Hence M, N, P are collinear.

The theorem in similarity corresponding to the converse theorem (2) is the following:

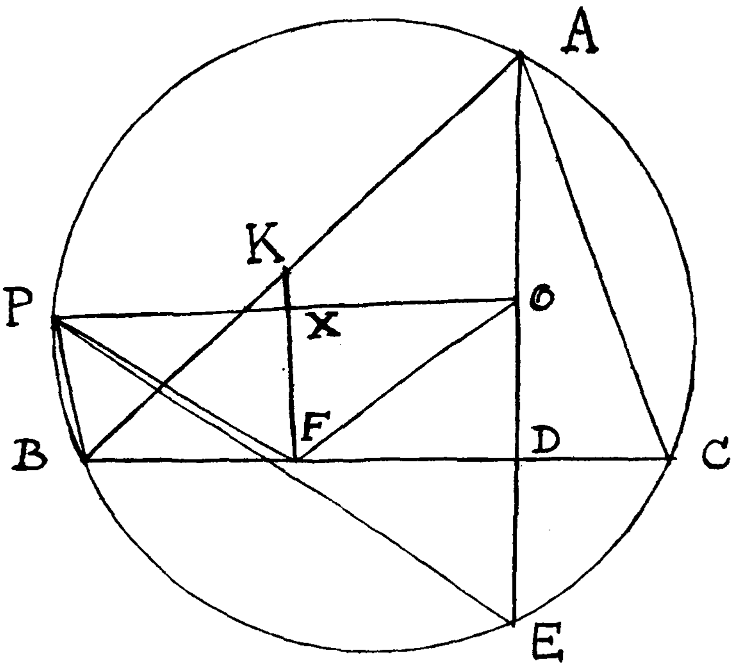
If two triangles have an angle of the one equal to an angle of the other and another pair of angles supplementary, then the sides opposite to the equal angles are proportional to those opposite to the supplementry angles.

Thus, in Euclid VI. 3 again, if AD bisect the angle BAC internally or externally, it readily follows from the triangles ADB, ADC (where one pair of angles are equal and another pair supplementary) that

$$AB : AC = BD : DC.$$

Note III.—Simson-Line.

The following is a new proof of the well-known theorem that "the pedal line of a point with respect to a triangle inscribed in a circle passing through the point, bisects the join of the point and the orthocentre of the triangle."



Let P be any point on the circum-circle of a triangle ABC, whose orthocentre is O. Let X be the middle point of OP and let the perpendicular bisector of OP cut BC at F and AB at K. Let AO cut BC at D and the circle at E. Join PB, PF, PE.

Since the perpendicular bisectors of OP and OE meet in F, F is the circumcentre of the triangle OPE. Hence

$$X\widehat{F}P \text{ or } \tfrac{1}{2}\, O\widehat{F}P$$
$$= O\widehat{E}P \text{ or its supplement (according to the figure)}$$
$$= A\widehat{B}P \text{ or its supplement (according to the figure)}$$

and in all cases the configuration of the equal angles is such that K, F, P, B are concyclic.

∴ The feet of the perpendiculars from P on AB, BC and KF are collinear.

But X is the foot of the perpendicular from P on KF

∴ The pedal line of P with respect to the triangle ABC bisects OP.

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