THE CONVENTIONAL GEOCENTRIC REFERENCE SYSTEM WITHIN THE FRAMEWORK OF RELATIVITY

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ABSTRACT Some basic conceptions of relativistic reference systems are given. The conventional solar-system barycentric reference system (B-system) and the conventional geocentric reference system (G-system) are discussed. It is suggested that the harmonic conditions be used in the definition of the B-system and the G-system be defined by the B-system and the coordinate transformation given by eqs (3-2), (3-3) and (3-4).

1. Introduction

With the improvement of accuracy of observational data, the relativistic effects have become more and more important. Time and distance will be no longer absolute quantities, but depend on the adopted reference system. Then, to construct self-consistent and suitable reference systems within the framework of relativity is one of main research fields in modern celestial mechanics and astrometry. Both of the solar-system barycentric reference system (B-system) and the geocentric reference system (G-system) are the important ones in solar-system celestial mechanics and astrometry. There are a large number of papers to deal with them (cf. Fukushima, 1988; Kopejkin, 1988; Brumberg, 1990; Damour, 1991). But there still exist different opinions on the choice of the B-system and, especially, on that of the G-system. In the XXIst General Asembly of the International Astronomical Union, a set of recommendations on relativistic reference systems were given. But, we believe, they are not satisfying and still have some drawbacks.

The present paper deals with the definitions of the reference systems, especially with that of the G-system. Section 2 gives some basic conceptions of relativistic reference systems. Section 3 gives discussions on the conventional reference systems. Some drawbacks of the IAU recommendations are discussed and a way of constructing the G-system is presented. Finally, a few suggestions and conclusions are given.

In this paper, greek indices are used to denote space or time components of a tensor, while the indices taken from the second part of latin alphabet (i, j, k, ...) to denote purely spatial components. However, the indices

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on a tensor which take from the first part of latin alphabet (a, b, c,...) do not represent components but are part of the notation for the tensor itself, much like the arrow used to denote a vector in ordinary three-dimensional space.

2.Basic Conceptions of Reference Systems in GRT

The general relativity theory (GRT) asserts that spacetime structure and gravitation are described by a spacetime (M, g_{ab}). The essence of it may be summarized as follows: Spacetime is a four-dimensional manifold M on which is defined a metric, g_{ab}, of Lorentz signature. The metric is related to the matter distribution in spacetime by Einstein's equation (Wald, 1984):

$$G_{ab} \equiv R_{ab} - \frac{1}{2} R\, g_{ab} = 8\pi T_{ab} \qquad (2\text{-}1)$$

where R_{ab} is the Ricci tensor, R the scalar curvature, T_{ab} the stress-energy tensor, and G_{ab} is called the Einstein's tensor. If we choose a coordinate system, the Einstein's equation (2-1) can be written in the form of coordinate basis components:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi T_{\mu\nu} \qquad (2\text{-}2)$$

and we will have the correct number of equations (six independent equations of Einstein's equation plus four coordinate conditions) and unknowns (ten components of the metric) to permit a good initial value formulation.

It should be note that for a given spacetime the metric tensor is determined by the matter distribution, but the metric components $g_{\mu\nu}$ are not unique. They are dependent on the choice of the coordinate system. If there are two different coordinate systems $\{x^\mu\}$ and $\{\bar{x}^\mu\}$ constructed in the same space time, there will be two different sets of metric components $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ correspondingly,

$$g_{\mu\nu} = g_{ab}\left(\frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial x^\nu}\right),$$

$$\bar{g}_{\mu\nu} = g_{ab}\left(\frac{\partial}{\partial \bar{x}^\mu}, \frac{\partial}{\partial \bar{x}^\nu}\right),$$

and they have the following relation:

$$g_{\mu\nu} = \frac{\partial \bar{x}^\alpha}{\partial x^\mu} \frac{\partial \bar{x}^\beta}{\partial x^\nu} \bar{g}_{\alpha\beta}. \qquad (2\text{-}3)$$

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That is to say, $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ represent a same physical solution of Einstein's equation.

A coordinate system is usually defined by the coordinate conditions, which are used in solving Einstein's equation, and their initial conditions, such as, the origin and the orientation of the coordinate bases at some point of spacetime. In the theory one uses rather often the harmonic conditions. It can not only simplify significantly the Einstein's equation, but make the coordinate system "quasi-Minkowski" in the weak gravitaional field (cf. Winberg, 1972; Brumberg, 1990). The latter character may be beneficial to elucidating the meaning of the relevant physical events.

In principle, any real coordinate system implies an idealized reference system since the metric components in the coordinate system are uniquely determined. But in order to make it available a conventional metric form must be given, which is an approximate solution of Einstein' s equation under the very coordinate conditions. Obviously, in the case that a conventional reference system is given, any other reference system can be defined by the coordinate transformation between them.

Another important conception in the theory of reference systems is the local inertial reference system. Since spacetime is locally flat, one can construct, for any point (P), a local Lorenz frame in the vicinity of the point, which satisfies:

$$g_{\mu\nu}(P) = \eta_{\mu\nu} , \qquad (2\text{-}4)$$

$$g_{\alpha\beta,\gamma}(P) = 0 \quad \text{or} \quad \Gamma^{\gamma}_{\alpha\beta}(P) = 0 , \qquad (2\text{-}5)$$

where $\eta_{\mu\nu}$ is the components of Minkowski metric in Cartesian coordinates ($\eta_{\mu\nu}$=diag (-1, +1, +1, +1)), $\Gamma^{\gamma}_{\alpha\beta}$ is the Christoffel symbol. Condition (2-4) sets the coordinate bases rectangular each other. Condition (2-5) makes the coordinate axes as straight as possible (generally, there doesn't exist such a coordinate system that makes $g_{\alpha\beta,\mu\nu}(P)$ vanish.).

For a freely-falling observer, one can go further to construct a comoving local Lorentz frame or called local inertial reference system, which satisfies, besides eqs (2-4) and (2-5), the following conditions:

$$e^{a}{}_{(0)} = U^{a} \qquad (2\text{-}6)$$

$$\nabla_{U^{a}}\, e^{a}{}_{(i)} = 0 \qquad (2\text{-}7)$$

where U^{a} is th 4-velocity of the observer, $e^{a}{}_{(\alpha)}$ are the coordinate bases at the observer. Condition (2-6) guarantees the observer is at rest with respect to the very reference system, and condition (2-7) does the spatial

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bases show no rotation in the sense of dynamics. In a background reference system denoted by $\{x^\alpha\}$, the coordinates of the observer-comoving local inertial reference system denoted by $\{\xi^\alpha\}$ can be expressed with:

$$\xi^0(x^\alpha) = c\tau \qquad (2\text{-}8)$$

$$\xi^i(x^\alpha) = [x^\mu - x_0{}^\mu(\tau)]\, e_{(i)\mu} + \frac{1}{2}\Gamma^\lambda_{\mu\nu}(x_0{}^\alpha)\, e_{(i)\lambda}\, [x^\mu - x_0{}^\mu(\tau)]\,[x^\nu - x_0{}^\nu(\tau)] + \cdots \qquad (2\text{-}9)$$

or

$$x^\alpha = x_0{}^\alpha(\tau) + \xi^i e^\alpha{}_{(i)} - \frac{1}{2}\Gamma^\alpha_{\mu\nu}(x_0{}^\beta)\, e^\mu{}_{(i)} e^\nu{}_{(j)}\, \xi^i \xi^j + \cdots \qquad (2\text{-}10)$$

where τ is the proper time of the observer, and $x_0{}^\alpha(\tau)$ its position. It should be note that the local inertial reference system is not uniquely determined for a given observer. Besides the orientation, the spatial coordinates (ξ^i) may be defined differently by the third order terms of $(x^\alpha - x_0{}^\alpha(\tau))$.

3.The Conventional Geocentric Reference System

In the practice of astronomy different reference systems are needed. We describe the motions of planets in a solar barycentric reference system, the motions of Earth satellites in a geocentric reference system, and make observations in a topocentric reference system, etc.. No matter what kind of reference system is used, it must be self-consistent. In a reference system, the metric components should satisfy Einstein's equition and be consistent with the adopted coordinate system. For any two different reference systems, the transformation of coordinates and/or the metric components should satisfy the equation (2-3).

At the XXIst General Assembly held in Buenos Aries, the International Astronomical Union passed a set of recommendations on reference systems within the framework of general relativity. But, we believe, they still have some drawbacks. The first remark is that the metric form given by Recomendation I (Rec. I) is not sufficient precision for modern celestial mechanics using the post-Newtonian equations of motion of the solar system bodies (Brumberg, 1990). In order to give equations of motion of particles in the post-Newtonian approximation (PNA), one must take account of higher order metric terms, i.e. c^{-3} terms in g_{0i} and c^{-4} terms in g_{00}.

The second remark, to be more important, is that the conventional reference systems are lack of clear conceptual definitions. the recommendations have given only the approximate metric form of spacetime (Rec. I) and the coordinate initial conditions (Rec. II for the spatial coordinates and Rec. III for the time coordinates), but not the coordinate conditions used in the solution of Einsten's equation. Then, one do not know how to add the higher order terms to the metric form when more accurate spacetime metric is needed.

By the way, in Rec. II, the spatial coordinate grids of the B-system and the G-system are demanded to show no global rotation with respect to a set of distant extragalactic objects. The words seems not to be much scientific. In the curved spacetime, the vectors located at different points can not be compared directly. If the solar system is recognized as a isolated system, which is not valid only for the range far from it, the spacetime will be flat at infinity, and hence we can define the spatial axes of the B-system show no rotation with respect to the extragalactic objects at infinity. But it is not allowed for us to do so for the G-system. What we can do in the similar meaning is only to define the spatial coordinate bases show no rotation in some sense with respect to those of the B-system at the geocenter. However, this condition is obviously dependent on the definition of the B-system.

Based on the above discussions, we suggest that the harmonic coordinate conditions be added to the definition of the B-system and the metric form of it be given in PNA under the very conditions.

As for the conventional G-system, there are three major choices at present:

1) the geocentric Fermi coordinate system, which is proposed by Berttoti in 1954 and recently developed and improved by Ashby and Berttoti (1986), Boucher (1986), Fujimoto and Graferend (1986), and Fukushima (1986, 1988);
2) the geocentric harmonic coordinate system developed by Brumberg and Kopejkin (Kopejkin, 1988; Bromberg and Kopejkin, 1989; Brumberg, 1990);
3) the geocentric conformally-Cartesian coordinate system given by Damour, Soffel and Xu (1990, 1991).

We have proved that the geocentric Fermi coordinate system and the geocentric harmonic coordinate system are different in theory, but their difference is so small that it can be neglected completely in practice (Han, et al., 1991). The third one is defined only in PNA and permitted arbitrary choice of a time-dependent rotation matrix. Actually, the first and the second ones can be included in the third. As a matter of fact, the time coordinates can be recognized as the same for the three coordinate systems, i.e. all the time axes are the world line of the geocenter, and the spatial coordinates are different only by terms of $|x^i - x_E{}^i|^3/c^2$ if the third one is demanded to show no rotation dynamically, where x^i and $x_E{}^i$ are the spatial coordinates of the event to be considered and the geocenter in the B-system respectively.

Therefore, we suggest that the conventional G-system be defined by a fixed coordinate transformation between the G-system and the B-system which contains no third or higher order terms of $(x^i - x_E{}^i)$. Indeed, we don't think it has any more evident advantages to consider the third or higher order terms in the transformation used to define the G-system. In addition, considering the astrometric applications, we think it is more convenient to cast away the small rotation whose main term is the geodesic precession.

If the harmonic conditions are used in the B-system, the components of the metric may be represented in PNA by the following form (cf. Brumberg, 1990):

$$\left.\begin{aligned} g_{oo} &= -1 + 2\varphi/c^2 + 2\psi/c^4 \\ g_{oi} &= -4U^i/c^3 \\ g_{ij} &= \delta_{ij}(1 + 2\varphi/c^2) \end{aligned}\right] \qquad (3\text{-}1)$$

where φ is the Newtonian scalar potential, ψ the nonlinear part of the scalar potential and U^i the vectorial potential.

In the case that the metric components of the B-system take the form of eq. (3-1), the conventional G-system can be defined by the coordinate transformation as follows (Han et al., 1991):

$$x^i = x_E{}^i + \xi^i - [(\delta_{ij}\varphi^*(x_E) - V_E{}^i V_E{}^j)\xi^j +$$

$$+ (\delta_{ij}\varphi^*_{,k}(x_E) - \frac{1}{2}\delta_{jk}\varphi^*_{,i}(x_E))\xi^j\xi^k]/c^2, \qquad (3\text{-}2)$$

$$TCB = t_E + \xi^i V_E{}^i/c^2 + \{[(3\varphi^*(x_E) + \frac{1}{2}V_E{}^2)V_E{}^i - 4U^{*i}(x_E)]\xi^i$$

$$+ [V_E{}^i\varphi^*_{,j}(x_E) - 2U^{*i}_{,j}(x_E) - \frac{1}{2}\delta_{ij}c\varphi^*_{,o}(x_E)]\xi^i\xi^j\}/c^4, \qquad (3\text{-}3)$$

and

$$t_E = TCG + \frac{1}{c^2}\int_{\tau_o}^{\tau}[\varphi^*(x_E) + \frac{1}{2}V_E{}^2]d\tau + \frac{1}{c^4}\int_{\tau_o}^{\tau}[-\frac{3}{2}\varphi^{*2}(x_E)$$

$$+ \frac{5}{2}\varphi^*(x_E)V_E{}^2 + \frac{3}{8}V_E{}^4 + \psi^*(x_E) - 4U^{*i}(x_E)V_E{}^i]d\tau + \cdots \qquad (3\text{-}4)$$

where (t or TCB, x^i) and (τ or TCG, ξ^i) are the coordinates of an event in the B-system and in the G-system respectively, $x_E{}^i$ and $V_E{}^i$ are the position and the velocity of the geocenter respectively at the moment t_E, τ_o is the moment at which TCB and TCG is defined to be equivalent at the geocenter, and the index star denotes that the mass of the Earth is taken to be zero in the corresponding terms.

The metric components in the given coordinate system can be obtained from the B-system or any other geocentric reference system, such as the geocentric harmonic coordinate system, by using the relation eq. (2-3). At present the metric components of the "kinematically non-rotating geocentric reference system" (KGRS) named by Brumberg (1990) can be used (Huang, et al., 1990).

4. Conclusions

Based on the above discussions, we suggest that the harmonic conditions be used in the definition of the B-system, and the G-system be defined by the B-system and the coordinate transformation given by eqs (3-2), (3-3) and (3-4).

The conventional G-system defined in this way has the following advantages:

1) no ambiguities in the definition;

2) a fixed coordinate transformation between the B-system and the G-system;

3) the spatial coordinate bases show no rotation with respect to those of the B-system at the geocenter, which is advantageous in astrometric applications; and

4) the most important is that it is different from a local inertial reference system at the geocenter, in PNA, only by a small spatial rotation whose main term is the geodetic precession. While a local inertial reference system has many advantages in dynamics, for instance, the potential of external bodies manifests itself only by tidal terms which vanish at the space origin.

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