Grechuk for bringing up all these interesting diophantine equations through his investigations. Last but not least, we are indebted to the referee for suggesting a number of amendments of a linguistic nature as well as for pointing out that it is better to mention elliptic curves in a gentler manner.

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107.16 An expression for the prime-composite characteristic function

Introduction

We define the prime-composite characteristic function as the function on the set of natural numbers that is equal to 1 for all primes, 0 for all composites, and is not defined otherwise.

Proposition

For all naturals $n \neq 4$, the prime-composite characteristic function is equal to:

$$C(n) = \frac{n}{n-1}\left\{\frac{(n-1)!}{n}\right\}$$

where $\{x\}$ is the fractional part of x [1].

Proof

The proof hinges on the evaluation of the fractional part :

- if n is a prime, then $\left\{\frac{(n-1)!}{n}\right\} = \frac{n-1}{n}$ by virtue of Wilson's theorem [1],
- if n is a composite different from 4, then n divides $(n-1)!$ and the braces evaluate to 0 [2].

Consequences

The set of composite numbers can be expressed, save for 1 and 4, as: $\{n \in \mathbb{N} : n^2 \mid n!\}$.

The prime counting function can be expressed for all $n \geqslant 4$ as:

$$\pi(n) = \frac{4}{3} + \sum_{j=3}^{n-1} \frac{j+1}{j} \left\{ \frac{j!}{j+1} \right\}.$$

Discussion

The number 4, which is the smallest composite number, is exceptional as 'it is the only composite n that does not divide $(n - 1)!$'[2]. This is the reason $C(n)$ indicates all primes and composites apart from 4. If C is used as a measure of *primeness*, then $C(4) = 2/3$ indicates that 4 is the 'least composite' composite in that measure.

The behaviour of $C(n)$ for large n can be considered through the following plausibility argument. First, note that $n \neq 4$ is a prime number only if $n!$ has no divisor that is a multiple of n^2. Erdős et al. [3] have shown that the number of divisors of $n!$ grows faster than any power of n for large n. This entails that the probability that $n!$ has no divisor that is a multiple of n^2, which is the probability that n is prime, tends to 0 as $n \to \infty$.

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107.17 Two curios related to lattice polygons

A *lattice polygon* is a planar polygon in $\mathbb{R}^d$ all of whose vertices have integer coordinates. It is regular if its sides and angles are all equal. The fundamental theorem in this area is:

Theorem A

(i) In $\mathbb{R}^2$, the only regular lattice polygons are squares.

(ii) In $\mathbb{R}^d$, $d \geqslant 3$, the only regular lattice polygons are triangles, squares and hexagons.

Theorem A is blessed with several neat proofs such as the ingenious geometric one in [1] and the one using algebra and trigonometry in [2]. The