KEY PROBLEMS OF FLAT OBJECTS DYNAMO THEORY AND WAYS OF THEIR SOLUTION

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ABSTRACT. The present status of galactic dynamo theory is discussed. A new concept which allows the determination of marginal dynamo numbers for axisymmetric as well as non-axisymmetric large-scale magnetic field modes in axisymmetric disks is applied to a simple galaxy model. The results obtained so far show that a preference of non-axisymmetric fields can only be expected if the α-effect is highly anisotropic and the differential rotation is not too strong. Mostly axisymmetric-spiral fields have to be expected.

1. INTRODUCTION

By the discovery of large-scale magnetic structures in a number of nearby galaxies the dynamo theory of flat objects became a field of high actuality. Beyond that it is of special interest since galaxies apparently have to be divided into two groups with basically different magnetic fields: One observes some with axisymmetric-spiral (ASS) fields (M 31, IC 342) and some with non-axisymmetric, the so-called bisymmetric- spiral (BSS) fields (M81). The physical reason for this difference is at present the most stimulating question for investigations in frame of dynamo theory.

Dynamo theory of today is mainly based on the investigation of the kinematic problem, i.e. the induction equation

$$\frac{\partial B}{\partial t} = \mathrm{curl}(u \times B) + \mathrm{curl}E - \mathrm{curl}\frac{1}{\mu\sigma}\mathrm{curl}B \qquad (1)$$

is considered and conditions providing for non-trivial solutions are determined. B denotes here the (mean) magnetic field, u the (mean) velocity field, E the turbulent electromotive force due to the correlated magnetic and velocity fluctuations and μ, σ the permeability and the conductivity.

Separating an exponential time dependence one obtains a denumerable set of eigenmodes. Some of them may grow. In this case the system

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represents a dynamo. Fig. 1 illustrates the situation. The induction action of the motion is characterized by the dynamo number C. There is a first (in the sense of growing C) eigenvalue C_1 where a non-decaying eigenmode B_1 appears. Beyond C_1 the system represents a dynamo and for larger C additional modes will grow. Since these modes are derived from the linear equation (1) they are only valid for a description of small magnetic fields.

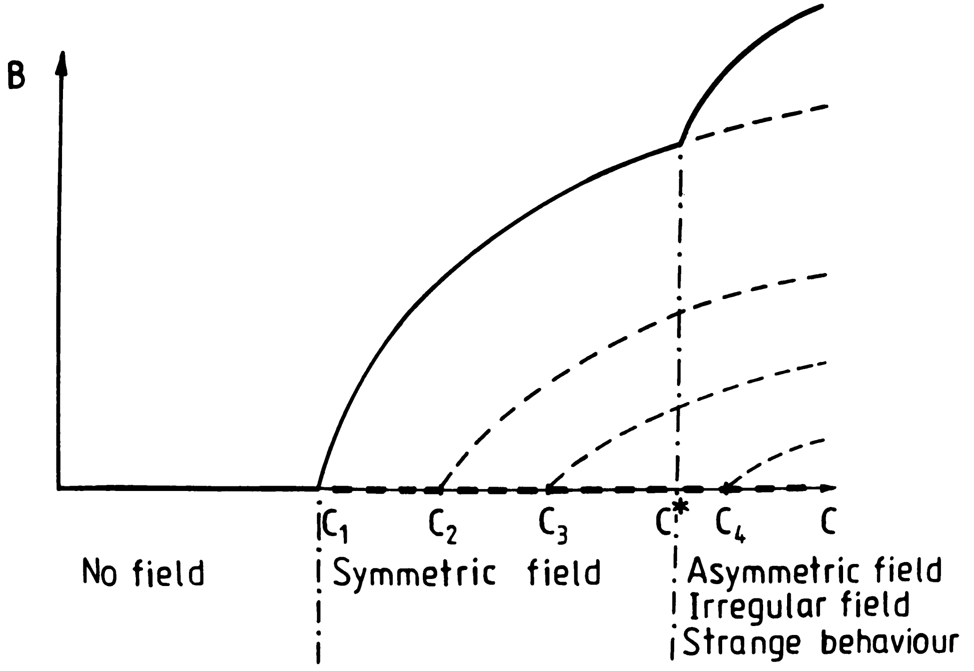

Figure 1. Excited magnetic fields in dependence on the dynamo number.

Nature is clearly nonlinear: The excited magnetic field reacts back to the motion and the final state - i.e. the observed state - will be determined in some way by equipartition of energy, indicated in Fig. 1 by the nonlinear extensions of the linear modes.

Because of the high mathematical complexity nonlinear dynamo theory is till now weakly developed only. However, simple models provide for some insight and we identify here ourselves with the standpoint (Krause and Meinel (1988 a,b)) that only that mode, which is most easily excited, is of physical interest. The nonlinear extensions of all higher modes, i.e. of the modes which need stronger induction action for becoming excited than the marginal mode, are first (in the neighbourhood of the bifurcation) unstable and so of less physical significance. Investigations show also rather complicated (cf. Brandenburg et al. (1989), Rädler et al. (1989)) behaviour in the far nonlinear regime, however, that cannot be explored by considering the solutions of the kinematic (linear) equation.

Fig. 1 demonstrates our view: The nonlinear extension of the first eigenmode is the attractor in a certain interval $C>C_1$. This attractor is generally of simple structure, i.e. steady or oscillatory and showing a certain spatial symmetry. More far in the nonlinear region symmetry breaking will appear and an attractor of complicated structure has to be expected.

What we stress here is that this behaviour can hardly be described by considering the higher (linear) modes. In this way we do not accept explanations of observational facts based on higher modes!

Realistic models of disk dynamos depend at least on two parameters: C_α describes the induction action of the turbulent motions undergoing the influence of Coriolis forces. These motions provide for a mean electromotive force **E** which is - in the simplest case - parallel to the mean magnetic field, i.e. $\mathbf{E}=\alpha\mathbf{B}$. The other parameter C_Ω is a measure of the induction action of the differential rotation. In addition, we introduce a third parameter by which we take into account an anisotropy of the α-effect due to the big differences between the horizontal and the vertical scales in a disk.

We model the galaxy by an axisymmetric disk, i.e. we assume that the electrical conductivity σ, the angular velocity Ω and the α-parameter do not depend on the azimuth. In addition, we assume σ and Ω to show symmetry and α antisymmetry with respect to the galactic plane.

The eigensolutions of Eq. (1) - the eigenmodes - in a system with the above symmetries are either symmetric (S) or antisymmetric (A) with respect to the central plane of the disk and depend on the azimuth ϕ according to $e^{im\phi}$. Correspondingly the magnetic field modes are denoted by Sm or Am.

A dipole in the centre of the disk and aligned to the axis of the rotation is typical for an A0-field. The axisymmetric quadrupole parallel to the axis of rotation is of type S0. Combined with a toroidal field encircling the axis of rotation and arranged symmetric about the galactic plane it represents the symmetry type of the ASS-fields observed in M 31 and IC 342.

Furthermore, a dipole in the centre but aligned to the galactic plane represents a magnetic field of type S1. Deformed by the shear of a differential rotation it takes the shape of a BSS-field as observed in M 81.

2. FROM SPHERES TO DISKS

Physical theories of a certain phenomenon may have imperfections for two reasons: Either is the modellation imperfect or the available mathematical tools are insufficient. To mention this truism seems useful if considering the dynamo models for galaxies presented so far (Krause, 1990).

Clearly, the mathematical description of the physics in thin and finite disks is highly complicated. The only clean models are probably those of oblate spheroids embedded in insulating space which were considered by Stix (1975) and White (1978). Concerning the excitation conditions, Soward (1978) was able to show that the eigenvalues asymptotically approximate those which were derived by Parker (1971) for a plane layer model. Thus with respect to the eigenvalues the plane layer seems to provide for a sufficient description. The field structure, however, is quite different for ellipsoids and slabs. In the former case the magnetic fields decrease with $(r^2+z^2)^{-3/2}$; whereas in the latter with $e^{-k|z|}$ and behave periodically in the galactic plane. For this reason, in case of slabs there is not a discrete set of eigenvalues. It becomes discrete if a rather artificial assumption concerning the r-direction is introduced. Here, the case of the slab, allows not for a correct description of the edge of the disk.

These above mentioned models are two-dimensional, in case of Stix and White by the assumption of axisymmetric solutions, in case of Parker the solutions are considered to be independent of the direction simulating the direction of rotation. Consequently, it cannot, in the frame of these considerations, be decided whether there are competing non-axisymmetric modes. According to our strict view --only the marginal mode is of physical interest -- this imperfection of these models left an important question open.

The mathematical tools used do not simply, for different reasons, allow for a generalization to three-dimensional models. Therefore, the way of Stix and of White has not been further followed; beside of this the convergence becomes more and more bad if more flat ellipsoids are considered. In case of the slabs the differential rotation is represented by a shear flow. So the equations allow a representation by Fourier modes for those magnetic fields, which do not depend on the coordinate in the direction of the flow, but in the three-dimensional case complications appear.

The main shortcoming of these models lies in the consideration of a restricted set of competing modes; namely of two-dimensional ones. This restriction is still stronger in the modellation of Fujimoto (e.g. Fujimoto (1987)) who considered a special ansatz of a bisymmetric field. The question of competing (e.g. axisymmetric) modes is not analyzed, although all other investigation indicate an easier excitation of axisymmetric modes. Besides of that the fields are singular for $r = 0$ and $r = \infty$. So the solutions of Fujimoto, although practical for fitting to observations, seem physically and mathematically not well founded.

A well-elaborated concept for the investigation of axisymmetric as well as non-axisymmetric fields in thin and finite disks was presented by the Moscow group (cf. Ruzmaikin et al., 1988). Taking into account the big difference in the vertical z-direction and the radial r-direction they used an asymptotic method for integrating the dynamo equation in two steps, first in the z-direction, then in the r-direction with an r-dependent thickness of the disk. In this way they arive at a denumerable set of eigenmodes with their growth rates.

Some simplification leaves the question unanswered whether the correct problem is really approximated. In the first step, the integration with respect to z, a boundary condition is used which assumes a vanishing radial wave number. It has to be suspected that in this way the marginal mode will not be found (Rädler and Bräuer, 1987). Further it is unclear whether the central region of the disk, where the r-dependence of the solutions is much stronger, is correctly taken into account. We think that future results will clarify the situation.

As it concerns the central problem - the excitation of non-axisymmetric magnetic fields in axisymmetric disks - in these investigations the non-axisymmetric mode is never the marginal mode (Baryshnikova et al., 1987).

In order to overcome these difficulties we will use in the following a concept (Elstner et al., 1990) which allows for an unambique determination of the marginal mode.

As a first step towards flat objects we calculated within this concept the marginal dynamo numbers for an oblate spheroid with α = const. embedded in (a) space of the same conductivity as the spheroid and (b) insulating space. We choosed a=2b with a, b being the semi-major and semi-minor axes. The results for the modes m=0 and m=1 are given in Tab. 1.

TABLE 1. Marginal dynamo numbers $C_\alpha = \mu\sigma\alpha b$ for an α=const. - ellipsoid

	m=0	m=1
insulating surroundings	2.9	3.1
conducting surroundings	2.3	2.35

Compared with the analytically known results for α-spheres (Krause and Steenbeck, 1967, see also Krause and Rädler, 1980) where the results do not depend on m we find here a preference of m=0 which is due to only the flat geometry. This effect is the stronger the lower the conductivity of the surroundings.

In section 4 we will discuss a more realistic model including differential rotation and inhomogeniety and anisotropy of the α-effect.

3. DYNAMO-RELEVANT PROPERTIES OF GALAXIES

Galaxies are the only known celestial bodies with observable inner *rotation* law. In particular this fact favours them as subjects of the dynamo theory because of the inducting action of differential rotation. Compared with stars the galactic rotation exhibits essential differences. In oposition to stars galaxies cannot be imagined as non-rotating as their over-all structure emanates from the equilibrium of gravitation and centrifugal force. The resulting angular velocity is generally non-uniform -- in good accordance with the (Doppler-) observations. What is observed is a nearly rigid-body rotation only in the core region (corresponding to the regularity condition at r = 0) up to a few kpc while for greater distancies the characteristic rotation law $\Omega \sim r^{-1}$ appears.

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Kepler rotation has never been found in visible disk regions normally extending up to a few tens of kpc. Dark coronal masses must be responsible for this phenomenon. We are, however, confronted with the situation that distinct non-axisymmetric configurations such as (inner) bars and (outer) spirals often rotate "almost rigidly". The global spiral patterns of galaxies having neither bar nor companion end where the rotation curves become flat (Kormendy and Norman, 1979).

The vertical profile $\Omega(z)$ behaves rather uniform throughout the whole disk. But also the above situated layers up to 2-3 kpc are corotating while only beyond this distancy the angular velocity slowly tends to zero.

Also the *turbulence* as the second input parameter of the dynamo equation can be observed in situ. Detailed descriptions of the current knowledge of the spectral properties of the interstellar turbulence are given by Ruzmaikin et al. (1988) and Henning (1990). We must be satisfied here with a brief discussion of those characteristic scales valid for giant molecular clouds. If only orders of magnitudes are concerned we find the numbers

$$u' \approx 10 \text{ km/s}, \quad l \approx 100 \text{ pc}, \quad \tau \approx 10^7 \text{ yrs} \tag{2}$$

for velocity dispersion, correlation length and correlation time . From these values the turbulent magnetic diffusivity η_T, magnetic Reynolds number $R_m = \eta_T/\eta$ and the Coriolis number $\Omega^* = 2\tau/\tau_{rot}$ can easily be estimated:

$$\eta_T \approx 10^{26} \text{ cm}^2/\text{s}, \quad R_m \approx 10^5, \quad \Omega^* \approx 0.1 \tag{3}$$

These quantities reveal the interstellar turbulence (molecular clouds) as a perfectly conducting material -- despite the low temperatures of only few tens of Kelvin -- under the action of a slow rotation. Then and if a density gradient exists there is as a second induction mechanism in the dynamo the α-effect of the general form

$$\alpha \approx -l^2 \,\Omega \;\; d \log\rho/dr \quad . \tag{4}$$

As in any flat disk the gradients in vertical direction (z) dominate those in the radial direction, the α simplifies to

$$\alpha \approx \pm \, l^2\Omega/H \quad \approx \pm \, l\Omega \leq \pm \, 5 \text{ km/s}, \tag{5}$$

with H as the vertical density scale height and the upper (lower) sign for the northern (southern) hemisphere. The given estimates yield for the normalized alpha number

$$C_\alpha = \alpha \, H/\eta_T \quad \approx (\Omega R/u') \, H/R \approx 10^2 \, H/R \approx 1 - 10 \tag{6}$$

(H disk half-thickness, R disk radius), i.e. just the same order of magnitude as for planets and stars.

There is, however, an important objection. Due to the disk geometry the α-effect must be expected as different for different field compo-

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nents. For subsonic externally driven (e.g. by explosions) random motions the calculations lead indeed to a highly anisotropic α-tensor (Rüdiger, these proceedings). The α-effect in vertical direction differs markedly from that in the radial direction. It will be a main topic of our dynamo calculations to find out the consequences of this sort of anisotropy for the functions of the dynamo.

A similar consideration as in (6) leads to

$$C_\Omega = H^2 \Delta\Omega/\eta_T \approx 1 - 10 \tag{7}$$

as well where $\Delta\Omega$ denotes the variation of Ω across the disk.

Kulsrud (1989) suggested the interstellar material having a magnetic Prandtl number much larger than unity because of the relatively high molecular viscosity for very thin gases. Either a detailed discussion of the microscopic diffusivities for the various modes as well as the concerted influence on the dynamo mechanism are not yet worked out so far.

4. NON-AXISYMMETRIC FIELDS IN AXISYMMETRIC DISKS?

The observation of large-scale non-axisymmetric magnetic field configurations in galaxies stimulates the search for an answer to an interesting question within the frame of kinematic mean-field dynamo theory: Is it possible that non-axisymmetric magnetic fields are excited preferrably in axisymmetrically structured disks?

If, under realistic conditions, the answer is "no", and the dynamo explanation is not doubted at all, the magnetic non-axisymmetry will probably be a consequence of some basic deviation from axisymmetry related to the spiral structure of galaxies.

Our results for a simple galaxy model (defined in Meinel et al., this volume) as well as all previously obtained results (see e.g. Ruzmaikin et al., 1988) support the answer "no".

Tab. 2 gives the obtained sequence of marginal dynamo numbers C_α for the first three modes A0, S0 and S1 for our model (ii) with and without differential rotation ($C_\Omega = 10$, $C_\Omega = 0$), and with and without anisotropy of the α-effect ($\alpha_\| = 10\alpha_\perp$, $\alpha_\| = \alpha_\perp$).

TABLE 2. Magnetic field modes ordered according to the values of their marginal dynamo numbers C_α.

C_Ω	$\alpha_\|/\alpha_\perp$	mode sequence
0	1	A0 < S1 ≈ S0
0	10	A0 ≈ S1 < S0
10	1	S0 < S1 < A0
10	10	S0 < S1 < A0

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These results show that only an extremely weak differential rotation combined with a high degree of anisotropy of the α-effect may possibly lead to a preference of BSS fields. Under more realistic conditions it is always a field of S0 type which is preferred in axisymmetric disks. This symmetry type corresponds to the observed ASS fields.

Our calculations revealed a further important point. In order to obtain the desired marginal dynamo numbers and field modes we used a direct method incorporating some artificial nonlinearity (Elstner et al., 1990). In this way we got some insight into relevant time-scales. The typical time-scale for reaching the stationary *large-scale* state is of an order of $\mu\sigma R^2 \approx 10^{12}$ yrs which is large compared to the world's age, cf. section 3. Therefore it seems to be questionable wether there was enough evolution time leading to a complete formation of the preferred large-scale field structure. However, in nature dynamical processes provide for a much more rapid propagation of magnetic fields in galaxies than that described by the diffusive time-scale.

Fig. 2 shows examples of calculated S0- and S1-, i.e. ASS- and BSS-fields. Note that the large-scale field structure (in particular its r-dependence) is still not completely stationary, although the corresponding marginal dynamo numbers are already approached with sufficient accuracy.

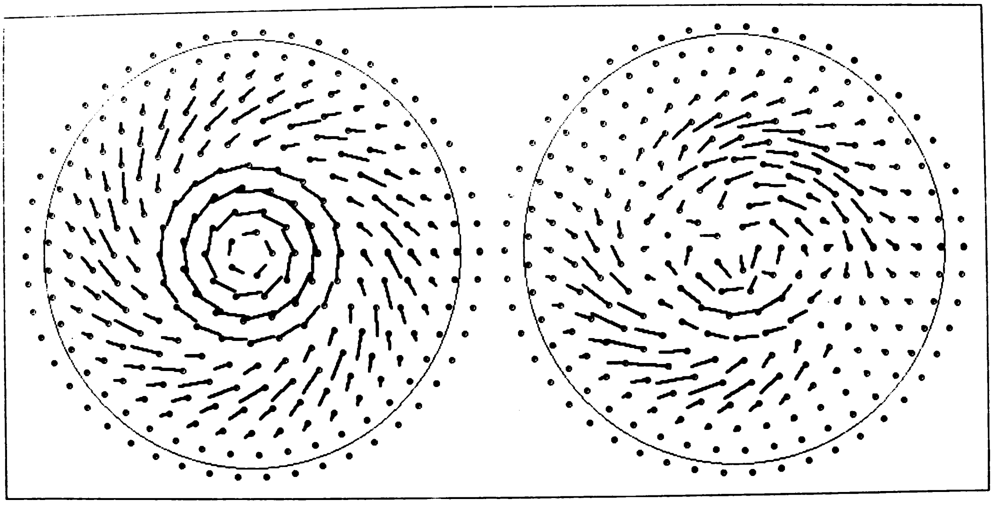

Figure 2. Field vectors in the galactic plane z=0 for the model with $C_\Omega=10$, $\alpha_\parallel=10\alpha_\perp$, cf. Tab. 2. Left: The preferred ASS-mode (S0), Right: The BSS-mode (S1)

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KAHN: The interstellar medium is really very complex. The clouds interact with the hot intercloud medium and are (relatively) frequently overtaken by stellar winds and supernova blast waves. How will a realistic dynamo theory cope with such difficulties?

F. KRAUSE: I fully agree with you that my model here is a crude one. What I wish to make clear is that the α-effect is a very elementary effect appearing in turbulences undergoing the influence of Coriolis forces. It is as elementary as the circulations (clockwise or counter-clockwise) around the lows or highs in the Earth's atmosphere.

SHUKUROV: What is the spatial structure of the "forgotten modes" mentioned by you? Are they concentrated near the disk center or in the outer parts of the disk?

F. KRAUSE: There is probably a misunderstanding: As "forgotten modes" I understand to be those modes which have been overlooked due to the imperfections of a model or the applied methods. There arises a serious problem if the marginal mode, i.e. the mode which is excited most easily, was not found and the following philosophy was based on a higher mode. Our model excludes a priori this possibility. Probably you wish to know the structure of the marginal modes which we determined. Our pictures show that, dependent on the assumed structure, fields may concentrate near the centre but also in more distant parts.

STIX: Which of the α's is larger according to the work of Rüdiger, the horizontal or the vertical?

F. KRAUSE: The analysis by Rüdiger results in a value of the vertical α, i.e. α_v, which is ten times the horizontal α.

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