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A Problem in Reversions

Sirs,

In reversionary business, one occasionally comes across a reversion expectant on the death of the survivor of two life tenants, say (s) and (t), contingent on the reversioner (r) surviving (s) only. The problem is to obtain a practical approximation to the factor for valuing this and the sum assured under the supporting survivorship assurance, (r) against (s).

There is, quite understandably, no symbol for the factor required. I suggest

$$A_{\left(\overset{1}{rs}\text{ or }\overline{st}\right)},$$

since the sum assured is payable if (r) predeceases (s) *or*, failing that, the reversion falls in at the death of the survivor of (s) and (t).

The following scheme should make the position clear; it gives all possible orders of deaths. Capital letters denote deaths on which payments are received.

R	s	t
R	t	s
s	r	T
s	T	r
t	R	s
t	S	r

By suitably manipulating the contingent assurances represented above, it is found that

$$A_{\left(\overset{1}{rs}\text{ or }\overline{st}\right)} = A_t - A_{st} + A_{rs} + \left(A_{\underset{1}{r}\overset{2}{s}\underset{3}{t}} - A_{\underset{12}{rs}\overset{3}{t}}\right)$$

Both terms in the bracket relate to the same order of deaths (an unlikely one, since in practice (r) must be comparatively young), and their net effect is merely the present value of interest from the second death to the last. Neglecting this, we have a simple and generally close approximation to the factor required. Since the

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bracket is necessarily positive, the approximation is always too small, i.e. on the safe side from the point of view of a purchaser.

Denoting the error by e, it can easily be shown that de/dr is positive and de/dt is negative for all practical cases. In general de/ds is at first positive, becomes zero for some value of (s) slightly before the peak of the curve of deaths, and is thereafter negative. In other words the error is greatest when (r) is comparatively old, (t) is comparatively young and (s) has some value which may, in the case of the $a(f)$ table, be near 80.

The results of two numerical examples, based on the $a(f)$ 5% ultimate table, are given below. The first is included only in order to give some indication of the maximum error. The second is more typical, and the error here may be considered negligible for practical purposes.

Example	Ages			$A(\overset{1}{rs} \text{ or } \overline{st})$		Error
	r	s	t	True	Approximate	
1	55	80	55	·373	·350	·023
2	40	70	70	·477	·472	·005

Yours faithfully,
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