THE FILAMENT INSTABILITY IN A SHEARED FIELD

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Filaments and prominences are classical examples of condensations of nongravitational origin, their formation being due to the thermal instability. The physics of this instability in a uniform plasma at coronal temperatures, which exhibits increasing radiative output as it cools, is well known (Field, 1965). Filaments, however, form in regions of sheared magnetic fields, as evidenced by their location above photospheric polarity-inversion lines and their occurrence after a period of increasing fibril inclination (Tandberg-Hanssen, 1974). Physically, the importance of the field structure is readily understood if one recalls the capability of magnetic field of strongly collimating the local heat conduction. Thus, the preferred locations for the development of the thermal instability will be those where the field configuration inhibits the stabilizing effects of thermal conduction on the growing temperature perturbation.

In this brief report we summarize the results of a self-consistent calculation of this instability in a non-uniform field, showing how the dynamic response of density and temperature to the competing effects of optically-thin radiation and field-collimated thermal conduction leads to the formation of characteristic "knife-blade" filaments (Chiuderi and Van Hoven, 1979).

The equilibrium magnetic field in the region of filament formation is assumed to have the model force-free form:

$$\vec{B}_o(x) = B_o\left[\vec{e}_y \tanh(x/a) + \vec{e}_z \operatorname{sech}(x/a)\right] \quad . \tag{1}$$

This field exerts a constant magnetic pressure and is therefore consistent with a uniform equilibrium state, yet it exhibits a field reversal. The calculation is based on the use of the standard ideal MHD equations, to which the following energy equation is added:

$$\frac{dp}{dt} = -\gamma p(\nabla\cdot\vec{v}) + (\gamma-1)\left[H-\rho^2\phi(T) + \nabla\cdot(\kappa_o T^{5/2}\vec{e}_b\vec{e}_b\cdot\nabla T)\right] \quad , \tag{2}$$

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where H is a generalized heat function, $\phi(T)$ specializes the temperature dependence of the radiative loss, κ_o is the heat conduction coefficient and $\vec{e}_b = \vec{B}/B$ is a unit vector along $\vec{B}$. Every quantity appearing in the equations is now written as $A(\vec{r},t) = A_o(x)+A_1(x)\exp(\nu t-ky)$, $|A_1| << |A_o|$, and the whole set of equations is linearized. This leads to a system of seven equations, five being algebraic and the remaining two first-order coupled differential equations in the variables $\xi_x = v_{1x}/\nu$, the transverse displacement, and $q = p_1 + (\vec{B}_o\cdot\vec{B}_1)/\mu_o$, the total pressure perturbation. H is considered to operate only in equilibrium. The equations contain a number of characteristic frequencies:

$$\Omega_\rho = -(\gamma-1)\frac{\partial}{\partial p}(\rho^2\phi(T)|_\rho \quad ; \quad \Omega_p = (1/\gamma)(1-2\frac{T_o}{\phi}\frac{d\phi}{dT})\Omega_\rho;$$

$$\sigma_\kappa = |(\gamma-1)\kappa_o T_o^{7/2}/p|k_\parallel^2 << \Omega_\rho .$$

In the range $10^{5.9} < T < 10^{6.8}$, $\phi \sim T^{-1}$ (Hildner, 1974) and we have

$$\Omega_\rho \simeq 10^{-3.6}\, n_9 T_6^{-2},\ \Omega_p = 1.8\,\Omega_\rho .$$

Finally, introducing the variable $\eta \equiv \tanh\zeta \equiv \tanh(x/a)$ and normalizing wavenumbers and frequencies as follows, $\alpha = ka$, $\tilde{\nu} = \nu a/c_a$ ($c_a^2 = B_o^2/\mu_o\rho_o$), we arrive at the second-order equation ($' \equiv d/d\zeta$):

$$q'' - 2\frac{\alpha^2\eta(1-\eta^2)}{\tilde{\nu}^2+\alpha^2\eta^2}q' - [\alpha^2+\alpha_\nu^2(\tilde{\nu},\alpha^2;\eta)]q = 0 \quad , \qquad (3)$$

where α_ν^2 can be negative and contains all the physical parameters of the problem. Eq. (3) has the form of a non-standard eigenvalue equation and must be supplemented by suitable boundary conditions. It is assumed here that q is a localized, even-parity, monotonic function of ζ. It is then possible to show that these boundary conditions imply certain limitations on the possible frequencies and wavenumbers of the purely growing solutions. More precisely,

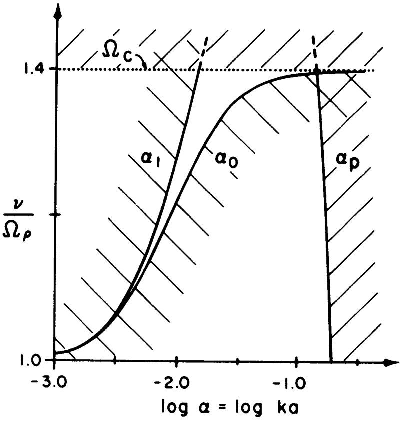

Figure 1. The allowed growth rate (ν) vs. wavenumber (α) parameter space, for monotonic filament profiles, is shown in the unshaded central window.

$$\Omega_\rho < \nu < \Omega_c \equiv (\Omega_\rho + \tilde{\beta}\Omega_p)/(1+\tilde{\beta}) \quad ; \quad \alpha_1^2(\nu) < \alpha^2 < \min[\alpha_o^2(\nu), \alpha_p^2(\nu)]$$

where $\tilde{\beta} = \gamma\beta/2 = \gamma\mu_o P/B_o^2$, and $\alpha_1^2(\nu)$, $\alpha_o^2(\nu)$, $\alpha_p^2(\nu)$ can be explicitly given in terms of β and the characteristic frequencies. The above conditions define a limited region in the $(\alpha,\tilde{\nu})$ plane where solutions may be found, as shown in Fig. 1.

A first set of numerical calculations have been performed, with the following choice of physical parameters, illustrative of the ambient coronal conditions before the start of filament condensation: $n_o = 10^{9.8}$ cm^{-3}, $T_o = 10^{6.2}$ K, $B_o = 7.5$ G and $a = 10^{8.5}$ cm. These give $\tilde{\beta} = 1$, $\Omega_\rho = 10^{-3.2^o}$ s^{-1}. The computed eigenvalues turn out to satisfy $\tilde{\nu}(\alpha) \sim \tilde{\nu}(\alpha_1)$ (Fig. 1) for $\nu < \Omega_c$. Concentrating on the fastest growing solutions, $\nu \simeq \Omega_c$, we can make the following comments.

(i) The computed eigenfunctions, $q(x)$, have the general aspect of decreasing exponentials (except for very small x) with a typical half-width of 10 a. This, however, is not the important empirical transverse scale, which is given by the variation of the physical observables ρ_1 and T_1. From the linearized MHD equation we find, in our regime,

$$\rho_1 = (\alpha_\nu/\nu a)^2 q \quad ; \quad T_1/T_o \simeq -(\gamma/\tilde{\beta} + 1)(\rho_1/\rho_o) \quad . \tag{4}$$

Since $\alpha_\nu^2(\eta)$ varies with x much faster than q, it actually determines the shape of ρ_1 and T_1 near the origin. We can now establish that the overall filament geometry corresponds to the empirical requirements that the horizontal width, δ (determined by the shape of α_ν^2) is much less than the vertical width, $\ell(\simeq \pi/\kappa)$, which is much less than the horizontal length ($\sim \infty$ in this model). For our choice of parameters and $\Omega_c - \nu = 0.1\,\Omega_\rho$, we obtain $\delta/\ell \sim 10^{-2}$.

(ii) The <u>initial</u> growth time of the fastest growing mode, $\tau_c \simeq \Omega_c^{-1}$ is estimated to be $\tau_c \simeq 10^{3.5}\, n_9^{-1}\, T_6^2$, which predicts a typical e-folding time somewhat shorter than is normally quoted. However, what is observed is the <u>nonlinear</u> stage of filament formation which usually has a much longer timescale.

(iii) Sheared magnetic fields are intimately connected with flares, both observationally and theoretically. The excess magnetic energy stored in the nonpotential field can be released due to finite resistivity effects, but the timescale of the process appears to be too long. The thermal instability timescale is much faster and it is therefore conceivable that the magnetic reconnection will take place in the nonlinear stage of filament development, at a considerably lower temperature and at a much faster rate, due to the strong temperature dependence of normal resistivity.

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DISCUSSION

Nakagawa: The effect of gravity is one of the crucial problems in the formation of condensations because the increased density required to induce thermal instability is immediately subject to gravitational acceleration downward. Your field configuration does not provide any support.

Van Hoven: I agree; this is only the initial step in a filament model, but it is the first dynamic calculation to reflect the sheared magnetic field and the finite geometry.

Foukal: The density assumed in calculations of thermal instability is critical in determining the growth rates, and the spatial wave numbers of the growing modes. Were you able to find reasonable growth rates for densities of the order of the background corona, from which such a filament would be expected to condense?

Van Hoven: The density in the example quoted here is $10^{9.8}$ cm^{-3}, and the growth rate is approximately proportional to this parameter.