The roots of (1) are therefore real and distinct, real and two equal, one real and two imaginary, according as

$$(3\alpha)^2 - 4(3\alpha^2 + p) \gtreqless 0,$$

$$i.e. \quad \text{according as } 3\alpha^2 + 4p \lesseqgtr 0.$$

Taking the first of these cases along with the condition $\alpha^3 + p\alpha + q = 0$, and writing it $3\alpha^2 + 4p = -k^2$, to avoid trouble with the odd indices and the inequality, we find

$$\sqrt{\frac{-4p-k^2}{3}}\left(\frac{-4p-k^2}{3} + p\right) = -q,$$

which, on simplifying and squaring, becomes

$$-(4p+k^2)(p^2 + 2pk^2 + k^4) = 27q^2,$$

or, $$-4p^3 - (3pk + k^3)^2 = 27q^2,$$

or, $$4p^3 + 27q^2 = -(3pk + k^3)^2,$$

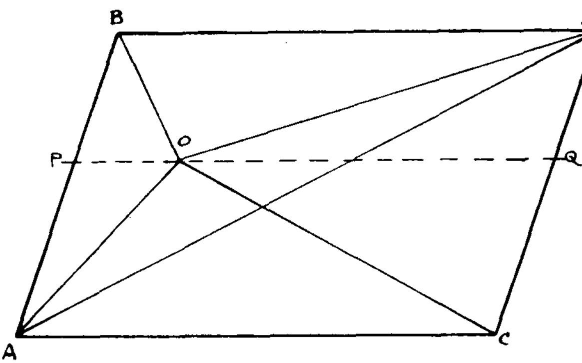
or, $$4p^3 + 27q^2 < 0.$$

Similarly, the necessary and sufficient conditions for the other two cases, viz., three real roots, two equal, and one real root only, are

$$4p^3 + 27q^2 = 0, \quad 4p^3 + 27q^2 > 0 \text{ respectively.}$$

J. M'WHAN.

The Moments Theorem.—If $ABDC$ be a parallelogram, of which AD is a diagonal, and if O be any point which, by joining



to A, B, C, D, makes three triangles, OAB, OAD, OAC, each of these being taken as positive in sign so long as O lies within the region selected (*i.e.* within ABD), then :

$$\triangle OAC - \triangle OAB = \triangle OAD.$$

For $\quad \triangle OAD + \triangle OAB + \triangle OBD =$ half the parallelogram.

But $\quad \triangle OAC + \triangle OBD =$ half the parallelogram.

(seen by drawing the line POQ)

From these equals, removing the $\triangle OBD$ we have

$$\triangle OAD + \triangle OAB = \triangle OAC$$

or, $$\triangle OAC - \triangle OAB = \triangle OAD,$$

if AB, AC, AD represent two forces and their resultant, becoming :

moment of AB
+ moment of AC
= moment AD, all about O.

This is the moments theorem, and involves the usual attribution of signs.

If O moves away so as to cross either a diagonal or a side of the parallelogram, it in so doing makes one of the moments vanish, the sign of that moment changes thereby, and the moments theorem is confirmed in all positions.

G. E CRAWFORD.

A simple nomogram for the solution of quadratic equations.—There are many well-known graphical solutions of the quadratic equation, in which the roots are obtained as the intersections of a circle with a straight line. The practical objection to these solutions is that a fresh diagram has to be drawn for each equation that is to be solved : and the time occupied in constructing the diagram is greater than the time required to solve the equation by the ordinary arithmetical method.

This objection no longer applies if the diagram is of such a nature that, when once constructed, it can be used for *any* quadratic, whatever be the values of the coefficients. Such a diagram, in which the construction is made once for all and is applicable to any number of special cases, is called a *nomogram*. A nomogram for the solution of the quadratic has been devised by Monsieur d'Ocagne, which depends on the intersection of a straight line with a hyperbola.