Then, since $f(x)$ is positive,

$$\frac{f'(x)}{f(x)} \leqq -k, \quad -\frac{f'(x)}{f(x)} \geqq k,$$

and so

$$\log\{1/f(x)\} \geqq kx + \text{constant}.$$

That is to say,

$$f(x) \leqq B \exp(-kx), \tag{6}$$

where B, k are positive constants.

In any possible application of Theorem 2 it will be easier to see whether (6) is satisfied than it will be to see whether (5) is satisfied.

Raabe's test. The analogue of Raabe's test is obtained by putting $\phi(x) = x$. If we suppose that, for some positive k and X,

$$\frac{d}{dx}\{f(x) \,.\, x\} \leqq -kf(x) \text{ when } x > X, \tag{7}$$

then, as a little calculation shows,

$$f(x) \leqq Ax^{-1-k}, \tag{8}$$

where A is a positive constant. No one would prefer (7) to (8) as a criterion of convergence and (8), like (6), is a well-known test for the convergence of infinite integrals.

The next test, in the usual order, is given by taking $\phi(x) = x \log x$ in Theorems 1 and 2. That the test is useless may be seen from the fact (mildly interesting in its proof) that

$$\frac{d}{dx}\{(x \log x) f(x)\} \leqq -kf(x) \text{ when } x > X$$

implies

$$f(x) \leqq A/x (\log x)^{1+k}.$$

Hertford College, Oxford.

A further note on differentials

By E. G. Phillips.

Since the publication of my article[1] on "The advantage of differentials in the technique of differentiation" both Dr H. A. Hayden and Prof. A. Oppenheim have kindly pointed out to me that

[1] *Math. Notes* 30, May 1937.

there is a much shorter solution of the problem by partial derivatives than the one which I gave as Solution 2. The solution is as follows:— If y and z are the independent variables we have, since

$$\frac{\partial x}{\partial z} = \frac{1}{p}, \quad \frac{\partial x}{\partial y} = -\frac{q}{p},$$

$$\begin{aligned} \frac{\partial^2 x}{\partial y\,\partial z} = \frac{\partial}{\partial y}\left(\frac{1}{p}\right) &= -\frac{1}{p^2}\left\{\frac{\partial p}{\partial y} + \frac{\partial p}{\partial x}\,\frac{\partial x}{\partial y}\right\} \qquad (1) \\ &= -\frac{1}{p^2}\left\{s + r\left(-\frac{q}{p}\right)\right\} \\ &= \frac{1}{p^3}(rq - sp), \end{aligned}$$

which gives the result required.

Since my Solution 2 was, quite unintentionally, rather unfair to the method of partial derivatives, I feel that I ought to draw attention to this shorter solution.

The fact that the above solution is merely *shorter* than the one which I gave does not however detract from the practical advantages of the differential method. Any experienced teacher knows that the step which presents real difficulty to the beginner is the obtaining of equation (1) above. Although in the case of the example which I happened to choose for illustration (and it may not have been the best for the purpose) the above solution by partial derivatives happens to be quite as *short* as the solution by differentials, the fact remains that, while the technique of differentiation, when once understood, is almost "fool-proof," the pitfalls for the beginner in the solution given above are well known to every teacher of the subject. While the solution of a problem by partial derivatives may be quite a difficult piece of manipulation, exactly the same technique is required for the solution of a problem by differentials, however simple or complicated the problem in question may happen to be.

On pedal tetrahedra

By R. T. Robinson.

1. In a tetrahedron $ABCD$ with its opposite edges perpendicular there are two tetrahedra which can be described as pedal tetrahedra.

(1) the tetrahedron $A_0 B_0 C_0 D_0$ where these points are the feet of the perpendiculars from $ABCD$ on to the faces BCD. ACD .... called here the face-pedal tetrahedron.

https://doi.org/10.1017/S095018430000255X Published online by Cambridge University Press