

## THE CLASSIFICATION OF CATEGORIES GENERATED BY AN OBJECT OF SMALL DIMENSION

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The goal of this thesis is to attempt the classification of unitary fusion categories generated by a normal object (an object commuting with its dual) of dimension less than 2. This classification has recently become accessible due to work of Morrison and Snyder, which shows that any such category must be a cyclic extension of an adjoint subcategory of one of the ADE fusion categories. These ADE fusion categories are the categories whose graph for fusion by the generating object gives one of the  $A_N$ ,  $D_{2N}$ ,  $E_6$  or  $E_8$  Dynkin diagrams. Our main tool is the Etingof–Nikshych–Ostrik classification of graded categories, which classifies graded extensions of a fusion category in terms of the Brauer–Picard group and Drinfeld centre of that category.

We compute the Drinfeld centres and Brauer–Picard groups of the adjoint subcategories of the ADE fusion categories. Using this information, we apply the machinery of graded extensions to classify the cyclic extensions that are generated by a normal object of dimension less than 2 of the adjoint subcategories of the ADE fusion categories. Unfortunately, our classification has a gap when the dimension of the object is  $\sqrt{2 + \sqrt{2}}$  corresponding to the possible existence of an interesting new fusion category. Interestingly, we prove the existence of a new category, generated by a normal object of dimension  $2 \cos(\pi/18)$ , which we call the DEE fusion category. We include the fusion rules for the DEE fusion categories in an appendix to the thesis.

These results of the thesis have appeared in the author’s papers [1, 2]. In particular, a full classification result can be found in [2], which includes the missing gap at dimension  $\sqrt{2 + \sqrt{2}}$  appearing in the thesis.

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### References

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