above suggests that the best choices for B are perhaps HHT and TTH. With these choices, the probability B wins is ½. However, if B chooses HTH and TTH then the probability B wins is greater - it is $^9/_{16}$!

Both these results can be obtained in exactly the same way as the results above: by drawing the path diagram and solving a simple set of linear equations which are obtained by conditioning on the outcome of the next step.

Reference

1. M. Gardner, Mathematical games, *Scientific American* **231** (4) (1974), p. 120.

R. J. REED

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Correspondence

DEAR EDITOR,

At the end of his Note 79.36 in the July 1995 *Gazette*, Nigel Backhouse states that he does not know the full pedigree of the result

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx = \tfrac{22}{7} - \pi.$$

It first appeared, together with some refinements, in a short paper by D. P. Dalzell titled 'On $\tfrac{22}{7}$', published in the *Journal of the London Mathematical Society* ***19*** *(1944) pp. 133–34.*

I first encountered this result, and an inequality deduced from it, as a question set in the Oxford & Cambridge Schools Examination Board's Higher Certificate Mathematics, Group III, viz. 1949, Paper 7, no. 4. Not until the early 1950s did I learn of its source from the author himself, who was then a much-valued part-time colleague.

It was quoted on p. 29 of the Mathematical Association booklet *50% Proof* from the December 1983 *Gazette* p. 247; but no basic reference was included, even though I had supplied one to the editors. Note 79.36 seems to elucidate the somewhat cryptic comment (ibid. p. 29) by Des MacHale.

Yours sincerely,

FRANK GERRISH

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DEAR EDITOR,

I cannot believe that what follows is original, since the basic facts are contained in my 1947 reprint of Rouse Ball, but I should be interested to learn whether any of your readers can explain the following conundrum.

In many ways, e and π are similar – they are both transcendental and they often occur together, primarily because of the Euler(?) formula $\exp i\pi + 1 = 0$ – so why is it that their continued fraction expressions are so different?

Specifically,

$$e = 2 + \frac{1}{1\,+}\,\frac{1}{2\,+}\,\frac{1}{1\,+}\,\frac{1}{1\,+}\,\frac{1}{4\,+}\,\frac{1}{1\,+}\,\frac{1}{1\,+}\,\frac{1}{6\,+}\,\frac{1}{1\,+}\,\frac{1}{1\,+}\,\frac{1}{8\,+}\,\frac{1}{1\,+}\,\frac{1}{1}\,\cdots$$

which shows a clear and simple pattern, whereas

$$\pi = 3 + \frac{1}{7\,+}\,\frac{1}{15\,+}\,\frac{1}{1\,+}\,\frac{1}{292\,+}\,\frac{1}{1\,+}\,\frac{1}{1\,+}\,\frac{1}{1\,+}\,\frac{1}{2\,+}\,\frac{1}{1\,+}\,\frac{1}{3}\quad\frac{1}{+\,1}\,\cdots$$

does not, and indeed Rouse Ball states that no law has (yet) been found for π.

Yours sincerely

ALAN D. COX

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DEAR EDITOR,

Note 78.12 (*Math. Gaz.* **78** (482) July 1994 p. 190) is of additional interest in that it provides a delightfully simple and elegant way of printing a list of all primitive Pythagorean Triples having their numbers all less than any specified maximum. The Logo procedure, pytrip, to do this is defined by

```
to pytrip :max
  pt 2 1 :max –l
end
```

with the procedure pt defined by

```
to pt :m :n :max
  if (:m * :m + :n * :n) > :max [stop]
  (pr :m * :m + :n * :n 2 * :m * :n :m * :m – :n * :n)
  pt 2 * :m + :n :m :max
  pt 2 * :m – :n :m :max
  pt 2 * :n + :m :n :max
end
```

This program runs very quickly since no unnecessary repetitions are involved in the recursions. To print out only those triples with pairs differing by 2, replace the third (print) line by

```
if :n = 1 [(pr :m * :m + :n * :n 2 * :m * :n :m * :m – :n * :n)]
```

and to print only those triples in which the largest two numbers differ by 1, replace it by

```
if (:m – :n) = 1 [(pr :m * :m + :n * :n 2 * :m * :n :m * :m – :n * :n)]
```

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Finally, and perhaps rather remarkably, to print only those triples in which the smaller two numbers differ by 1 [N. B. article by Gillian Hatch, *Math. Gaz.* **79** (484) March 1995, p. 51], it is sufficient to omit the last two lines of the procedure pt. Readers will have no difficulty in satisfying themselves of the truth of each of these statements.

Yours sincerely,

ROBERT MACMILLAN

43 Church Road, Woburn Sands MK17 8TG

DEAR EDITOR,

I much enjoyed reading Tony Crilly's account of the life of Arthur Cayley in the July 1995 *Gazette*. It is pleasing to see that it has been recognised that the history of mathematics is a worthwhile source for an article.

I should like to draw attention to Cayley's contribution to Projective Geometry. Those of us who were brought up on the subject will remember being told that he was the first to demonstrate that metrical ideas (measurement of length and angle) were natural developments from the projective subject which was based on the propositions of incidence (concurrency and collinearity). The statement 'Projective Geometry is all Geometry' is attributed to Cayley.

Incidentally, classes in Rudolf Steiner schools still learn some Projective Geometry. I have recently taken several such classes and made sure that I mentioned Cayley's contribution seeing that most of the mathematicians associated with the subject were French or German.

Yours sincerely,

LESLIE T. J. BARR

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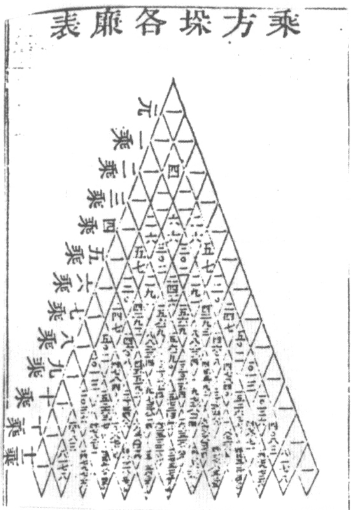
DEAR EDITOR,

Delighted as I was to see *The Cayley Arms Brompton (by Sawdon)* on the cover of the July 1995 *Mathematical Gazette*, I was puzzled that the article to which this picture apparently referred concerned *Arthur* Cayley and not the father of aviation Sir *George* Cayley who is celebrated at Brompton. I have never seen it claimed that the two Cayleys were related; however a little research in the University Library here shows that indeed they were: Sir George was the sixth baronet and the algebraist Arthur was decended from the fourth son of the first baronet. Thus (if my genealogical calculations are correct) each was the fourth cousin of the other. But perhaps you and/or Tony Crilly (and maybe everybody else) already know this!

The July issue also contains notes by Steve Abbott (79.31) and Richard Grassl (79.33) both of which lead to a certain Pascal-like triangle of natural numbers (feet of p. 357 and p. 363 respectively). Exactly this triangle was given by the Chinese mathematician Lǐ Shànlán (1811 – 1882) in his book *Sums of Piles of Various Types*, this particular triangle appearing in Chapter

II: 'Square Piles'. So far as I am aware Lǐ Shànlán's book has never been translated; however Lǐ Yan and Dù Shíràn summarise this particular chapter in their history of Chinese Mathematics, and this book has been translated by Crossley (no relation) and Lun [1]. It is interesting to compare the method of computing the table outlined by Abbott (p. 358 of 79.31) with that of Lǐ Shànlán, for which Crossley and Lun offer the translation: 'Each entry depends on the entries immediately above to left and right, the left entry governs the number of layers along the left diagonal and the right entry governs the following layers along the right diagonal and each multiplies it according to the layer number. Combining them gives the present entry.' Passing over one or two shortcomings of translation this is exactly the method given by Abbott so the triangles are indeed identical. Professor Rongbin Wang of Northwest University, Xi'an, China, has very kindly provided me with a photocopy of Lǐ Shànlán's book, enabling me to reproduce Lǐ Shànlán's triangle (figure); no doubt the reproduction could be better, but a modest grasp of Chinese numerals together with the eye of faith should enable the reader to make out at least the first ten rows of the triangle.



Lǐ Shànlán has received probably much less than his due in the West, largely because he adhered to ancient Chinese methods, in particular using figurate numbers (see e.g. [2]) rather than binomial coefficients, and in general presenting results without proof. A brief and readily accessible article about him has been published by Martzloff [3].

References

1. Lǐ Yan and Dù Shíràn (trans. J. N. Crossley and A. W-C. Lun), *Chinese mathematics: a concise history,* Clarendon Press (1987).
2. A. W. F. Edwards, *Pascal's arithmetical triangle,* London: Griffin 1987.
3. J-C. Martzloff, Lǐ Shànlán (1811-1882) and Chinese traditional mathematics, *Mathematical Intelligencer* **14/4**, pp. 32-37 (1992).

Yours sincerely,

RICHARD CROSSLEY

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Editor's Note: Tony Crilly supplied a Cayley family tree with his article showing the relationship. Tony agrees that the two Cayleys were indeed fourth cousins.

DEAR EDITOR,

When I quoted my second conjecture in the Centenary Issue of the *Gazette*, I was honest enough to say that I was not the author although I could not recall where I found it. I also offered a very modest prize (in comparison with that for the first conjecture) given my suspicion that it would not be all that difficult to prove. How wise I was in both respects! – since the ensuing correspondence had thrown up some most interesting history.

A proof of the 'if', part of the conjecture came almost at once from W. R. Brakes while the 'only if' part came a little later and very elegantly from Prof. I. G. Macdonald F.R.S. They shared the prize! Other contributions came from Philip Spain, Dr Günter Rote, B. R. H. Boys and K. Robin McLean who variously treated the slightly ambiguous wording originally printed. None of these correspondents appeared to know of the conjecture already.

Peter Butt referred to a class-room 'discovery' game called Diffy (which considered the algorithm with four initial numbers) and mentioned that he had set it in some of the original course-work for MEI.

Dr M. S. Klamkin of Edmonton, Canada, however, produced the most interesting response by sending a photocopy of pp. 297-300 of the *SIAM REVIEW* **12** (1970) in which J. M. Hammersley used the conjecture as 'Problem 69–1, Sequences of Absolute Differences' which he had incorporated in his IMA Bulletin article (pp. 66-85, 1968) entitled 'On the enfeeblement of mathematical skills by "modern mathematics" and by soft intellectual trash in schools and universities'.

I was heavily involved in this most controversial paper and had the satisfaction of disposing in no uncertain fashion with my old friend in a debate on that subject held at Winchester College at the time. It is worth recording that I have always credited John Hammersley with the start of the 'New Math' in the UK through his initiative in establishing in 1957 the first of the three biennial conferences involving mathematicians from schools, universities and industry, the third such conference being the Southampton Mathematical Conference 1961 which led directly to the foundation of the SMP.

To come back to the *SIAM REVIEW*. After the solution printed, the editor added a note which gave other earlier references to the problem, the earliest being B. Freeman, *The four numbers game*, Scripta Math., **14** (1948), pp. 35-47.

All in all, therefore, my brief reminiscences brought forth some most interesting responses.

SIR BRYAN THWAITES
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DEAR EDITOR,

An alternative approach to the special Pythagorean triples which Gillian Hatch investigated in *Math. Gaz.* **79** (484) March 1995, p. 51 is to use Pell's equation.

For the integer triple $[a, (a+1), b]$ the Pythagorean relationship implies that b must be odd and reduces to

$$a = \left[-1 + \sqrt{(2b^2 - 1)}\right]/2,$$

which implies that $(2b^2 - 1)$ must be an odd perfect square, m^2 say, and gives

$$m^2 - 2b^2 = -1.$$

This is a Pell equation with the simplest solution $m = 1 = b$, which gives the artificial triple (0, 1, 1) and also the general solution

$$2m = \left(\sqrt{2} + 1\right)^n - \left(\sqrt{2} - 1\right)^n$$

$$2\sqrt{2}b = \left(\sqrt{2} + 1\right)^n + \left(\sqrt{2} - 1\right)^n$$

with $n = 1, 3, 5, \ldots$.

Her starting point, the triple (20, 21, 29) is given by $n = 5$ and its derivation is a useful calculator exercise for using memories.

Yours sincerely,

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New evidence that boxing damages the brain

With a £4 million purse awaiting him, he was quick to draw attention to the challenger's £20 million pay cheque. 'I'm not a greedy man, but Tyson is getting 16 to 20 times more than I'm getting,' Bruno said. 'Would you be happy if you were the champion and were being treated like the challenger?'

From *The Times* 29 February 1996 sent in by Frank Tapson.

Clever things these starlings

"They have to measure intervals between captures to know how good the pickings are in each place," Dr Kacelnik said. "Starlings have a tremendous ability for judging accurate time intervals from within a few seconds to at least a few minutes and they are capable of using this information to form averages, and to compare these averages in a specific way.

"But it is more complex than than that. They also need to do statistics to extract the average and compare the pickings between each place."

From *The Daily Telegraph* 21 February 1996
and sent in by A. Robert Pargeter, Devon.

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