CORRESPONDENCE

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Valuation of annuities with a guaranteed term

Sirs,

The following short and accurate method of valuing these annuities makes use of such a simple device that I am sure many members of the Society must be familiar with it. However, judging by the articles and correspondence on the subject that appeared in the Society's journals a few years ago, there are certainly some members who do not know the method and may be interested.

The factor used in the valuation is of the form

$$a_{\overline{n}|}+\frac{N_{x+n+1}}{D_x}\quad\left(\text{where } N_{x+n+1}=\sum_{t=n+1}^{\infty}D_{x+t}\right).$$

This is equal to

$$\frac{1}{i}-\frac{v^n}{i}+\frac{N_{x+n+1}}{D_x}$$

$$=\frac{1}{i}-\frac{v^{\text{year of final `certain' instalment}}}{i\times v^{\text{year of valuation}}}+\frac{N_{x+n+1}}{D_x}.$$

Each valuation card should therefore have two functions in the following form:

(1) Annuity p.a. $\times\, v^{\text{year of final `certain' instalment}}$ (say V function),

(2) Annuity p.a. $\times\, N_{x+n+1}$ (say N function).

For valuation it is then only necessary to sort the cards according to attained age. The 'deferred' portion is valued by multiplying, for each age group, the total of the N functions by a factor of the form $1/D_x$.

The 'term certain' portion is valued simply by

$$\frac{\text{Total annuity p.a.}}{i}-\frac{\text{Total } V \text{ functions for all age-groups combined}}{i\times v^{\text{year of valuation}}}.$$

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extent that the assumptions underlying it reflect the facts of the transaction. Baden assumes income receivable annually, and the sum assured at the end of the year of death. These assumptions are not realized in practice. When a reversionary company purchases an annuity charged on a life interest it is usual to stipulate that the annuity be payable in quarterly instalments, and I think it unlikely that the sum assured would be received, on average, as late as six months after the death of the life tenant. In this latter respect Nightingale has a nicer theoretical basis than Baden, but I have never heard of anyone using Nightingale—his formula is too complicated. In the less frequent case of the outright purchase of a life interest all the common formulae go even further astray—the question turns partly on whether income is apportionable at the beginning and at the end. In practice, of course, the choice of any particular formula scarcely matters; the dominant factor is the margin taken on the income.

My second point is more a question of opinion, and I will approach it indirectly by telling how I came to write the original note. It was suggested to me by Mr W. P. Goodchild that Baden might not be an appropriate formula at an advanced age, since, depending on the point in the year when the life tenant died, the overall yield would fluctuate and a capital loss might even be sustained. I set to and 'invented' Jones, only to find that I was a century or so too late. Even so, I considered that publication was justified, because of the intrinsic merits of the formula and its apparent neglect.

If we adopt Mr Pegler's definition of expected yield, and imagine a frequency distribution of all possible yields from a given investment, then the greater the standard deviation of that distribution, the higher should be the mean. In other words, with a more speculative security an investor should be entitled to a higher *average* return. Such is my opinion, and such my justification of the lower comparative price brought out by Jones at the older ages.

Yours faithfully,

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