SECULAR PERTURBATIONS OF ASTEROIDS AND COMETS

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In 1962 the author published a paper on secular perturbations of asteroids with high eccentricity and inclination (Kozai, 1962) and found that for such asteroids the secular perturbations are more complicated than those estimated by the classical linear theory. Before that the author(Kozai, 1954) tried to extend the classical theory to include terms of squares of disturbing masses, eccentricities and inclinations in expressions of secular motions of angular variables and found that the sum of the longitudes of the perihelion and the ascending node is not as stable quantity as the classical theory predicted, and, therefore, it cannot be used to estimate the ages of asteroid families. Then Williams(1969) and Yuasa(1973) extended the classical theory further.

In this paper the author computes the secular perturbations of numbered asteroids and short-periodic comets whose semi-major axes are less than 30 a.u. by assuming that (1) the disturbing planets are Jupiter, Saturn, Uranus and Neptune, (2) their orbital semi-major axes are not much disturbed, and (3) the disturbing planets are moving along circular orbits on Jupiter's orbital plane. According to the assumption (2) any asteroid or comet with commensurable mean motion and any comet which can approach to any disturbing planet very closely should be excluded.

According to the assumption (2) the mean anomaly of the disturbed body and the mean longitude of each of the disturbing planets can be eliminated from the disturbing function by averaging it with respect to each of the angular variables independently. The averaging is carried out not by any analytical way but by a numerical harmonic analysis with 5° as one step for the variables. After the elimination the semi-major axis can be assumed to be constant. By the assumption (3) the longitude of the ascending node of the disturbed body does not appear in the averaged disturbing function when the orbital plane of Jupiter is adopted as the reference plane, and, therefore, the z-component of the angular momentum, $(a(1-e^2))^{1/2}\cos i$, is conserved, where i is the inclination with respect to Jupiter's orbital plane. And as the averaged Hamiltonian as well as the disturbing function do not depend on time explicitly, the Hamiltonian F is conserved.

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Now the system of the equations of motion is reduced to that of one degree of freedom, and it is the equation for the secular perturbations. The variables are $G = (a(1-e^2))^{1/2}$, the angular momentum, and g, the argument of perihelion, and the Hamiltonian is regarded as a function of x $=(1-e^2)^{1/2}$ and 2g with a and $H = x \cos i$ as parameters. Therefore, x can be solved as a function of 2g, in fact, a periodic function of 2g, and as H is given, both the eccentricity and the inclination can be graphically expressed as functions of g. As the values of a and H are given for each of the asteroids and the comets, the values of the Hamiltonian can be computed for various sets of x and g, and then equi-F-value curves are drawn in g-x plane, where g-axis extends from 0° to 90° and x-axis extends from H to 1. There are symmetries with respect to g = 0°, 90°, 180° and 270°. Two lines, x = 1 and x = H, are always equi-F-value curves and x = 1 corresponds to a circular orbit whereas x = H corresponds to an orbit on Jupiter's orbital plane. The two orbits are not disturbed due to the secular perturbation unless the argument of perihelion is of libration. As H is decreased, the variation of x becomes wider, and if the value of H is below a certain value around 0.8, a libration region appears in the g-x diagram. If the value of F corresponding to the concerned asteroid or comet is that in the libration region, the argument of perihelion does not make a complete revolution but makes a libration around 90° or 270°.

Except for the libration case the eccentricity of any of the asteroids takes its minimum and the inclination takes its maximum at g = 0° and 180° and vice versa at g = 90° and 270°. For the libration case both maximum and minimum take place at 90° or 270°. Therefore, the orbits become the least eccentric when the perihelion is on Jupiter's orbital plane and can avoid any very close approach to Jupiter if the orbit is inside of Jupiter. For most of the asteroids it is found that x is larger than cos i and for such cases the variation of the eccentricity is wider than that of the inclination. In Figure 1 the maximum, the minimum and the present values of the eccentricities and the inclinations of eight asteroids belonging to a family, Pallas family by Hirayama(1928) and family No. 28 by Brouwer(1951), are plotted with asteroid numbers. For each of the asteroids a narrow libration region appears in the g-x diagram as the values of H are nearly equal to 0.8, although for non of them the argument of perihelion is of libration. In the figure the crosses in the upper left
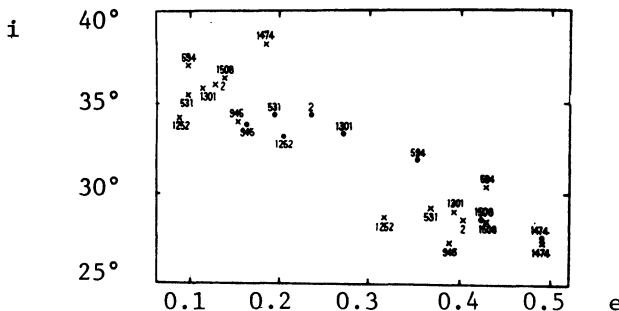

Figure 1. Variations of the eccentricities and the inclinations for asteroids belonging to Pallas family.

corner correspond   to the values of the eccentricity and the inclina-
tion at g = 0° and 180° and those in the lower right corner correspond
to those at g = 90° and 270°. The circles correspond to the present val-
ues. For these asteroids the eccentricities change by more than 0.3 and
the inclinations change by 10° as functions of the argument of perihelion.
Differences between the proper eccentricities and inclinations and oscu-
lating ones are much smaller than the variations. The asteroids 1474 and
1508 were not included in this family in Brouwer's paper, however, it is
found that they are real members of the family.

   There are four asteroids among the numbered ones, for which the ar-
guments of perihelion are of libration. They are 944 Hidalgo, 1373 Cin-
cinnati, 1866 Sisyphus and 1981. Kozai(1962) found that the argument of
perihelion of Cincinnati(a = 3.411 a.u.) is of libration between 69° and
111° by a simpler computation. The eccentricity and the inclination
change between 0.318 and 0.559 and between 40.°2 and 29.°2, respectively,
as one sees in Figure 2,where the scales of the ordinate are not given in
x but in e and i although it is scaled with equal interval of x. The
present place is marked with the cross on one of the equi-F-value curves.
Because of the libration both the perihelion and the aphelion of this
asteroid cannot be placed on Jupiter's orbital plane, and, therefore, it
cannot approach to Jupiter very closely even though the aphelion distance
can become larger than the semi-major axis of Jupiter. There are three
asteroids, for which the minimum aphelion distances are larger than 4.4
a.u. One of them is this asteroid, and the other two belong to Hecuba
group. For the latter two the critical arguments are of libration around
0°(Franklin et al., 1975) and, therefore, they can avoid any close ap-
proach to Jupiter.
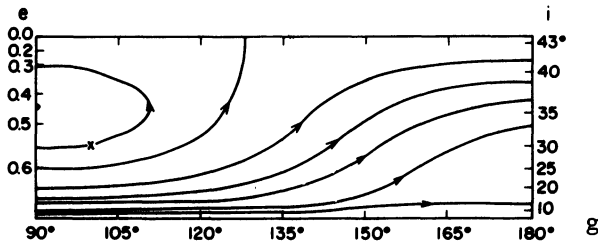

Figure 2.   The g-x diagram for 1373 Cincinnati.

   According to the present computation the argument of perihelion is
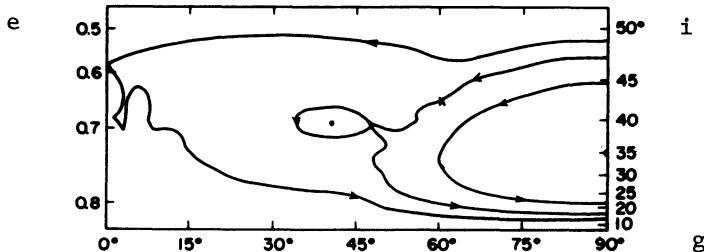of libration for a peculiar asteroid, Hidalgo, (a = 5.82 a.u.) between

Figure 3.   A part of the g-x diagram for 944 Hidalgo.

48° and 132°, and the eccentricity and the inclination change between 0.
568 and 0.811 and between 47.°3 and 17.°2, respectively, whereas the pres-
ent values are e = 0.658 and i = 42.°26. Though the argument of perihelion
is now decreasing from 59.°5, after reaching to 48° it will increase and
when it will become 90° the perihelion distance will be as small as 1.1
a.u. Then the orbit will be quite similar to that of a comet. As Hidalgo
has a possibility to approach very closely to Jupiter, the present com-
putations might have some errors. However, Nakano's numerical integration
results(1977) suggest that the errors are not so large. For the other
two asteroids, for which the arguments of perihelion are of libration,
the semi-major axes are 1.89 and 1.78 a.u. and their orbits are quite
similar to those of comets and can avoid any very close approach to Mars.
Besides the four asteroids there appear librations regions in g-x dia-
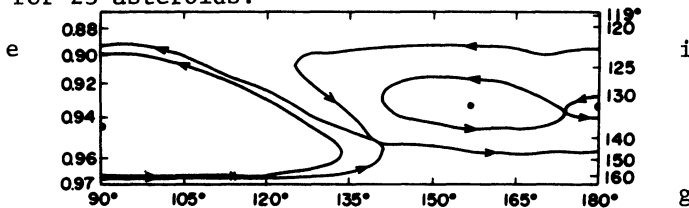grams for 23 asteroids.

Figure 4. A part of the g-x diagram for Halley's comet. The pres-
    ent place is marked by the cross which is on one of the equ-F-
    value curves.

The situation is quite contrary for short-periodic comets. Of 104
comets treated here 78 comets have their semi-major axes less than that
of Jupiter. Of them 68 comets have libration regions in their g-x dia-
grams because the values of H are usually small. However, for none of
them the argument of perihelion is of libration. For most of them x is
less than cos i and the variation of the inclination is wider than that
of the eccentricity. However, of the short-periodic comets the semi-
major axes of 26 comets are between that of Jupiter and 30 a.u. Of the
26 comets 25 have libration regions in g-x diagrams, the only one excep-
tion being P/Schwassma-n-Wachmann I. And the arguments of perihelion are
of libration for 15 comets. However, even for the libration cases the
eccentricities do not change so much as those of some asteroids. For
Halley's comet, for an example, the argument of perihelion is of libration
between 47° and 133° and the eccentricity and the inclination change be-
tween 0.899 and 0.968 and between 123.°3 and 164.°4, respectively. The
present values are e = 0.967, i = 161.°2 and g = 114.°4 which is increas-
ing. Therefore, the eccentricity of this comet is now nearly maximum and
the perihelion distance will not decrease in near future. However, for
this short range of the variation the perihelion distance is changed by
1.2 a.u. which is large enough to affect the evolution of the comet.
Also there are a few comets, for which the eccentricities are changed in
wider ranges like Hidalgo.

Since the eccentricity and the inclination do change very much for
asteroids with small values of H, the proper eccentricity and inclination
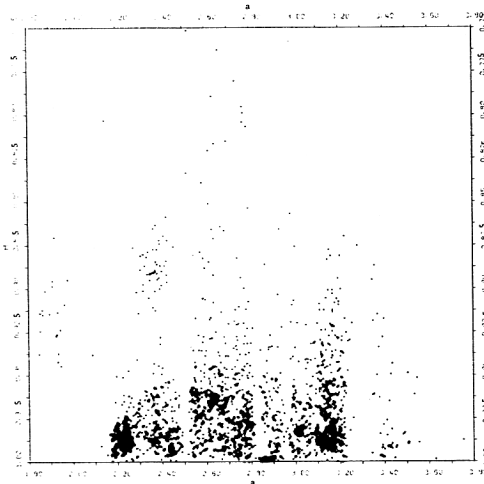which are derived by the classical linear theory are not adequate to clas-
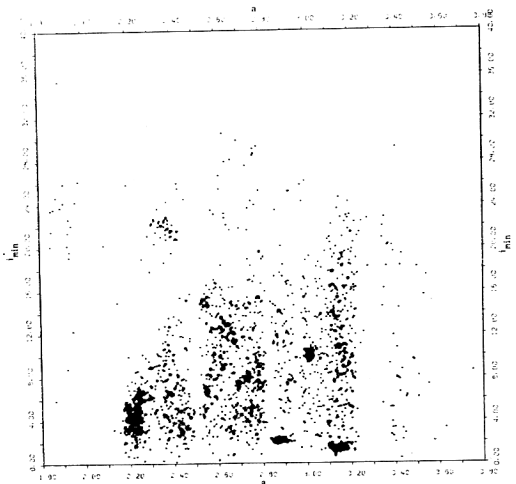
Figure 4. a-H diagram.   Figure 5. a-i_{min} diagram.

sify such asteroids into families. The author believes that the minimum
value of the inclination(or the maximum value or the eccentricity) and H
as well as the semi-major axis are more adequate and stable quantities
which can be used for classification of the asteroids with not only small
values of H but also H nearly equal to 1. In Figures 4 and 5 the numbered
asteroids are plotted in a-H plane and a-i_{min} plane, respectively. Even
in these figures 6 famous families discovered by Hirayama(1923 and 1928)
as well as the family in Figure 1 clearly show up. More clear pictures
can be obtained when the asteroids are plotted in a-H-i_{min} space. By
this way the seven families in the following table are derived. In the
table the names of the families after Hirayama(1923 and 1928), the numbers
of their members by the present analysis, Hirayama(1928) and Brouwer(1951)
in this order, and the mean values of the semi-major axes, H and the min-
imum inclinations as well as their mean deviations are given. Also the
mean values of the maximum eccentricities are given for a reference.

Name	Numbers			a	H	i_{min}	e_{max}
Themis	77	32	53	3.140$\mp$ 0.024	0.9881$\mp$ 0.0033	1.°40$\mp$ 0.°21	0.108
Eos	89	27	58	3.015$\mp$ 0.007	0.9821$\mp$ 0.0013	10. 01$\mp$ 0. 27	0.052
Coronis	48	20	33	2.875$\mp$ 0.018	0.9984$\mp$ 0.0007	2. 14$\mp$ 0. 14	0.030
Maria	20	14	17	2.543$\mp$ 0.014	0.9609$\mp$ 0.0023	14. 93$\mp$ 0. 30	0.074
Flora	228	63	125	2.229$\mp$ 0.032	0.9860$\mp$ 0.0060	4. 58$\mp$ 1. 40	0.104
Phocaea	34	11	21	2.371$\mp$ 0.033	0.8882$\mp$ 0.0093	21. 85$\mp$ 0. 60	0.207
Pallas	8	3	6	2.723$\mp$ 0.057	0.8023$\mp$ 0.0167	32. 16$\mp$ 2. 43	0.229

As one can see, some families are very compact whereas others are
not. The scattering of H is small for the families whose eccentricities
are small. This may suggest that the neglecting the eccentricities of
the disturbing planets gives stronger effects on more eccentric asteroids
such as those belonging to Phocaea and Pallas families. The effects are
very small for those belonging to Eos, Coronis and Maria families. Even

though these effects are taken into account, there are still some loose
families such as Flora and Pallas. Therefore, the author has an idea
that these loose families were not originated from single mother aster-
oids as the other compact ones were probably originated from single ones.
Probably many asteroids were pushed out from regions where secular com-
mensurabilities exist and gathered there as loose families as Williams(
1969) pointed out. In fact several gaps seen in Figures 4 and 5 are sit-
uated near regions of secular commensurablity.

The computations were made with the computor FACOM 230-58 at the
Computing Center of the Tokyo Astronomical Observatory by the orbital
elements of the asteroids and comets provided by Dr. K. Hurukawa.

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DISCUSSION

Kiang: 1. Did I understand that comet Halley is a librator?
        2. Hirayama and later Brouwer use proper e, i and a to find
families. What are your defining parameters? Do you get better
defined families? Have you found any new families?
        3. Would not the functional relation between e and i in your
simplified model not explain the elongated form of the families
in the proper (e, i)-plane?
Kozai: 1. Yes, it is, according to my computation. However, as you
can see in Fig. 4 the equi-F-value curve for Halley's comet is
just inside a limiting curve and, therefore, if there is a small
error in my computation, it is possible that the argument of peri-
helion is not of libration.
        2. My parameters are a, $1-e^2 \cos i$, and the minimum values of
i. I can get better defined families at least for the Pallas case.
        3. No.

Marsden: Don't you think there is a significant difference between
        (1373) Cincinnati and the other objects for which you now find
        argument-of-perihelion libration? The libration of (1373) very
        successfully keeps this object away from Jupiter, and the orbit is
        thus rather stable. The comets and the other minor planets for
        which you find libration all make close approaches either to
        Jupiter or to the Earth, so it would seem that these librations
        can have little effect on the stability of the orbits.
Kozai: The two asteroids can avoid any very close approach to Mars
        due to librations.