# 624 HEKTOR: A BINARY ASTEROID?

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Dunlap and Gehrels (1969) have published lightcurves of the Trojan asteroid 624 Hektor. They proposed a conventional explanation in which Hektor is regarded as having the shape of a cigar. Two circumstances suggest, but do not prove, that Hektor is a binary asteroid. (1) The cigar shape at the conventional density of stony meteorites (3.7 $g\text{-}cm^{-3}$) appears to produce stresses that may well exceed the crushing strength of meteoritic stone. (2) The lightcurves exhibit an asymmetry changing with time that suggests librations of two ellipsoidal components. Observations are clearly required to look for these periodicities when we shall again be nearly in the plane of Hektor's revolution (or rotation) in 1973. An additional supporting lightcurve is desirable in 1972 and also in 1974. The periods of libration are probably nearly 1 day, if they exist, so that observations should be made from more than one geographic longitude in 1973. The present paper is an exposition on these considerations.

## THE CIGAR-SHAPED MODEL

Dunlap and Gehrels (1969) employed a geometric albedo of 0.28 ± 0.14 and a cigar shape consisting of a right circular cylinder capped by two hemispheres at the ends. The radius of the cylinder and of the hemispheres is 21 km, and the height of the cylinder is 70 km. Mathematical convenience is served by replacement of this model by an ellipsoid of Jacobi with the same ratio of end-on to side-view cross sections. The ratio of the intermediate semiaxis to the largest semiaxis is as follows:

$$\frac{b}{a} = 0.32$$

A convenient graph for finding the ratio of the smallest semiaxis $c$ to the largest has been published by Chandrasekhar (1965). His figure 2 (p. 902) yields

$$\frac{c}{a} = 0.23$$

The density of this ellipsoid at which equilibrium occurs so that no stresses are applied, i.e., so that the pressure is everywhere isotropic, can also be found from another graph by Chandrasekhar (1965, fig. 3, p. 903). The abscissa in this case is Arccos $(c/a) = 77°$, whence the ordinate is

$$\frac{\Omega^2}{\pi G \rho_e} = 0.17$$

where $\Omega$ is the angular velocity of rotation,

$$\Omega = \frac{2\pi}{P}$$

$P$ is the period of rotation ($2.492 \times 10^4$ s according to Dunlap and Gehrels, 1969), and $G$ is the universal constant of gravitation. Solution for the density $\rho_e$ of the asteroid in equilibrium yields 1.7 g-cm$^{-3}$. It follows that if Hektor is a single body, either it is of lower density than a carbonaceous chondrite of type I or it is not in equilibrium.

## STRESS IN THE CIGAR-SHAPED MODEL

Computation of a representative stress at the density of meteoritic stone is required to assess the viability of the Jacobi ellipsoid as a large meteoritic stone. Jardetzky (1958) provides the appropriate mathematical discussion. His equation (2.2.13) on page 31 can be transformed to read

$$\frac{\Omega^2}{G} = L_y - \frac{c^2}{b^2} L_z = L_x - \frac{c^2}{a^2} L_z = \frac{\pi \rho \Omega^2}{\pi G \rho} \tag{1}$$

where $\rho$ without subscript refers to the actual density, and the potential takes the form

$$V = \frac{C'}{G} - \frac{1}{2}\left(L_x x^2 + L_y y^2 + L_z z^2\right) \tag{2}$$

where $x$ is taken along the largest semiaxis, $z$ along the shortest, and $y$ along the intermediate one; the origin lies at the center of the ellipsoid; and $C'$ is an arbitrary constant. Poisson's equation takes the form

$$\nabla^2 V = L_x + L_y + L_z = 4\pi\rho \tag{3}$$

Solution of equations (1), (2), and (3) for $L_z$, $L_x$, and $L_y$ yields

$$L_z = 2\pi\rho \, \frac{1 - \Omega^2/\pi G \rho}{1 + c^2/b^2 + c^2/a^2} \tag{4}$$

$$L_x = \frac{c^2}{a^2} L_z + \pi\rho \frac{\Omega^2}{\pi G \rho} \tag{5}$$

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$$L_y = \frac{c^2}{b^2} L_z + \pi\rho \frac{\Omega^2}{\pi G\rho} \tag{6}$$

The pseudopotential including the centrifugal term is

$$\frac{C_I}{G} = \frac{C'}{G} - \frac{1}{2}\rho \left[\left(L_x + \pi\rho \frac{\Omega^2}{\pi G\rho}\right)x^2 + \left(L_y + \pi\rho \frac{\Omega^2}{\pi G\rho}\right)y^2 + L_z z^2\right] \tag{7}$$

where $C_I/G$ is the pseudopotential. The pressure $p_s$ at the surface is given by

$$\begin{aligned} p_s &= C_{I_s} \\ &= C' - \frac{1}{2}G\rho \left[\left(L_x + \pi\rho \frac{\Omega^2}{\pi G\rho}\right)x_s^2 + \left(L_y + \pi\rho \frac{\Omega^2}{\pi G\rho}\right)y_s^2 + L_z z_s^2\right] \end{aligned} \tag{8}$$

$$\left(\frac{x_s}{a}\right)^2 + \left(\frac{y_s}{b}\right)^2 + \left(\frac{z_s}{c}\right)^2 = 1 \tag{9}$$

where $x_s$, $y_s$, $z_s$ refer to a point on the surface.

At the equilibrium value of the density $\rho_e$, $p_s$ vanishes all over the surface. We compute the difference due to a different value of $\rho$ and consider only the differences in pressures along the principal axes, whence

$$\begin{aligned} \Delta p_{s_a} &= -\frac{1}{2} G(\rho - \rho_e)\left(L_x + \pi\rho \frac{\Omega^2}{\pi G\rho}\, a^2\right) \\ \Delta p_{s_b} &= -\frac{1}{2} G(\rho - \rho_e)\left(L_y + \pi\rho \frac{\Omega^2}{\pi G\rho}\, b^2\right) \\ \Delta p_{s_c} &= -\frac{1}{2} G(\rho - \rho_e) L_z c^2 \end{aligned} \tag{10}$$

In terms of $a$, $b$, $c$, and $\Omega^2/\pi G\rho$, these expressions become

$$\begin{aligned} \Delta p_{s_a} &= -\pi G\rho(\rho - \rho_e)\left(1 + \frac{a^2}{b^2} + \frac{a^2}{c^2}\right)^{-1}\left[1 + \left(\frac{a^2}{b^2} + \frac{a^2}{c^2}\right)\frac{\Omega^2}{\pi G\rho}\right] a^2 \\ \Delta p_{s_b} &= -\pi G\rho(\rho - \rho_e)\left(1 + \frac{a^2}{b^2} + \frac{a^2}{c^2}\right)^{-1}\left[1 + \left(\frac{b^2}{a^2} + \frac{b^2}{c^2}\right)\frac{\Omega^2}{\pi G\rho}\right] a^2 \\ \Delta p_{s_c} &= -\pi G\rho(\rho - \rho_e)\left(1 + \frac{a^2}{b^2} + \frac{a^2}{c^2}\right)^{-1}\left(1 - \frac{\Omega^2}{\pi G\rho}\right) a^2 \end{aligned} \tag{11}$$

Next we subtract the hydrostatic part or mean to find

$$\Delta p'_{s_a} = \frac{1}{3}\Omega^2(\rho - \rho_e)\left(\frac{1 + b^2/a^2 + b^2/c^2}{1 + a^2/b^2 + a^2/c^2} - 2\right)a^2$$

$$\Delta p'_{s_b} = \frac{1}{3}\Omega^2(\rho - \rho_e)\left(1 - 2\,\frac{1 + b^2/a^2 + b^2/c^2}{1 + a^2/b^2 + a^2/c^2}\right)a^2 \tag{12}$$

$$\Delta p'_{s_c} = \frac{1}{3}\Omega^2(\rho - \rho_e)\left(1 + \frac{1 + b^2/a^2 + b^2/c^2}{1 + a^2/b^2 + a^2/c^2}\right)a^2$$

These are the hydrostatic pressures that would be required on the surface to keep the internal pressures isotropic. In their absence, an anisotropic pressure will appear at the center with signs opposite to those in equations (12). At $\rho = 3.7$ g-cm$^{-3}$ as for meteoritic stone, we have

$$\left.\begin{aligned} -\Delta p'_{s_a} &= 7.6a^2 \\ -\Delta p'_{s_b} &= -3.3a^2 \\ -\Delta p'_{s_c} &= -4.6a^2 \end{aligned}\right\} \quad \text{nN-m}^{-2} \tag{13}$$

(or $7.6 \times 10^{-8}\ a^2$, $-3.3 \times 10^{-8}\ a^2$, and $-4.6 \times 10^{-8}\ a^2$ dyne-cm$^{-2}$, respectively). This loading resembles that in a conventional unidirectional compression test of

$$P' \simeq 12 \text{ nN-m}^{-2} \tag{14}$$

(or $1.2 \times 10^{-7}\ a^2$ dyne-cm$^{-2}$).

The cross sections in side view and end-on of Dunlap and Gehrels' (1969) model impose $a = 77$ km whence $P' \simeq 0.7$ MN-m$^2$ (7 bars), compared with a crushing strength of about 1 MN-m$^{-2}$ (10 bars) for the Lost City meteorite (R. E. McCrosky, private communication, 1971). A geometric albedo of 0.14 (as for the brightest parts of the Moon) makes Hektor larger in dimension by a factor of $\sqrt{2}$, whence $P' \simeq 1.4$ MN-m$^{-2}$ (14 bars). Finally, a geometric albedo of 0.07 (upper limit for the dark part of Iapetus according to Cook and Franklin, 1970) introduces a further factor of $\sqrt{2}$ in dimension and raises $P'$ to about 3 MN-m$^{-2}$ (30 bars). A large body like Hektor will have weak inclusions and thus have a lower bulk strength than a small body like the Lost City meteorite. Moreover, meteoritic bombardment will tend to induce failures as well. All this casts doubt on the model of Dunlap and Gehrels (1969) and suggests that a binary model may be more satisfactory.

## THE BINARY MODEL

The lightcurves of Gehrels (Dunlap and Gehrels, 1969) are a heterogeneous lot obtained with different telescopes, photometers, and skies. The best quality in observations occurs at the largest and smallest amplitudes. The largest amplitude was observed on April 29 and May 1, 1968, with the 152 cm reflector at Cerro Tololo. The zero point of magnitude was obtained only on the second night. The smallest amplitude was observed on February 4, 1965, with the 213 cm reflector at Kitt Peak.

The author has carried out an analysis that can be called only a reconnaissance. An unusual amount of work has been required compared to the usual solution for an eclipsing binary. Interim light elements were derived to plot the lightcurves of 1965 and 1968 against phase. There is no evidence for any differences between successive half-periods, so each night's observations were plotted on a single half-period. A notable feature of the 1968 observations is an asymmetry such that the maxima occur 0.012 period early. The descent into the minimum is slower than the rise from it. In 1957 this asymmetry appears to be at the limit of detection but in the opposite direction. The 1957 observations are thin and were made at the Radcliffe Observatory, Pretoria, at the 188 cm reflector.

The most obvious explanation for the asymmetry is libration of the components about the radius vector joining them. This hypothesis can be tested by extensive observation at the next opposition in which Earth is near the plane of Hektor's revolution (or rotation).

The libration was taken into account in the rectification. The formula used for rectification of the intensity was

$$I^R = \frac{I}{[1 - z\cos^2(\theta - \theta_0)]^{1/2}} \tag{15}$$

where $I$ is the observed intensity, $I^R$ the rectified intensity, $z$ the photometric ellipticity, $\theta$ the phase angle, and $\theta_0$ the phase angle at which we look along the major axes of the components. We use here the standard preliminary model of two similar ellipsoids similarly situated. Rectification for phase was effected by the formula

$$\sin^2\Theta = \frac{\sin^2\theta}{1 - z\cos^2(\theta - \theta_0)} \tag{16}$$

where $\Theta$ is the rectified phase angle. Solution for $z$ in the standard graphical plot of $I^2$ versus $\cos^2(\theta - \theta_0)$ employed

$$\bar{I}(\sin^2\theta) \equiv \tfrac{1}{2}[\bar{I}(\theta) + \bar{I}(\pi - \theta)] \tag{17}$$

and yielded

$$z = 0.745 \pm 0.015 \qquad \theta_0 = 0.012P = 4.32^\circ \tag{18}$$

Execution of the usual procedures using the tables of $\chi$ functions of Merrill (1950) produced the following solution:

$$l_0 = 0.777$$

$$k = 0.30 \pm 0.02$$

$$a_g = 0.42 \pm 0.04$$

$$i_r = 71^\circ \pm 2$$

where $l_0$ is the brightness at minimum in units of that outside eclipse, $k$ is the ratio of radii of the components, $a_g$ is the largest semiaxis of the larger component in units of the distance between the asteroids, and $i_r$ is the inclination of the relative orbit in the rectified model in which the components are spheres. The most extreme solution pointing to the highest density of the components is that for equal bodies, $k = 1.00$. In this case $a_g = 0.38$ and $i_r = 70\overset{\circ}{.}1$.

The lightcurve for February 4, 1965, shows an apparent elliptic variability only with $z = 0.154 \pm 0.002$. The unit of intensity corresponds to an absolute magnitude of 7.63 compared with 7.70 for May 1, 1968. Combination of the results from 1968 and 1965 yields for the inclinations of the orbit and the ratios of principal axes the following results:

| *Year* | $i$ | |
|---|---|---|
| 1965 | $28\overset{\circ}{.}35$ | $b/a = 0.493$ |
| 1968 | $83\overset{\circ}{.}24$ | $c/a = 0.454$ |

Here $k = 1.00$ has been chosen to present the most extreme case. The mean density of the components comes to 9.6 g-cm$^{-3}$. Correction for finite sizes of the components in ellipsoidal shape yields a drop of a few tenths, and adoption of $k = 0.60$ would push the density down to that of iron, about 8 g-cm$^{-3}$. Before one blithely proposes that Hektor is a binary composed of two solid iron ellipsoids, it seems only prudent to question the rectification, which is large. The extreme form of doing this is to consider a contact binary as a model.

## THE CONTACT-BINARY MODEL

In the case of a contact binary, the rectification cannot be considered separately. In this reconnaissance, a start was made by assuming (1) that $\sin^2 \Theta_e \equiv 1$ where $\Theta_e$ denotes the rectified phase angle at external contact; (2) that the ratio of radii $k \equiv 1$; and (3) that the fractional depth of greatest eclipse $\alpha_0 \equiv 1$; i.e., that central eclipse occurred. This representation failed, so that two progressions away from this model were next considered. One such sequence of models had $k = 1$ but $\alpha_0$ decreased successively. No models fitting the observations could be found. Next, a sequence of grazing annular and total eclipses was tried ($\alpha_0 \equiv 1$), so that $k$ was varied. This sequence yielded a satisfactory representation of the observations at $k = 0.80$:

| *Year* | $i$ | $\alpha_0$ | $z$ |
|---|---|---|---|
| 1965 | 31°16 | 0.020 | 0.1403 |
| 1968 | 86°27 | 1.000 | 0.5944 |

$$a_g = 0.5556$$

$$b/a = 0.635$$

$$c/a = 0.582$$

$$\rho = 2.4 \text{ g-cm}^{-3}$$

Correction for finite sizes of components reduced $\rho$ to 2.0 g-cm$^{-3}$.

The two directions of Hektor in 1965 and 1968 lie some 110°4 apart, whence it appears that the pole of revolution of the binary must lie near the plane of Hektor's heliocentric orbit. The librations will cause the epochs of the shallower minima to be not very reliable in derivations of this pole.

Better light elements have been derived by bootstrapping across successively longer intervals on the assumption that the pole lies in the plane of the orbit. The corresponding sidereal period is

$$P = 0^{\mathrm{d}}2884483 \pm 0.0000002$$

The ellipsoids can be replaced individually by two point masses of spheres to yield the same moment of inertia about the smallest semiaxis. Then the period of libration can be calculated in the field of the other represented as a point mass. The results indicate a period of libration of two to four periods of orbital revolution. The near equality of the heights of maxima in 1968 during two long nights at a 2 day interval strongly suggests periods of about 1 day for the librations.

## FURTHER STUDIES NEEDED

Observations can only be planned efficiently if the above analysis is completed by using many values of the photometric ellipticity $z$ for 1968. It is to be expected that there will be two groups of solutions–those at high densities described above and belonging to a narrow range of $z$ near 0.745 and those at low densities and belonging to a wider range of $z$ near 0.60. It is to be hoped that this latter group will reach up to or approach more conventional densities like that of meteoritic stone.

Observations will be needed in 1972, 1973, and 1974 to settle the choice between the cigar-shaped and binary models by seeking the periodicities in the librations and to improve the accuracy of the photometric solutions either by observation of annular and total eclipses or by observation of very deep partial eclipses on either side of the orbital plane (or equatorial plane). This implies an extensive campaign in 1973 at one observatory coupled with an international campaign during the dark of the Moon closest to opposition. The best available range of photometric solutions will be required for intelligent planning of the extensive campaign at one observatory. The international campaign would be aimed at covering the suspected 24 hr periods of the librations. Good lightcurves at single epochs would be desirable in 1972 and 1974.

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## DISCUSSION

HARTMANN: I wish to make a comment on irregular shapes of asteroids. Cook's evidence that Hektor would not retain an irregular shape rests on the crushing strength he assigns to the material. It appears that Cook's value of 1 MN-m$^{-2}$ (10 bars), based on the Lost City chondrite, is unusually low. Wood (1963) lists compressive strengths of eight chondrites; they range from about 6 to 370 MN-m$^{-2}$ (60 to 3700 bars) although Wood notes that some more crumbly chondrites are known. The one iron listed has a strength of about 370 MN-m$^{-2}$. Thus, according to the 0.7 MN-m$^{-2}$ (7 bar) stress found by Cook for a Jacobi ellipsoid of Hektor's dimensions and chondritic density, the asteroid could be quite irregular.

How large an asteroid can be irregular? A simple estimate comes from the size of a nonrotating spherical asteroid whose central pressure $P_c$ is just equal to the crushing strength. Under this condition the central core begins to be crushed and hence lacks rigidity. Larger asteroids would have a nonrigid interior and could thus deform to an equilibrium shape. For typical chondritic strengths we have

$$P_c = \frac{2\pi\rho^2 G}{3} r^2 = 1 \text{ to } 370 \text{ MN-m}^{-2} \text{ (10 to 3700 bars)}$$

Thus, the diameter = 46 to 880 km (if $\rho = 3.7$).

It is concluded that asteroids substantially larger than Hektor (42 by 112 km) can be irregular in shape. Such irregularity is indeed evidenced by lightcurves and is theoretically expected because many if not most asteroids are probably fragmentary pieces that have resulted from collisions.

## DISCUSSION REFERENCE

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