# COMMENTS ON THE EFFECT OF ADOPTING NEW PRECESSION AND EQUINOX CORRECTIONS


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ABSTRACT. The new astronomical constants being introduced for 1984 affect the derived values of UT1 and terrestrial longitudes differently for the space techniques than for the optical techniques. The new sidereal time equation has been defined to avoid a discontinuity in UT1 as obtained by the established techniques. However, this definition does introduce a discontinuity in both the terrestrial longitude system and the UT1 rate derived by the space techniques which have an inertial celestial reference frame rather than a stellar catalog frame. The use of consistent expressions for precession and Greenwich Mean Sidereal Time will not eliminate differences between the inertial and classical optical techniques. Only improvements in the accuracy of the precession constant and equinox offset and drift will bring about consistency. Improvements in the precession constant can be expected in the next few years. The inertial techniques will exhibit shifts in the derived UT1 rate and terrestrial longitude system with each change in the precession constant if Greenwich Mean Sidereal Time is referenced to the mean equinox of date, as is the present practice, but would be stable if the reference is a fixed equinox. The latter choice is recommended.


## I. CHANGES FROM NEW CONSTANTS

The hour angle, H, of an observed body is given by

$$H = \alpha - \alpha_G - \lambda \tag{1}$$

where

$\alpha$ = right ascension (true of date) of the celestial body

$\alpha_G$(UT1) = Greenwich Apparent Sidereal Time




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$\lambda$ = terrestrial east longitude of the observing station (or baseline direction in the case of radio interferometry)

The quantity H can be considered as determined by the ensemble of observations made over time and hence invariant to changes in convention or in definition of the quantities $\alpha$, $\alpha_G$, and $\lambda$. This leads to the constraining equation linking the zero points of the terrestrial and celestial frames

$$\delta\alpha - \delta\alpha_G - \delta\lambda = 0 \tag{2}$$

which must hold over time. We will ignore the effect of nutation in this discussion since the effects are periodic with position, although it should be recognized that changes in the nutation model will lead to changes in derived longitude and UT1 values that depend on the technique and on the distribution of observations. Subsequently, $\delta\alpha$ will be treated as a change in the mean of date right ascension and $\delta\alpha_G$ as a change in the Greenwich Mean Sidereal Time (GMST). We shall also not attempt to discuss the artificial earth satellite case since parameters other than those in equation 2, particularly the zonal harmonics, come into play.

The celestial reference frame defined by distant radio quasars and the dynamical frame of the lunar and planetary ephemerides are inertial frames. The space techniques of radio interferometry and lunar and planetary ranging must use inertial frames. The rotation into the true of date system gives sensitivity to the luni-solar and planetary precession corrections $\Delta p_1$ and $\Delta\dot{X}$. For the purpose of this analysis, we may express the change in right ascension rate $\dot{\alpha}_p$ due to the precession corrections as follows:

$$\dot{\alpha}_p = \Delta p_1 \ (\cos \varepsilon + \sin \varepsilon \sin \alpha \tan \delta) - \Delta\dot{X} \tag{3}$$

where $\varepsilon$ is the obliquity of the ecliptic and $\delta$ the declination. For the changes adopted for the 1976 IAU expressions for precession

$$\Delta p_1 = 1\overset{''}{.}1/\mathrm{cy}$$

$$\Delta\dot{X} = -0\overset{''}{.}029/\mathrm{cy}$$

Classical optical astrometry differs from the space techniques. In changing from the FK4 to the FK5, new proper motions are introduced to null the changes in right ascension and declination due to precession. In addition, the new FK5 will have a catalog equinox correction that has been given by Fricke (1981) as

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$$\delta\alpha = E_o + \dot{E}t \tag{4}$$

where

$$E_o = 0''.525$$

$$\dot{E} = +1''.275/cy$$

and the time t is measured from B 1950.0. The constant and rate terms in the polynomial expression for $\alpha_G$(UT1) have been modified by the addition of the above coefficients for the catalog equinox corrections. In general, the change in $\alpha_G$ may be written as

$$\delta\alpha_G = \Delta\alpha_G + \omega_e\Delta(UT1) \tag{5}$$

where $\Delta\alpha_G$ results from the change in polynomial coefficients, $\Delta$(UT1) is a possible change in UT1, and $\omega_e$ is the rotation rate of the earth.

For the optical techniques, the effect of the above changes on the right ascension of a star is given by

$$\delta\alpha = E_o + \dot{E}t \tag{6}$$

For the inertial techniques, the situation is not so straightforward. We may express the change as

$$\delta\alpha = E_I + \langle\dot{\alpha}_p\rangle t \tag{7}$$

where $E_I$ is a change in the zero point of the inertial frame that we are free to choose and time t is measured from a nominal epoch, for example B 1950.0. The shift $E_I$ could be used to align a radio source catalog or an ephemeris with either a particular star catalog at a particular time or with the dynamical equinox. $\langle\dot{\alpha}_p\rangle$ is the value of $\dot{\alpha}_p$ average over the observations.

If we now apply these changes in $\alpha$ and $\alpha_G$ to the space and optical techniques we can determine the changes in $\lambda$ and UT1 that result from invoking the constraining equation (2) with the adopted changes in $\alpha$ and $\alpha_G$

$$\delta\lambda + \omega_e\Delta UT1 = \delta\alpha - \Delta\alpha_G \tag{8}$$

This constraint leads to two conditions or transformations between the terrestrial and celestial frames: one for the zero point offset, and one for the rotation rate between the frames. In practice, we do not wish to have drifts in $\lambda$ so the rate offsets must be put into the UT1 change. The space techniques can align their UT1 systems with BIH at some effective time $t_u$

after the nominal epoch ($\Delta UT1(t_u)=0$). This may be done by making the values equal at some particular date or by making the average difference zero over some span of time centered on $t_u$. The rates cannot be adjusted. Carrying through the above process, we arrive at the results given in the following table. The upper half of the table represents choices made, the lower half the consequences.

Table 1
Changes in Longitude and UT1 Due to Changes in Precession and Catalog Equinox Corrections

| | Space Techniques | Classical Techniques | Comments |
|---|---|---|---|
| $\delta\alpha$ | $\langle\dot{a}_p\rangle t + E_I$ | $E_o + \dot{E}t$ | Full effect of precession must be applied to inertial frames. Precession nullified by compensating proper motion corrections in stellar frame. |
| $\Delta\alpha_G$ | $E_o + \dot{E}t$ | $E_o + \dot{E}t$ | New changes in Sidereal Time equation designed to keep UT1 invariant to catalog equinox corrections. |
| $\delta\lambda$ | $(\langle\dot{a}_p\rangle - \dot{E})\ t_u + E_I - E_o$ | 0 | For the classical techniques, $\Delta(UT1) = 0$, hence, $\delta\lambda = 0$. |
| $\Delta(UT1)$ | $(\langle\dot{a}_p\rangle - \dot{E})(t-t_u)/\omega_e$ | 0 | $t_u$ is the effective alignment epoch of the UT1 systems. |

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The symbol ⟨ ⟩ denotes the average value over all observations of the effects of the precession corrections in right ascension. It should be noted that $\langle\dot{\alpha}_p\rangle$ differs between radio interferometry and lunar laser ranging. In the former technique, radio sources are observed randomly over the sky and the $\sin\varepsilon \sin\alpha \tan\delta$ term of equation 3 averages to zero. In lunar laser ranging, the observations tend to be equally distributed along the ecliptic and this term differs from zero and has the effect of replacing the $\cos\varepsilon$ term by unity. Using RI to denote radio interferometry and LLR for lunar laser ranging

$$\begin{aligned} \langle\dot{\alpha}_p\rangle_{RI} - \dot{E} &= \cos\varepsilon\, \Delta p_1 - \Delta\dot{X} - \dot{E} = -0\overset{''}{.}237/\mathrm{cy} \\ \langle\dot{\alpha}_p\rangle_{LLR} - \dot{E} &= \Delta p_1 - \Delta\dot{X} - \dot{E} = -0\overset{''}{.}146/\mathrm{cy} \end{aligned} \tag{9}$$

and the changes in the UT1 rates are

$$\begin{aligned} \frac{d}{dt}\Delta(\mathrm{UT1})_{RI} &= (\langle\dot{\alpha}_p\rangle_{RI} - \dot{E})/\omega_e = -0.157\ \mathrm{ms/yr} \\ \frac{d}{dt}\Delta(\mathrm{UT1})_{LLR} &= (\langle\dot{\alpha}_p\rangle_{LLR} - \dot{E})/\omega_e = -0.097\ \mathrm{ms/yr} \end{aligned} \tag{10}$$

As lunar ranging has a decade of earth rotation data, this is a significant change.

## II. POSSIBLE CHANGES AND ERRORS

This paper has concerned itself with the changes in both the derived terrestrial longitude system and UT1 rate which result from the changes from the old to the new internationally adopted astronomical constants. That the space techniques experience discontinuities due to these changes is an acceptable inconvenience when 1) the changes are of such a nature as to remove systematic differences between the various techniques and 2) the several changes can be made at nearly the same time so that the analysis results distributed to the scientific community show a discontinuity at one time only. There is a disquieting implication from derivations of this paper due to the fact that the luni-solar precession constant is in principle an observable constant for radio interferometry and lunar laser ranging. When the inertial techniques are able to make a significant correction to the newly adopted precession constant, which is only a few years away, it will be necessary to continue to solve for precession in order to fit the data satisfactorily. It will be argued below that subsequent continued use of GMST fixed with respect to the mean equinox of date would lead to both a continually changing UT1 rate and a shifting zero point of terrestrial longitude for the inertial techniques.

The previously developed formalism will be used, but now the newly adopted astronomical constants will be taken as the nominal values and the corrections will be to these values. A table of corrections is given below for the case where GMST is held fixed, but all other parameters vary. As before the top half contains the choices made, and the bottom half is the consequences.

Table 2
GMST Referenced to the Mean Equinox of Date

| | Inertial Techniques | Classical Techniques |
|---|---|---|
| $\delta\alpha$ | $E_I + \langle\dot{a}_p\rangle t$ | $E_o + \dot{E}t$ |
| $\Delta\alpha_G$ | 0 | 0 |
| $\delta\lambda$ | $E_I + \langle\dot{a}_p\rangle t_u$ | $E_o + \dot{E}t_u$ |
| $\Delta$UT1 | $\langle\dot{a}_p\rangle(t-t_u)/\omega_e$ | $\dot{E}(t-t_u)/\omega_e$ |

First consider the changes to be corrections made to agree with perfectly known values of precession and equinox offset and drift. The expressions for the UT1 rates in the two columns differ by $(\langle\dot{a}_p\rangle-\dot{E})/\omega_e$ and the expressions for longitude are also different. It must be concluded that consistent use of adopted constants with different techniques does not guarantee either consistent UT1 rates or terrestrial longitude systems. Fricke (1977) quotes an error of $\pm 0\overset{''}{.}15$/cy for the precession constant adopted by the IAU so that the classical and space techniques should not be expected to agree better than 0.1 ms/yr. More satisfying is the expected difference between lunar laser ranging and radio interferometry $(1 - \cos \varepsilon)\Delta p_1/\omega_e$ with a precession induced error of 0.008 ms/yr.

A third table of changes can be developed for the case where the Greenwich Mean Sidereal Time is held invariant with respect to a fixed equinox, rather than a moving equinox, for changes in precession.

Table 3
GMST Referenced to a Fixed Equinox

| | Inertial Technique | Classical Technique |
|---|---|---|
| $\delta\alpha$ | $E_I + \langle\dot{a}_p\rangle t$ | $E_o + \dot{E}t$ |
| $\Delta\alpha_G$ | $(\Delta p_1 \cos\varepsilon - \Delta\dot{X})t$ | $(\Delta p_1 \cos\varepsilon - \Delta\dot{X})t$ |
| ////// | ////////////////////////////// | ////////////////////////////// |
| $\delta\lambda$ | $E_I + (\langle\dot{a}_p\rangle - \Delta p_1\cos\varepsilon + \Delta\dot{X})t_u$ | $E_o + (\dot{E} - \Delta p_1\cos\varepsilon + \Delta\dot{X})t_u$ |
| $\Delta$UT1 | $(\langle\dot{a}_p\rangle - \Delta p_1\cos\varepsilon + \Delta\dot{X})(t - t_u)/\omega_e$ | $(\dot{E} - \Delta p_1\cos\varepsilon + \Delta\dot{X})(t - t_u)/\omega_e$ |

For the same variables the differences between the inertial and classical columns remain the same for the different tables because they come strictly from the differences in right ascension, when $\alpha_G$ is consistent. Thus there is no change to GMST that will eliminate these differences. Only improved constants will bring about consistency. For radio interferometry and lunar laser ranging

$$\begin{aligned} (\langle\dot{a}_p\rangle - \Delta p_1 \cos\varepsilon + \Delta\dot{X})_{RI} &= 0 \\ (\langle\dot{a}_p\rangle - \Delta p_1 \cos\varepsilon + \Delta\dot{X})_{LLR} &= (1-\cos\varepsilon)\Delta p_1 \end{aligned} \tag{11}$$

The planetary precession is not a parameter which is observed and thus does not require correction to fit data. It could have been omitted from the expression for $\Delta\alpha_G$. It has been included for future convenience since there is already known (Bretagnon and Chapront, 1981) a small correction of $-0''.005$/cy which can be applied to the adopted value at some future time.

Consider the implication of Tables 2 and 3 at a time, a few years hence, when the space techniques have had to abandon the adopted precession constant but the classical systems remain fixed.

Using $\pm 0\overset{''}{.}15$/cy for the uncertainty of the precession constant, in the case of Table 2 the UT1 rate for the inertial techniques will have shifted $\pm 0.1$ ms/yr from the adopted system and, taking $t_u \sim$ 25 yrs, the derived terrestrial longitude will move about $\pm 0\overset{''}{.}035$. If GMST is made invariant with respect to a fixed equinox then the expected shifts are zero for radio interferometry, and $\pm 0.008$ ms/yr and $\pm$ $0\overset{''}{.}003$ for lunar laser ranging. Classical astrometry would still remain fixed. Finally consider the present estimates of realistic random errors from existing analyses of lunar laser data for the conditions of Table 3, $\pm 0.06$ ms/yr in UT1 rate and $\pm 0\overset{''}{.}004$ for terrestrial longitudes ($\pm 0\overset{''}{.}002$ if the UT1 rate is fixed). In addition, better than $0\overset{''}{.}01$ accuracy can be achieved in adjusting the right ascension zero points of the various inertial techniques to a common dynamical equinox (one use for the parameter $E_I$). These adjustments include shifting the zero point of the planetary ephemeris to the dynamical equinox, tying the lunar orbit to the earth's orbit, and linking the VLBI system to the planetary system (Newhall, 1981, private communication). It can be seen that defining the GMST expression as invariant with respect to the mean equinox of date causes systematic errors in the UT1 and longitude systems of inertial techniques which exceed the random errors inherent in the data. Though shifts of a few hundreths of an arc second may seem modest, it must be remembered that much of the motivation behind the development of the space techniques comes from their geodetic capabilities of a decimeter ($0\overset{''}{.}003$) or less.

If the expression for GMST were to be referenced to a fixed equinox then what would be the consequences for the classical techniques, in particular BIH? We are not asking for a change in the numerical values of the coefficients to be adopted for the new IAU system. We ask for agreement now on how a changing precession constant is to be incorporated in the future. The new system adopted for BIH and the FK5 would not be changed. Our recommendation would imply that for the next major change in the classical optical systems, these systems would be adjusted to agree with the inertial systems as given in table 3 and would assure that the inertial systems are not shifting by (to them) large amounts before then. We also note that BIH will soon face a decision on how to incorporate the UT1 derived by the inertial techniques. The implication of the increasing power at long periods in the spectrum of UT1(LLR) - UT1(BIH) of Fliegel et al (this volume) is that the BIH system of UT1 is more stable for short time spans than long spans, which is expected. If the long time stability of the inertial techniques is to be exploited in the near future, then the preservation of that stability should be considered.

Guinot (1979) suggested connecting the classical techniques to the inertial frame and several authors have discussed the philosophy of various frames including inertial frames. It is straight forward to make the expression for GMST invariant, in an inertial frame, against changes in precession. One identifies GMST (still measured from the mean equinox of date) with the secular terms of the differential equation for rotations about the pole and integrates to get

$$\begin{aligned} \mathrm{GMST}(t) &= A + Bt - X_A(t) + \Psi_A(t)\cos\omega_A(t) \\ &+ \int \Psi_A(t)\sin\omega_A(t)\ \dot{\omega}_A(t)\,dt \end{aligned} \tag{12}$$

where A and B are empirical constants to be chosen and $X_A$ is the accumulated planetary precession, $\Psi_A$ is the accumulated luni-solar precession of the mean equator of date from the fixed equinox, and $\omega_A$ is the obliquity of the mean equator of date to the fixed ecliptic. The three right most terms contain quadratic and cubic terms as well as linear contributions. The integral only contributes to the cubic. The very specific notation is that of the 1976 IAU precession paper (Lieske et al, 1977). It would probably be well to make both the planetary and luni-solar precession explicit in the definition to make future changes easier and to place the other terms of Eq. 12 into a fixed polynomial

$$\mathrm{GMST} = \text{fixed polynomial} + \Psi_A \cos\omega_A - X_A \tag{13}$$

but the stability of the inertial frames would be preserved by the incremental form

$$\mathrm{GMST} = \text{fixed polynomial} + \Delta p_1\ t\cos\omega_A \tag{14}$$

where this second fixed polynomial is identical with the expression already proposed for GMST by other scientists. The coefficients in the fixed polynomial would not change as the precession rate $\dot{\Psi}_A$ improves, but the overall expression would change. The total expression for the right side would take on the numerical values being choosen to go with Fricke's precession constant and equinox offset and the values for the fixed polynomials can be derived from them.

It can be mentioned that there is an alternative modification which preserves the stability of the inertial systems. If the precession matrix were reformulated to be a rotation about an axis in the equator at $\alpha$ = 6 hours, then precession would be orthogonal to obliquity (first axis) and UT1 rotations (third axis). For partials this orthogonality of infinitessimal rotations of the earth in space is very useful and is used in the lunar laser software. The increment about the second axis is $\Delta p_1$

sin ε and about the third $-\Delta p_1 \cos \varepsilon + \Delta\dot{X} + \dot{E}$. For finite rotations, though, this seems much more cumbersome than modifying the expression for GMST.

## III. SUMMARY

In summary, it is shown that the derived terrestrial longitude zero point and UT1 rate will be different when determined by classical optical and space (inertial) techniques so long as the precession constant and equinox offset and drift are imperfectly known. For the uncertainty of the new IAU precession constant, this inconsistency is expected to be nearly 0".04 in longitude and 0.1 ms/yr in UT1 rate. The consistency of constants does not guarantee consistent results. The classical practice of defining Greenwich Mean Sidereal Time as invariant with respect to a moving equinox must be replaced by invariance with respect to a fixed equinox if the longitude and UT1 results from the inertial techniques are to be stable against changes in the precession constant. Equation 13 gives one such form for GMST. The impact of this proposed change on optical techniques would come when a future precession constant (subsequent to 1984) and catalog are adopted.

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