stiffness, and in simulating damping forces other than those due to viscous damping

A review of some of the chief features of analogue computing is added to assist in the understanding of its application to the problem of design

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## Discussion

The **Chairman** said that they were very grateful to Mr Venning for his paper He would like, first of all, to invite the users, or potential users of analogue computers to put forward their views

**Mr C H Jones** (*Dynamics Engineer*, *Bristol Aircraft*) said that he would like to thank Mr Venning for his lecture which had helped his own understanding of analogue computors quite considerably

First of all he would like to ask a question about the coefficient correction which Mr Venning said that he had included The flutter equation had terms proportional to velocity squared and terms proportional to velocity The Coleman equation had an extra term, which was proportional to velocity and, using the Southampton University computer it was necessary to keep on changing the coefficients appropriately to change the rotor speed He wondered whether it was now possible to change the rotor speed readily

Secondly, he was very much in favour of analogue computers in the sense that if one desired to know the answer to a stability problem one gave the analogue an impulse and waited to see what it did when left to itself That gave him more confidence than having to plough through the whole field in order to find a brick in the middle Using other methods one had to cover the whole range of rotor velocity and frequency of the oscillation

He would like to know what other people's views were on the subject of stability criteria With the analogue computer it was possible to measure the logarithmic decrement at any particular speed and assess the stability by how rapidly the oscillation died out Other methods of solution, such as the classical Coleman method, gave no idea of how severe the oscillation was likely to be Instead, one found the extent of any unstable range

In the representation of stepped dampers, as the Author would probably know, there was a damper on each hinge which took up one plate after another to provide an increase in torque as the blade swung on the hinge It gave a torque displacement relation which was more easily represented by a straight line, as the Author had shown In his own calculations it had been assumed that the dampers could be represented by a viscous damper, having the same energy dissipation per cycle, together with a spring giving the required gradient Agreement between the Author's analogue using that assumption and the model rotor used at Bristol's some years previously, had not been very good He would be glad to have confirmation on the dynamic analogue In the model rotor investigation on blade motion, the blade motion was not sinusoidal and the blades swung on the drag hinges with a constant velocity until they stuck at one end and then swung back again He wondered whether that assumption of sinusoidal motion might be to some extent one of the troubles

His last point concerned the difficulty of representing helicopter impedance He still preferred to confirm impedance by test on the complete helicopter less blades He had more faith in that than relying solely on calculation of impedance based on coefficients obtained by component tests, for instance on tyres and oleos He would be glad to hear other people's experience on that point also

**Mr B H Venning,** in reply, said that on the original computer it had been necessary to change the settings of all the six coefficients in the Coleman equations which were dependent on rotor speed at each value of the speed chosen A determination of stability-rotor speed therefore meant a continual resetting of these coefficient potentiometers The improvement now made involved an additional switch control that made appropriate changes to the coefficients concerned for small variations in rotor speed up to ±20% about a mean value This speeded up the work enormously With coefficients of the form $[\Lambda_2 - \omega^2 (1 - \Lambda_1)]$, the constant part was added separately, and the switch catered for the $\omega^2$ term

Discrepancies had certainly arisen between the results from the model rotor at Bristol's, which was a mechanical analogue, and the electric analogue computer, when dealing with what was supposed to be the same problem This was thought to be due to the nature of the damping in the mechanical system, which was due to stepped friction dampers On the electric analogue this was represented by an equivalent viscous damping coefficient, but unless the motion on the mechanical system remained sinusoidal then the two systems were obviously not the same This view was supported by the observation that the blades were not swinging sinusoidally The attempt to simulate hysteretic damping was a step towards making the two systems more compatible

With regard to the point of hub impedance, it should be possible to simulate the motion of the hub by introducing a " black box " with a transfer function which agreed with the test results from actual measurements at the hub It was not essential to set up actual equations on analogue computers—a lot of work on servo design and auto-pilots was carried out using only units that simply had identical transfer characteristics to the actual system elements

**Mr J M Harrison** (*Chief Aerodynamicist, Westland Aircraft Ltd*) said that he would start off in an unusual manner by answering one of the Author's questions! The Author had asked how long it took to make a Coleman plot The answer is that it takes approximately one man day The analogue computer compared very well since it took about 5 minutes to extract the equivalent information (On second thoughts this is not strictly true To extract the equivalent information would require a series of say five or six runs each taking 5 minutes to execute ) The case referred to was a system of four degrees of freedom If one had to consider, say, eight degrees of freedom it would be impracticable to attempt it by hand, and therefore a computer would be essential

His Company's experience on ground resonance was limited to theoretical investigation They had never had any practical experience of it and simply represented the helicopter by a system of four degrees of freedom, two blade freedom, lateral displacement of, and roll about the c g They assumed that the longitudinal motion did not matter very much He would be glad if someone could confirm if this was so

With reference to aerodynamic loads and vertical motion, he thought that consideration of these would excessively complicate the problem and we would be batting on a sticky wicket The effect of blade flapping on the aerodynamic loads would lead to complicated equations of motion which would be non-linear in a big way The difference between the characteristics in the static and take-off cases could probably be explained for a conventional air/oil strut by variation in air pressure with extension, leading to variation in chassis stiffness

That led to the question of bounce He had not seen it happening He had only seen pictures of the dire results, and was not certain what happened first Did the helicopter become unstable and start to bounce, or was bouncing the destabilising agent ? If the former were the case, was it necessary to investigate non-linear stiffness ?

Coming to the analogue computer in general, his experience had been confined to stability and control investigation, but he was in favour of a special machine instead of a general purpose one

If one had an elaborate general purpose machine, an expert on electronics would be needed all the time With the Author's machine, after a few simple instructions, a child could play with it without getting into serious trouble He had seen experts in a great deal of trouble with a general purpose machine

There had been some slight reference to controversy between digital computers and analogue computers He thought it foolish to start this sort of argument because the two types of machine were complementary There were some jobs which each could do better than the other As an example one could consider helicopter stability and control That could be dealt with by classical linear theory within certain limits, and an analogue computer could do the work very well Unfortunately, however, one came across problems which involved somewhat violent manoeuvres such as transition from level flight to autorotation, or accelerating in the ground cushion and subsequent climb to 100 ft , in which the stability derivatives varied widely His Company had come to the conclusion that such problems must be dealt with on a non-linear basis, and they were well on the way to perfecting a programme for a digital computer So far they had had a small measure of success He hoped to be able to elaborate on this aspect of computing on a more suitable occasion

**Mr J G L Michel** (*National Physical Laboratory*) said that he spoke as a user of machines—both analogue and digital—and emphasised that he had no special knowledge of helicopters Firstly, he would like again to put the cat among the pigeons and refer to remarks by the Author and previous speakers on the relative advantages of digital and analogue computers

He was in general agreement with the views already expressed on the fields in which analogue equipment was most powerful , but though he agreed with the conclusions, he did not always agree with the reasoning whence they were adduced In particular, there had been much talk about the advantages of setting up the problem in the physical form in which it occurred—often even without explicitly deriving the differential equations This process was by no means impossible on most digital computers , a special type of digital computer, the digital differential analyser had been designed with just this object in view, and was obtainable commercially in the U S A , it was likewise possible to programme most general purpose digital computers so that, by means of so-called " interpretive programmes ", the user had only to go through processes similar to those involved in setting up analogue equipment The fact that most users of digital machines had not used them in this way, and, more generally, that not a great deal of use had been made of digital machines in problems of an exploratory nature, was largely due to matters of an organisational nature, connected with the incidence of electronic digital computers on the aircraft industry in this country Digital machines were relatively few , the industry had many important, well-formulated problems of an algebraic nature , whilst such work was occupying existing machines fairly fully people did not view the making available of machines to research engineers for an unspecified time in order to do experimental work in too kindly a light

In one other matter, that of cost, he was not in complete agreement with other speakers As soon as the designer of analogue equipment started thinking in terms of large problems, flexible machines, non-linear problems, the cost rose very rapidly , it was by no means difficult to approach the price of digital machines—indeed he thought that no commercially produced digital machine in this country approached TRIDAC in cost

The real advantages of analogue machines lay in the other points stressed by the Author It was possible to provide an analogue (graphical) output , and, moreover, any or all of the variables entering into the problem could be so represented, so that those concerned with the problem could study simultaneously the behaviour of all variables appearing in the problem and decide upon improvements in the system being solved in the light of those solutions Likewise (particularly on mechanical differential analysers) it was a simple matter to introduce any desired functional relationship between two variables appearing in the problem Lastly, although this again was not an essential point, the main application of analogue machines was in problems of an exploratory nature , in the administrative problems concerned with their utilisation there was not usually the pressure of a mass of routine problems awaiting solution the moment the current user had completed his work—there was freedom to " play " with the problem without the ever present need to justify research of this nature Investigation into the problem could be conducted in the presence (and with the help) of engineers concerned with the physical side of the problem, the solutions being immediately available for assessment To use a digital machine in such work it was necessary to employ a sufficiently flexible system of programming to enable *ad hoc* changes in the problem to be made in the light of the solution, and digital solutions had generally to be prepared in graphical form before an assessment could be made , consequently the present tendency was to employ digital machines for a firmly formulated problem and evaluate solutions over a range of predetermined parameters

He went on to comment on the Author's reference to mechanical differential analysers Such machines were by no means passe A large, servo-connected, mechanical differential analyser had been designed at the National Physical Laboratory since the war, and had been used on a wide variety of problems originating in the aircraft industry Its accuracy was appreciably higher than any electronic analogue computer (not that electronic machines, with errors of the order of 1%, were not entirely adequate for the vast run of engineering problems), and it even compared favourably in speed with the (admittedly rather slow) electronic computer described by the Author In the field of non-linear problems, and problems requiring graphical input, it was much more flexible than existing electronic analogue computers It also had more flexible arrangements for providing graphical output , in particular it was easy to plot any one variable in the problem against any other, a facility of which much use had been made in studying second order non-linear differential equations by means of the so-called phase-plane method

Lastly he would like to refer to remarks made by previous speakers about the use of impedance and transfer functions in non-linear problems He felt that little use could be made of such concepts since the essential point about non-linear responses was that they did not vary linearly with the amplitude of the input, whilst the whole idea of impedance and transfer functions depended essentially upon the concept of linearity

He would like to draw to the attention of members of the Association the wide variety of digital and analogue computing facilities available at the N P L and hoped that he would be able to assist them in their problems

**Mr W E Hooper** (*Bristol Aircraft*) (*Student Member*) said that Mr Harrison had made some remarks about the Southampton computer not being a general purpose computer and had said that he was glad that that was so He personally would go further and suggest that it was hardly specialised enough, or could be improved if further specialised

There was a distinct similarity between the ground resonance equations and the flutter equations The ground resonance equation might be written

$$[-A w^2 + [B\Omega + c] iw + [D\Omega^2 + E\Omega + F]] \left\{x\right\} = 0$$

which differs from the flutter equation only in the term $[E]\Omega$ Could not then the Ground Resonance problem be solved more quickly and easily by putting it on the flutter simulator which could solve the equation as it stood by simply adding the $[E]\Omega$ term to the structural stiffness [F] term as the speed $\Omega$ was changed ?

Alternatively if a special simulator was to be built anyway why not build one as an extension of the flutter simulator to include $[E]\Omega$ and so traverse the speed range very quickly ?

The **Author,** in reply, said that the initial design of the Southampton computer had to some extent followed that of the R A E Flutter Simulator, but that to keep the project reasonably simple no system of automatic coefficient switching similar to the "Airspeed " control had been included originally The ground resonance equations were similar to those of the flutter problem, and they could, no doubt, be solved on a flutter simulator, with suitable choice of zero coefficients, and rotor speed replacing airspeed The computer at Southampton was, however, available for use by, or for the helicopter industry for the analysis of ground resonance problems, or for that matter any others within its capabilities The advantage of having it there was that it could also be continuously developed to tackle other useful approaches to the problem, as they were now doing on the effects of different types of friction

**Mr D V Blake** (*Control Mechanisms and Electronics Division, National Physical Laboratory*) said that he would like to add one or two points to what had been said by Mr Michel He, personally, knew little about helicopters and would confine his remarks to the analogue computing aspect

First of all he thought that the author had given the impression that analogue computers were very cheap and he did not think that that impression should go too far The price of a complete flutter simulator equipment would be of the order of £10,000 If it was desired to deal with other types of problem it would probably be necessary to buy more equipment The cost of developing and making the equipment oneself is not much less if all the “ hidden costs ” are taken into account Comparing analogue and digital equipment, the analogue machine is cheaper to buy and much cheaper to maintain than digital Nevertheless the cost of using an analogue machine works out at several pounds per hour The commercial machine would work faster than the one described by the author, which offsets some of the extra cost

Quite a number of analogue machines will give a complete solution in a fraction of a second This means that the amplifiers must have a much higher frequency response than those which the author has described There is no fundamental difficulty about this , our amplifiers are tested for a few degrees of phase shift at 20 k/cs

He himself was dealing with a general-purpose machine and he still thought that, despite what had been said during the evening, there was a lot to be said in favour of the more general machine Of course, a specialised machine is smaller and easier to operate, but it is limited to a particular set of equations A small change in the design of the helicopter or another non-linearity considered may necessitate a change of set-up, extra units, more ironmongery This is not particularly troublesome when doing more or less routine studies of the solution of particular sets of equations It becomes important when doing more general studies such as comparing the performance of a helicopter with the simulator solution in an attempt to obtain a more realistic or more accurate simulator set-up Another example of a general study is the attempt to improve the performance of a helicopter by trial and error changes of parameters or design on the simulator

He imagined that in helicopter study one could not easily set up a direct analogue of the physical system This was worth considering as it had been found to be a useful scheme for many types of problem though it did need a general-purpose machine The advantage of the direct analogue was that the primary coefficient and parameters appeared explicitly and each could be changed by a single knob on the simulator In the Coleman and similar equations each coefficient was some function of the primary coefficients If, say, the stiffness of a shaft was altered, it was necessary to re-calculate and then alter nearly all the coefficients in the set of equations

He wished to say something about recorders It was possible to get $\frac{1}{2}$ per cent accuracy if one was prepared to pay for it At the National Physical Laboratory they had a high-speed repetitive display and a stroboscopic system for reducing the frequencies so that the response could be plotted on a paper recorder (about 1 per cent ) This speeds up the work considerably compared with a low-speed display and direct recording

With regard to drift, it was quite true that drift was not particularly troublesome when dealing with linear systems It altered the mean level but if one kept within saturation limits, it did not introduce error If, on the other hand, a lot of work was being done with non-linear systems, particularly with positive feedback where the drift of individual units was exaggerated, there would be trouble So far they had managed without automatic drift correction but if much serious work was to be done on non-linear systems it would have to be incorporated

**The Author,** in reply, said that he had not meant to give the impression that all analogue computers were cheap The one at Southampton was reasonably so, but their costs did not bear any relationship to commercial costs Commercial analogue computers were usually general-purpose machines because as many as possible had to be produced to keep down the cost , and as the sale of such machines was small, the firms were anxious not to restrict sales further The average user suffered therefor, from having to deal with a system of cords or plugs, but that was inevitable There was a lot to be said for a person, or organisation building their own if they knew what it was to be used for, as the result was often a simpler machine to handle

Dealing with the question of operating frequencies, amplifier bandwidths of 20 kc/s were quite normal, but unless the solution frequency was considerably below that, errors were introduced in the damping coefficient which became serious in the critical region of instability Considering the simple equation $x + \omega_0^2 x = 0$, the true undamped solution was multiplied by a term $\exp [\omega_0^2 (T_1 + \frac{T_2}{2}) - \frac{1}{T_0}]$ where $T_0$, $T_1$ and $T_2$ are all functions of the amplifier and integrator design $T_1$ and $T_2$ are the integrator and adding amplifier high-frequency bandwidths of the order of $10^{-5}$ secs , and $T_0$ is the integrator low frequency bandwidth, which may be 50,000 secs With these figures, taking the error in amplitude which occurs after, say, ten cycles of the solution, this error is less than 0 1% for $\omega = 1$ rad/sec , but increases to 20% at $\omega = 200$ (about 30 c/s ) The advantage of a slow computer becomes apparent

The strobing recorder was a very elegant solution to the problem of recording higher frequency traces, but, at present, it was not in common use The ordinary commercial pen recorder, whether servo-driven or not, did not have an amplitude response in its range of operating frequencies much better than ± 2%

**Mr J S Shapiro** (*Consulting Engineer*) (*Founder Member*) said that he had been doing some mental arithmetic Was he correct in thinking that one speaker had said that a machine cost £100 per hour ?

**Mr Blake** No, a few pounds per hour

**Mr Shapiro** said that he was glad to hear that because otherwise it would not have compared well with the ordinary calculating machine

He was sorry that so little had been said about helicopters He wondered whether anyone from Saunders-Roe had gained any knowledge through the work of the computer

The characteristics of the fuselage and the frequency of the hub were very complex factors He could not visualise a black box anything like that Another point which should be emphasised was that Coulomb friction actually gave an equivalent damping which went down with the amplitude, so that when applied to the so-called ground resonance there was always some magnitude at an initial impulse which would make it unstable, if stability was achieved by damping That was a very important point though a very simple one

He thought that one speaker had said that damping could not be calculated from the Coleman equation It could be done but it was very laborious He did not think the computer did anything in that respect which could not be done otherwise

He believed that the Author had come up against the question of hub frequency and one point there, which had probably been cured, was the constancy of that frequency or its erratic behaviour He did not know whether the Author could add anything on that point

The **Author,** in reply, said that the first problem put on the computer related to one of the earliest designs of the Skeeter, using data that referred to the very early stages The computer could not, unfortunately, be said to have helped in the development of that machine as the former was not completed until after the exhaustive trials of the company concerned had cleared away any troubles

Damping could in fact be calculated from Coleman-type solutions, but the computer solution was obviously much quicker, and no additional time was required, unlike the analytical work

Representation of fuselage characteristics by satisfactory parameters, or by suitable transfer impedance, was a continually recurring point, and only served to emphasise the difficulties of treating the system as a whole

**Dr O P Mediratta** (*Louis Newmark Ltd* ) said that his Company had built a flight simulator which they used for the design and development of Automatic Controls for aircraft and helicopters They had no experience of flutter or resonance simulation

The main difficulty with a simulator was not the limitations of the machine but what was put into it   it was indeed a very difficult problem to find a representative set of constants to introduce   They had found that it was better to break down the equations of the system to be simulated in terms of constants which could be identified with various elements in the loop   If there were a number of different components which contributed to inertia, damping, etc , it paid to break them down to have a clear concept of the individual contributions   One could then simulate the transfer function of each component separately   Provided one was prepared to accept the complication involved, there was not much which could not be simulated   Non-linearities could easily be introduced by the use of diodes

A considerable amount of simulation could be avoided by including the physical components themselves as part of the loop   His own Company's applications were in the field of Autopilots where it was a relatively easy matter to include such components but he was not sure if that could be done with flutter simulation and he wondered if any thought had been given to that aspect at Southampton

The **Author,** in reply, said that the introduction of physical links into a system was one of the great features put forward for the use of analogue computers, but that, of course, meant working in real time   Up to the present time this had been avoided as the solution frequencies were such that serious errors would be introduced   Future design would overcome these to some extent, and consideration would be given to the incorporation of physical links, although this was probably more difficult outside an aircraft factory, simply from the mechanical problems involved

**Mr C Faulkner** (*Saunders-Roe Ltd*) (*Associate Member*) said that previous speakers had made several references to his Company's activities, and he now felt there had been sufficient provocation to speak

In his Company's experience, the helicopter had often proved to be the most reliable computer, particularly in ground resonance analysis   In their case they had had a small helicopter with a big ground resonance problem   At that time they had found that the quickest and most convenient way to investigate and cure it was to conduct full scale experiments on the machine itself   They had measured and analysed the effects of external excitation with different combinations of blade drag and undercarriage damping, and had ultimately arrived at a satisfactory condition for the prevention of resonance   Analogue computer results available later had lined up moderately well with the test results

We were still suffering from a pronounced lack of fundamental knowledge when dealing with most advanced analytical problems on helicopters and, hence, a " purist " approach was not at present always the correct one, although possibly the most aesthetically satisfying

There was little doubt that in the majority of cases and, where convenient, it was preferable to conduct experiments on the actual components and/or the complete aircraft   One had to be extremely wary of producing extra equipment needlessly   For example, it was desirable on all counts to couple actual power control systems into the computer circuit rather than to attempt to simulate them electronically   This sort of argument could be extended to cover much of the " hardware " in the helicopter business

In conclusion, he would hasten to add that he was definitely not attempting to condemn the analogue computer as such, but merely emphasising that it was but a means to an end

The **Author,** in reply, agreed that the only fair test was on the design itself, because all the characteristics could never be completely simulated, and computer designers were only too well aware of the difficulties involved in deciding the magnitudes of the various parameters   If an existing machine was found to have any trouble the obvious thing to experiment with was the machine itself as far as this was possible   However, in the case of new projects, the problem was vastly different, as an analysis on a simulator or any other computer should at least narrow the field considerably, even if the solutions obtained referred to something much simpler than the actual machine, and the data was not perfect   The computer can at least indicate the most promising fields of design and eliminate the worst, with considerable economy of time and effort   This was the aim, and experience would show the ways in which the present over-simplified approach could be improved

The **Chairman** said that the essence of the problem, if one was engaged on the manufacturing side, was not necessarily to have a large digital computer waiting which required specialists and extensive programming and programming staff but to have available some means which allowed a reasonably wide but quick examination so that one could be sure of not running into trouble Hence, if trouble were encountered, one could tackle the problem on known broad lines, the computer having at least made it clear which lines of attack could be discarded as unproductive

On a small helicopter one could juggle with the physical parameters by actually making and trying out changes in the aircraft This approach was possibly acceptable with a small helicopter because the changes might not take an unreasonable amount of time But with a big aircraft this approach was quite out of the question New things had to be designed and one might spend six months on the redesign, stressing and manufacture—and not get the right answer after that Hence the computer offered great advantages in indicating which were the blind alleys to be cleared out of the way It might be too much to expect to find the precise modifications to introduce at the very first shot but one could at least find out the generally correct line

What appealed to him was that if there was available a reasonable amount of knowledge of the nature of the problem, together with the elements of an analogue computer at an early enough stage, the analogue computer could be brought up to date to deal with the specific problem , check tests could thus be done at the earliest possible moment One of the biggest attractions of the analogue computer to a firm was that it could be operated and looked after by the design staff who would know roughly what to expect and what was a ridiculous answer and what was not It avoided the recruitment of specialist staff for programming, which might be essential in the use of digital machines

On behalf of the Association he would like to thank Mr Venning for his most fluent and lucid exposition of a difficult subject He himself had come to the meeting knowing nothing about the design of analogue computers but he had learned something about how they were put together and on what principles they were based The emphasis which had been placed on applying the computer to the bugbear of ground resonance was most encouraging, as it showed how this new tool could be used with effect by designers in their endeavours and ensure that each design was clear of this serious trouble

The vote of thanks to the Author was carried by acclamation