0	.543	.736	2.95
12.0	0	0	Filter Cake

TARLE 3-WATER-LOSS DATA SOBHIM CARROXYMETHYLCELLILLOSE

FIGURE 5.-Water loss versus concentration function, sodium carboxymethylcellulose.

of the samples. Accordingly, values of C' were determined using the approximation technique of Equation 10. These data are given in Table 3. The relation between water loss and the reciprocal of the square root of C' for the sodium carboxymethylcellulose suspensions is presented in Figure 5. Deviation from the predicted linear relationship is again observed at the higher concentrations.

This material is of particular interest for several reasons:

a) While very hydrophilic, it is structurally and chemically unlike the clay minerals.

b) Its affinity for water is not attributable to defects in its structure.

c) **It** is chemically stable and produces a stable colloidal suspension.

d) **It** is frequently used to control water loss in oil-well drilling fluids.

With the single exception of its high affinity for water, colloidally dispersed sodium carboxymethylcellulose has little, if anything, in common with the clay minerals. As a result, the generality of the proposed filtration relationship is considerably extended.

Verification by Data from the Literature

Further confirmation is indicated in Figures 6 through 9. Water loss-concentration data for Figures 6, 7, and 9 (Tables 4, 5, and 7) were obtained from Rogers (1953, p. 290) while those for the Altwarmbeuchen clay (Table 6 and Fig. 8) were obtained from von Englehardt (1954, p. 7). Values of C' were estimated using Equation 10. While these curves extend over only a limited concentration range, the linear relationship is indicated or strongly suggested.

FIGURE 6. Water loss versus concentration function, clay "A" (Rogers, 1953, p. 290).

FIGURE 7.- Water loss versus concentration function, clay "B" (Rogers, 1953, p. 290).

FIGURE 8 .-- Water loss versus concentration function, Altwarmbeuchen clay (von Englehardt, 1954, p. 7).

FIGURE 9.-Water loss versus concentration function, Wyoming bentonite (Rogers, 1953, p. 290).

Concentration (grams)	1/C'	$(C')^{-1/2}$	Water Loss (m _l)
20	2.57	1.603	57
25	1.91	1.382	50
30	1.47	1.213	42
35	1.156	1.075	39
40	.92	.959	34
90	0	0	Filter Cake

TABLE 4.-WATER-Loss DATA, CLAY *"A"}*

} Rogers, 1953, p. 290.

Concentration (grams)	1/C'	$(C') -1/2$	Water Loss (ml)	
20	4.27	2.065	82	
25	3.29	1.814	79	
30	2.64	1.625	71	
35	2.18	1.475	63	
40	1.83	1.352	60	
45	1.56	1.248	55	
160	0	0	Filter Cake	

TABLE 5.-WATER-LOSS DATA, CLAY "B"1

} Rogers, 1953, p. 290.

TABLE 6.-WATER-LOSS DATA, ALTWARMBEUCHEN CLAY¹

Concentration (grams/gram Mud))	1/C'	$(C') -1/2$	Water Loss (ml at 20 atm and 60 minutes)	
0.153	10.00	3.16	54.4	
0.190	7.78	2.79	47.5	
0.229	6.07	2.46	41.5	
0.273	4.53	2.13	36.8	
0.307	3.85	1.96	33.6	
0.346	3.10	1.76	30.5	
0.383	2.54	1.60	27.9	

} von Englehardt, 1954, p. 7.

CONCLUSIONS

On the basis of a careful analysis of the data gathered during this investiga· tion, along with data available in the literature, the following conclusions have been formulated. It should be emphasized that conclusions, while possibly extendible to other systems, are formulated specifically for bentonite-water systems.

Concentration $(grame)$	1/C'	$(C')^{-1/2}$	Water Loss (m _l)
6.0	2.88	1.70	17
6.5	2.59	1.61	16
7.0	2.34	1.53	15
7.5	2.12	1.45	13.5
8.0	1.93	1.39	14
8.5	1.76	1.33	13
9.0	1.62	1.27	13
9.5	1.48	1.22	13
10.0	1.36	1.17	12
25.0	0	0	Filter Cake

TABLE 7.-WATER-LOSS DATA, WYOMING BENTONITE¹

1 Rogers, 1953, p. 290.

1. Filtration of hydrophilic dispersions cannot be described adequately by the filtration theories which do not make appropriate allowance for the film of water adsorbed on the surface of the dispersed solid.

2. The filtration theory allowing for the hydration of the particles, as derived above, adequately characterizes the filtration of bentonite dispersions.

3. The expression of concentration arising from the filtration theory, *C,* is an appropriate means of expressing concentration which more accurately describes the nature and importance of the dispersed phase.

4. The permeability (and compaction) of a bentonite filter cake is not significantly time-dependent at a given filtration pressure. Further, it does not depend on the initial concentration of the bentonite.

5. One hundred percent of the water may be considered to be adsorbed or fixed at the filter-cake concentration. Conversely, a simple means of estimating adsorbed water at a given filtration pressure is by measurement of the filtercake concentration and density.

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