EXPONENTIAL GROWTH OF DISTANCE BETWEEN NEARBY RAYS DUE TO MULTIPLE GRAVITATIONAL LENSING

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Abstract. We have investigated multiple gravitational lensing by numerical "ray tracing" simulations. We have found that the distance between rays grows exponentially, on average, until it reaches the projected mean separation of lensing objects ($\sim RN^{-1/2}$, where R is the system size and N is the number of scattering object). This nature may affect observations of high redshift objects or the anisotropy of the cosmic background radiation.

1. Basic Concept

The explanation is summarized as follows (Fukushige and Makino 1994). When the distance between the rays is small, the rays are scattered coherently by the same object. In the single scattering event, the distance increases in proportion to the distance before the scattering, since the differential acceleration or the "tidal force" is proportional to the distance between rays. Through multiple scattering, the distance increases exponentially. This exponential growth stops when the distance becomes so large that the scattering becomes incoherent. In the community of gravitational N-body simulations, the exponential divergence of initial conditions has been well known (e.g. Goodman et al. 1993).

Previous studies neglected the effect of the exponential growth. For example, Gunn (1967) expressed differential deflection between rays through multiple lensing by a superposition of single events, under an assumption that the differential deflection through single event is small. This assumption is, however, not appropriate. The average increase of the distance between rays by a single event is proportional to its original distance. Thus,

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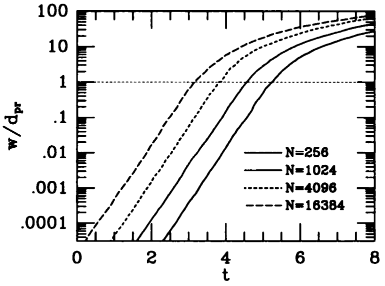

Figure 1. The distance w between rays grows exponentially until it reaches at the projected mean separation of scattering objects ($d_{\rm pr} = RN^{-1/2}$)

the distance grows exponentially through multiple scattering. The increase of the distance evidently cannot be expressed by the simple superposition.

Note that during this exponential growth the phase space volume is conserved and the "shape" of phase space volume is elongated. The exponential growth can be observed in a projection of this volume on position space or velocity space. Moreover, the exponential growth does not continues forever, since it stops at the projected mean particle separation. Note also that the exponential growth is not affected by the scattering event and that the distance converges, on average. If the scattering object exists between two rays, the two rays are scattered to approach each other. This is the more familiar case of strong gravitational lensing. However, this is very rare event when the distance between rays is small.

2. Ray Tracing Simulation

We have performed 3D "ray tracing" simulations (Fukushige et al. 1994). The lensing objects are distributed randomly in a cube, and are fixed. Figure 1 shows the distance w between rays grows exponentially until it reaches the projected mean separation $RN^{-1/2}$. We plot the median value of 1000 runs. In our units, the light velocity, the system size (R), and the gravitational constant (G) are unity, and the mean density (ρ) is 0.5. The lensing object is point a mass. We use a special-purpose computer (MD-GRAPE (Taiji et al. 1995)) for the force calculations.

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