On the History of Hellin's Law

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Theorems, proofs, laws and rules are commonly named according to the presumed investigator, but often earlier investigators have contributed substantially to the findings. One example of this is Hellin's law, which was named after Hellin, although he was not the first to discover it. In research on twinning and higher multiple maternities, the law has played a central role because it is approximately correct, despite showing discrepancies that are difficult to explain or eliminate. Several improvements to this law have been proposed. In this study, we reexamine some old papers to provide an overview of the scientists who have contributed to the genesis and the improvements of this law. In addition, we consider more recent contributions in which Hellin's law has been discussed and evaluated. It has been mathematically proven that Hellin's law does not hold as a general rule. However, most studies are based on empirical rates of multiple maternities, ignoring random errors. Such studies can never confirm the law, but only serve to identify errors too large to be characterized as random.

Keywords: historical demography, multiple maternities, monozygotic twinning, dizygotic twinning, recurrent twinning

Theorems, proofs, laws and rules are commonly named according to the presumed investigator. However, noteworthy contributions have also been made by other scientists (Stigler, 1980). In research on multiple maternities, Hellin's law has played a central role. In the literature, authors generally refer to Hellin (1895), and consequently, the law has used his name. The later contributions by Zeleny (1921) have resulted in the law also being called the Hellin-Zeleny law. Our article starts with a review of the common rules for national demographic registers and how these have improved information, enabling the scientists to contribute to the genesis of research on multiple maternities. Our intention is not to provide an exhaustive list of publications, but only to present papers in which the authors before Hellin have presented ideas leading up to Hellin's law, or where later authors have analysed and commented on the strengths and weaknesses of the law. The arguments for Hellin's law are usually based on stochastic models for multiple fertilizations (see, e.g., Allen & Firschein, 1957; Jenkins, 1927, 1929; Jenkins & Gwin, 1940; Zeleny, 1921). Observed deviations from Hellin's law are caused by disturbing factors such as differences in intrauterine survival.

The interest in Hellin's law is mainly the result of its being mathematically simple and approximately correct, but does show discrepancies that are difficult to explain or eliminate. Jenkins (1927, 1929), Jenkins and Gwin (1940), Bulmer (1970), and later Fellman and Eriksson (2004) have tried to modify the law in order to improve it. Fellman and Eriksson (1993) have given a mathematical proof that Hellin's law cannot hold as a general rule. One application of Hellin's law is to compare the twinning rate (TWR) and the square root of the triplet rate (TRR), the cubic root of the quadruplet rate (QUR) and so on (Eriksson & Fellman, 2007; Fellman & Eriksson, 2006). Fellman and Eriksson (2004) considered the correlation between the TWR and the square root of the TRR in Sweden. After elimination of temporal factors, they found that the correlation was positive, but not very strong. This finding also indicates that Hellin's law cannot be exact. Hellin's law presupposes strong correlations, but even strong correlations do not guarantee that Hellin's law will be upheld.

Results

Stigler's law of eponymy. Particularly important scientific observations are often associated with a person, as is the case with Gaussian distribution, Halley's comet, Planck's constant and Weinberg's differential rule. Historians of science, have however, noted that often the person associated with a particular finding was not its original discoverer. Historical acclaim tends to be allocated to people unevenly. Scientific observations and results are often associated with people who have high visibility and social status, and the results are named long after the discovery. Based on his studies on the history of statistics, Stigler (1980) proposed his own 'Stigler's Law of Eponymy'. In brief, the law says: 'No scientific discovery is named after its original discoverer'. Stigler attributes the discovery of Stigler's law to Merton (1973), which makes the law self-referencing. In addition, Raimi (1976) had already formulated Stigler's law in connection with his

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study of the first digit problem (Benford's law). He noted that Newcomb (1881) had formulated Benford's law 57 years before Benford (1938). Consequently, in this study, one must bear in mind Stigler's law.

Prerequisites for Twinning Research.

In the 19th century, a series of statistical congresses, most notable in Brussels in 1853 and in St Petersburg in 1872, were important for the start of demographic research and especially twinning research. Levi (1854) gave a detailed presentation of the suggestions accepted at the congress in Brussels.

> [T]here ought to be an annual registry of population, exhibiting the births by sex, by age of both parents, legitimate and illegitimate, number of twins, stillborn, marriages and divorces, by months. The deaths, by sex, by age, and by months, distinguishing among dead children, till three years of age, the legitimate from the illegitimate. The deaths by month, with the causes of death, and the profession of the deceased; marriages, with the age of the parties, their condition, profession, and number of children, distinguishing the legitimate and those acknowledged as such. Considering the extreme importance of a uniform nomenclature of diseases equally applicable to all countries, the attention of learned men is to be called to the question for further consideration at some future congress.

According to Brown (1872), the principal discussion at the St Petersburg congress centred around facts relating to the movement of the population and the mode in which they should be registered. Among the facts to be registered were that in multiple maternities the sex and number of the children, stillborn or born alive, whether legitimate or not, and the age and parity of the mother on the birth date. Westergaard (1932) has devoted a whole chapter in his history of statistics to the presentations of the statistical congresses in the middle of the 19th century and their importance.

In most countries, the registers were deemed to be lacking in essential facts; those of Belgium and Sweden were perhaps the most detailed for scientific inquiries. Already in the 18th century, Wargentin published demographic data for Sweden (cf. Hofsten, 1983). However, he did not pay any attention to twinning and higher multiple maternities. During the second half of the 19th century, Statistics Sweden published in Statistisk Tidskrift extensive time series of demographic data. The data were given separately for the different counties of Sweden and contained the size of the population, the number of births (live and stillborn) and twin, triplet and quadruplet sets. A list of these data is given in Table 1, indicating that Sweden has for the whole country the oldest continuous population statistics worldwide. We have used these data in different studies (e.g., Eriksson & Fellman, 2004; Fellman & Eriksson 2003, 2005).

Table 1

The Division of Sweden Into 25 Counties for Regional Data Concerning Population Size, Births and Multiple Maternities, 1749–1888 (ST = *Statistisk Tidskrift*)

County (län)	Period	Reference
City of Stockholm	1749–1858	ST, 1860–62:43–47
Stockholm	1749–1773, 1795–1858	ST, 1860–62:134–141
Uppsala	1749–1773, 1795–1859	ST, 1860–62:280–288
Södermanland	1749–1773, 1795–1859	ST, 1860–62:317–324
Östergötland	1749–1773, 1795–1860	ST, 1863–65:164–171
Jönköping	1749–1773, 1795–1862	ST, 1863–65:266–273
Kronoberg	1749–1773, 1795–1862	ST, 1863–65:274–281
Kalmar	1749–1773, 1795–1868	ST, 1870:211–220
Gotland	1759–1869	ST, 1870:27:221–231
Blekinge	1749–1773, 1795–1869	ST, 1870:232–240
Kristianstad	1749–1773, 1795–1871	ST, 1873:133–142
Malmöhus	1749–1773, 1795–1871	ST, 1873:153–152
Halland	1749–1773, 1795–1871	ST, 1873:153–162
Göteborg och Bohus	1749–1773, 1795–1859	ST, 1860–62:388–400
Älvsborg	1749–1773, 1795–1874	ST, 1875:127–136
Skaraborg	1749–1773, 1795–1876	ST, 1877:156–168
Värmland	1795–1865	ST, 1877:170–176
Örebro (Närke)	1749–1773	ST, 1877:166–169
Västmanland	1749–1773, 1795–1887	ST, 1888:159–170
Kopparberg	1749–1773, 1795–1887	ST, 1888:171–182
Gävleborg	1763–1773, 1793–1888	ST, 1888:161–172
Västernorrland	1792–1888	ST, 1888:173–184
Jämtland	1792–1888	ST, 1888:185–196
Västerbotten	1802–1860	ST, 1863–65:50–57
Norrbotten	1802–1860	ST, 1863–65:44–49

Influential Papers Before Hellin

The Veit data set from Prussia (1826–1849), presented in Table 2, consists of 13360557 maternities, including 13208868 single, 149964 twin, 1689 triplet and 36 quadruplet maternities (Veit, 1855). Veit analysed the temporal trend in the TWR and noted very small variations, but during the first half of the period the annual TWRs were almost constantly higher than during the last half of the period (except for the year 1849). The trend may be seen in Figure 1. Note that the TRR shows more marked fluctuations by chance than the TWR. For the total data set Veit noted the following rates: for twin pairs 1:89, for triplet sets 1:7910 and for quadruplet sets 1:371126. He did not give the relations between TWR, TRR and the quadruplet rate (QUR); that is, Hellin's law. He also presented the sex compositions within the twin, triplet and quadruplet sets and noted a lower sex ratio among multiple births than among singleton births.

The Wappäus data set was collected from different European countries and comprised 19698322 maternities, including 226807 twin and 2623 triplet maternities (Table 3). Wappäus (1859) presented the rates of multiple maternities, but did not discuss the relation between the number of twin, triplet and quadruplet maternities.

In our opinion, Bertillon (1874) foresaw Hellin's law. He considered multiple maternity data from dif-

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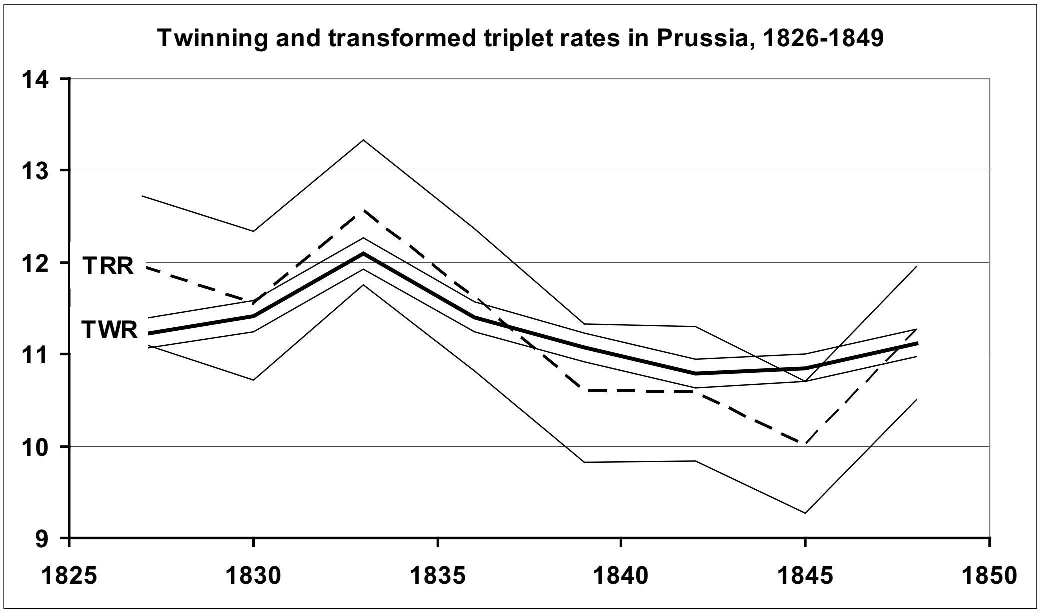

Figure 1

Comparison between twinning rate (TWR) and transformed triplet rate (TRR) in Prussia, 1826–1849. Veit stressed the high rates during the first half of the period and the low rates during the second half. The TRR shows stronger fluctuations, but the confidence bands indicate that the difference can mainly be ascribed to random errors.

Table 2

Data from Prussia, 1826–1849, According to Veit (1855)

	Maternities				
Year	All	Single	Twin	Triplet	Quadruplet
1826	519633	513727	5824	80	2
1827	485165	479724	5374	65	2
1828	493749	488060	5620	69	0
1829	489604	483796	5738	69	1
1830	491659	486141	5455	62	1
1831	484889	479281	5543	65	0
1832	476035	470175	5783	76	1
1833	530954	524525	6340	87	2
1834	549750	542947	6717	83	3
1835	527148	521156	5918	73	1
1836	544177	537805	6301	69	2
1837	551450	545084	6289	77	0
1838	560086	553837	6186	61	2
1839	568487	562065	6360	59	3
1840	580747	574293	6381	72	1
1841	585085	578738	6277	67	3
1842	616845	610058	6716	71	0
1843	597912	591420	6426	64	2
1844	616287	609452	6771	59	5
1845	640214	633123	7029	60	2
1846	619727	613101	6556	69	1
1847	577007	570766	6183	58	0
1848	570737	564633	6030	73	1
1849	683210	674961	8147	101	1
Total	13360557	13208868	149964	1689	36

ferent countries in central Europe. In his study, he presented the number of triplet maternities per year and per one million total maternities. He also presented the number of total maternities per one triplet maternity and the number of twin maternities per one triplet maternity; that is, he considered the relation between twin and triplet rates. However, he did not relate the number of total maternities to one twin maternity. Had he done this, he would have been close to Hellin's law. In Table 4, we present a translated version of his table (Bertillon, 1874, p. 285) and also include our calculations of the number of total maternities in relation to one twin maternity and the annual mean number of maternities. We believe that had Bertillon included the former of our columns in his table, he would have discovered Hellin's law.

Shortly after the congresses in Brussels and St. Petersburg, Neefe (1877) published his classical work. He stressed how important the above-mentioned statistical congresses were for the standardization of the demographic registers in different countries, and he used the new possibilities that the improved birth registers offered. Although other contemporaneous studies were published, in our opinion the history of twinning research starts with his publication. Neefe analysed a long series of problems connected to twinning and which have been shown to be central in later studies. He considered inter alia (1) the rates of twin and higher multiple maternities, (2) the crude birth rates among single and multiple maternities, (3) the regional and seasonal variations in TWRs, (4) the rates of live and stillbirths among twins, (5) the sex composition of

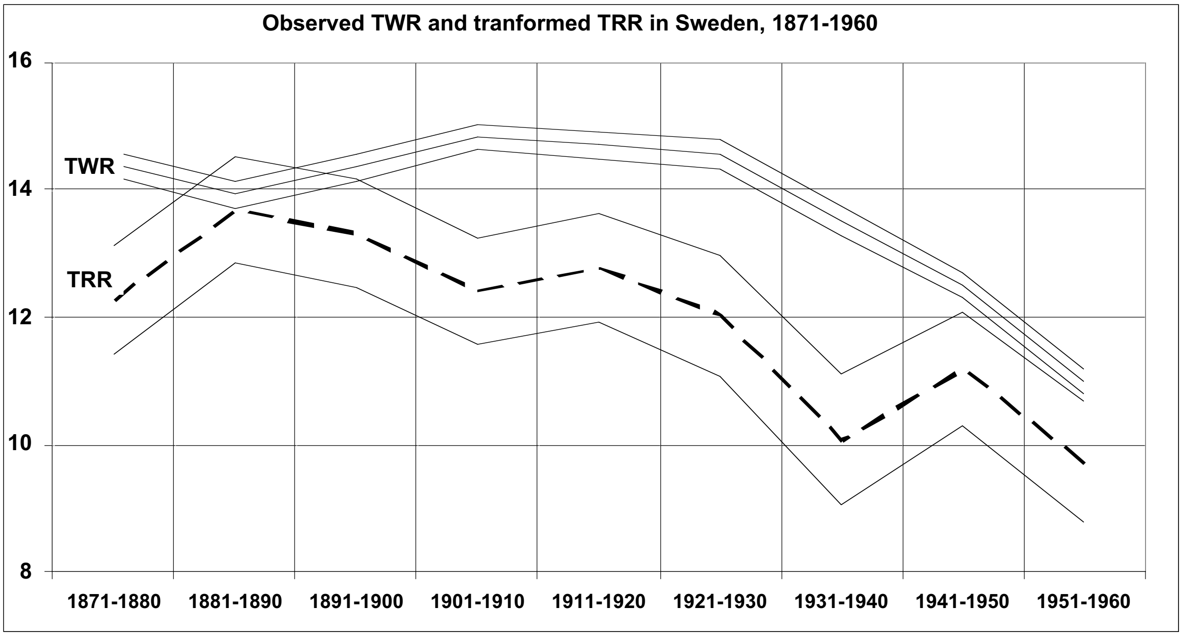

Figure 2

Temporal trends in twinning rate (TWR) and transformed triplet rate (TRR) in Sweden, 1869–1960. 95% confidence bands are included. For the whole period, the TRR shows a deficit compared with the TWR. This finding supports the model presented in the text.

sets of multiple maternities, (6) the sex ratio among single and multiple maternities and (7) the effect on the number of multiple maternities of the age of the parents, of the marital status and confession of faith of mothers, of urban and rural regions, and of seasonality. In addition, he considered the weight and prematures among multiples and mortality among multiples and mothers. The list presented indicates clearly that Neefe introduced a thorough research program for twinning studies. It is noteworthy that Neefe did not comment on the relation between the rates of multiple maternities, and consequently, despite his extensive study, he did not explicitly foretell Hellin's law.

Somewhat later, Berg (1880) published a comprehensive study of multiple maternities. He analysed the rates of multiple maternities in Sweden from 1776 to 1878. He also presented corresponding data for several European countries and analysed the sex combinations of twin, triplet and quadruplet sets in Sweden from 1869 to 1878. His study was published in Swedish, and thus few scientists paid attention to this paper. Since Swedish is our native language, Berg's results have been of great value in our studies (Eriksson, 1973; Eriksson & Fellman, 2006; Fellman & Eriksson, 2006).

Strassmann (1889) noted the findings in Veit (1855) and Wappäus (1859) and concluded, using Veit's total data set, that there is one twin maternity per 89^1 and one triplet maternity per 89^2 total maternities. Note that Strassmann related the number of multiple maternities to the number of all maternities, in contrast to Hellin (1895) who related the number of multiple maternities to the number of single maternities. However, both used the same relation, 1:89.

Hellin's Contributions

Hellin (1895) observed an empirical relationship between the rates of twin and triplet maternities. He stated on page 25 that

> Während man sagen kann, dass beim Menschen durchschnittlich eine Zwillingsgeburt auf etwa 89 einfache Geburten vorkommt tritt eine Drillingsgeburt auf $(89)^2$ einfache Geburten auf, eine Vierlingsgeburt auf $(89)^3$; überhaupt, soweit dies in Grenzen der Möglichkeit liegt, erscheint eine x fache Geburt auf $(89)^{x-1}$ einfache Geburten.

The translated statement reads

> Among human beings there is on average one twin maternity per 89 singleton maternities, one triplet maternity per $(89)^2$ singleton maternities, one quadruplet maternity per $(89)^3$ and in general, within the range of the possibility, one x-tuplet maternity per $(89)^{x-1}$ singleton maternities.

Two points should be stressed. Hellin cited the articles by Veit (1855), Wappäus (1859) and Strassmann (1889), but did not mention that Strassmann had already presented Hellin's law. On page 26 he only stated that, according to Strassmann, in Germany the number of single maternities per one twin maternity varies between 70 and 84. Hellin related the frequencies of multiple maternities to the number of singleton maternities. Today, one usually follows Strassmann and considers rates of multiple maternities with respect to all maternities. Although Strassmann appeared to have noted the law before Hellin, Hellin's additional contribution was that he gave the law a general form.

Table 3

Birth Data for Different European Countries During the 19th Century According to Wappäus (1859)

			Maternities				
Country	Year	Births	All	Singleton	Twin	Triplet	Quad
Belgium	1841–50	1356515	1343831	1331287	12409	130	5
Prussia	1826–49	13512710	13359260	13207571	149964	1689	36
Norway	1846–55	464309	458696	453151	5477	68	0
Hanover	1853–55	173995	171869	169775	2062	32	0
Sachsen	1847–56	790383	780496	770696	9715	83	2
Holstein	1845–54	169045	166920	164818	2080	21	1
Württemburg	1846–56	653564	645129	636786	8258	78	7
Austria	1851	1157309	1142269	1127441	14624	197	6*
Sweden	1841–50	1061469	1046885	1032501	14186	196	2
Denmark	1845–54	463688	457284	450992	6180	112	0
Iceland	1849–54	12284	12110	11940	166	4	0
Schleswig	1845–54	115285	113573	111874	1686	13	0
Total		19930556	19698322	19468832	226807	2623	59*

Note: The data from Prussia is for the period 1826–1849, and consequently, should be identical to the total data given by Veit, as presented in Table 2. A small discrepancy can be observed for the number of singleton maternities (cf. Table 2).
* in addition one quintuplet set

Table 4

Bertillon's Table of Triplet Rates (Bertillon, 1874, p. 285) – The Number of Total Maternities per One Twin Maternity (column 7) and the Total Number of Maternities (column 8) are our Estimates

	Period	No. of triplet maternities per year (absolute nos.)	No. of triplet maternities per one million total maternities	No. of total maternities per one triplet maternity	No. of twin maternities per one triplet maternity	No. of total maternities per one twin maternity[2]	Estimated annual mean no. of maternities[2]
France	1858–68	120.0	116.7	8570	85.9	99.8	1028278
Italy	1868–70	130.0	136.0	7359	76.2	96.6	955882
Prussia	1858–67	107.0	139.5	7170	89.4	80.2	767025
Hungary	1851–59	62.5	175.2	5700	74.6	76.4	356735
Austria	1851–70	215.0	183.0	5460	64.8	84.3	1174863
Galicia[1]	1851–59	36.0	193.6	5160	61.7	83.6	185950

Note: [1] Galicia (Bertillon called it in French Gallicie) is a historical region currently divided between Poland and Ukraine.
[2] This column is calculated and included in the table by us.

Nowhere have we seen the name Strassmann's law or the Strassmann-Hellin law.

Drejer (1895) was apparently unaware of the results of Hellin (1895), but referred to Strassmann (1889), and stating that Strassmann had noted the relation between the rates of twin and triplet maternities. Drejer felt dubious about the regularity between the rates. He stressed that under such circumstances the rule had to hold also for higher multiple maternities, but he could not find any clear indication of that.

Studies after Hellin

In this section, we included only papers in which Hellin's law has been analysed in detail and the strengths and weaknesses of the law discussed.

Zeleny (1921) examined Hellin's law in a short note. He considered the Strassmann version that the rates are related to the total number of maternities. His analyses yielded him the honour that some authors later renamed the law the Hellin-Zeleny law. He also referred to the data of Veit (1855) and found remarkable good agreement with the law. Zeleny stated that from the statistical relations it would appear that triplets are produced by the coincidence of two independent processes occurring with equal frequencies. One of these processes by itself gives rise to twins. This relation would apply to any mode of origin of multiple births or to different combinations of these provided that each followed the rule.

Jenkins (1927) stated that Hellin's law should be considered as a first approximation. He based his own model on the rates of monozygotic (MZ) and dizygotic (DZ) twinning. The rate of DZ twinning is strongly dependent on maternal age, and consequently, he stressed that the relation between TWR and TRR should hold only for age-specific rates. He assumed that the age-specific TRRs are the squared age-specific TWRs. Consequently, for the total TRR he obtained the formula

$$TRR = \frac{1}{n}\sum_i (TWR_i)^2 n_i, \quad (1)$$

where n_i is the number of mothers in age group i and n is the total number of mothers. The stochastic model indicated at least that Hellin's law holds approximately within the maternal age groups, but for the total rates Hellin's law does not hold. Discrepancies within a maternal age group he ascribed to differential mortality of twins and triplets in utero. In addition, Jenkins stated that the proposed model did not give the correct proportion of same-sexed and opposite-sexed triplet sets. Later, Jenkins (1929) revisited to the model. He gave some suggestions about how the model could be improved. Jenkins and Gwin (1940) attempted a rigorous treatment of live-birth statistics, with classification of the data according to the age groups of the mothers. The rule so-derived deviated markedly from Hellin's law.

Komai and Fukuoka (1936) studied the rates of multiple maternities in Japanese and related populations. Their contributions are interesting for a variety of reasons. The Asian populations are characterized by very low TWRs. The MZ TWRs are comparable with the MZ TWRs in other populations. The low DZ TWRs cause the low TWRs. As noted above, Jenkins also connected Hellin´s law to MZ and DZ TWRs.

The data caused large problems for Komai and Fukuoka. They considered data published by the Bureau of Statistics of the Japanese Government, hospital data and data from the records of midwives. Comparisons between these three data sets showed marked discrepancies. They considered that the data reported by the midwives to be the most accurate. The authors applied Weinberg's differential rule and noted the low proportion of DZ twin maternities. For some hospital data, they had information about the state of the placenta, and this information supported Weinberg's differential rule. When they applied Hellin's law, without explicit reference to Hellin (1895), they found an excess of triplet maternities in the population statistics and hospital register data, but a good agreement with the midwife records. Komai and Fukuoka stressed that the discrepancies obtained indicate inaccuracies in the population statistics and hospital registers because the intrauterine death of triplets must be considered more frequent than that of twins. This fact should, however, reduce the relative number of triplet sets, not increase it.

Sarkar (1944–1945) examined the TWR in India and on Ceylon (Sri Lanka). One central problem in his study is if the TWR level was associated with skin color. At least in some African and Afro–American populations with dark skin also have high TWRs (Bulmer, 1970; Little & Thompson, 1988). Sarkar's study is based on hospital data. This reduces the relevance of the study, a fact which Sarkar recognized. However, in connection with our study, his paper is interesting because he defined the TWR as $1 : n$ and the triplet rate as $1 : m^2$, that is, he indirectly used Hellin's law without any explicit reference to Hellin (1895). In analysing his results, one finds a deficit of triplet maternities ($m > n$). In addition, one observes that on Ceylon the TWR was low ($1 : 161.1$), yielding a TWR of 0.62%. On Ceylon, the TRR followed Hellin's law because it was $1 : 154.4^2$. The Ceylon findings must be considered reliable because the total number of maternities was as high as 1620077.

In his study of the rates of multiple maternities for total, 'white' and 'colored' populations in the United States (1922–1936), Strandskov (1945) also investigated how well his data satisfied Hellin's law. Applying χ^2 tests, he found that in none of the three populations tested did the observed plural birth frequencies agree closely with Hellin's law.

Peller (1946) stressed that deviation of the actual figures from expectation must not obscure the fact that the Hellin(-Zeleny) law comes pretty close to reality. Neither should the rule be condemned just because it cannot be explained. On the contrary, correction and explanation should be attempted. Peller based his study on models that he constructed for recurrent multiple maternities within sibships. As an application of this model, he obtained an alternative rule for the relationship between the number of twin sets and sets of higher multiple maternities. Although his starting point was different, his formulae differed only slightly from Hellin's law. Peller is the first, at least indirectly, to connect Hellin's law to inter-individual variation in mothers' chances for multiple maternities. Later, Eriksson (1973) considered recurrent twin maternities in families from Åland (Finland) and gave a modified model (in the paper called Fellman's law). When he applied this law on his Åland data, he obtained better congruence with Hellin's law than if Peller's version was applied.

Das (1953) formulated Hellin's law such that 'the frequency of twin confinements bears to that of total confinements a ratio which is equal to the ratio borne by the frequency of the triplet confinements to that of the twin confinements'. This modified definition is in congruence with Strassmann's version of the law. He gave a review of earlier studies concerning Hellin's law and stressed the discrepancies presented therein. Furthermore, he considered data for more than 300 millions births and showed that Hellin's law was inexact for triplets and did not hold for quintuplets. Consequently, Das concluded that Hellin's law has no sound basis and that exceptions to the rule have been the rule. In a later paper, Das (1955) developed his model first given in Das (1953) for the frequencies of twins and higher multiple maternities. Based on this theory, he also considered the relation $TRR = (TWR)^2$. His mathematical analyses did not support Hellin's law.

Allen and Firschein (1957) gave a theoretical argument for the law, but continued to modify it. Their argument for the law was that it is a mean of a stochastic model. If the probability for an extra ovulation, yielding a twin set, is p, the probability for two extras, yielding a triplet maternity, is p^2, and so on. They stated that the agreement of Hellin's law with birth statistics is not entirely coincidental, and that it fits data because of an underlying truth. They improved the formula to include both MZ and multizygotic maternities. However, the obtained expected frequencies were not in concordance with the observed ones, and thus they paid thorough attention to possible disturbing factors and how their effects could be identified.

Alternative models. Assuming that Hellin's law holds, sometimes the number of higher multiple maternities is too high, sometimes too low. Our opinion is that this problem is generally minimized in the literature. As mentioned above, Komai and Fukuoka (1936) found an excess of triplet maternities and stressed the unreasonableness of this finding. Using data from England and Wales, the United States and Italy, Bulmer (1970) constructed an alternative to Jenkins's model. Bulmer obtained the empirical formula $R = 1.36M^2 + 2MD + 0.47D^2$, where R is the TRR, M is the rate of MZ and D is the rate of DZ twinning rates. This formula differs from Hellin's law, which should be formula $R = M^2 + 2MD + D^2 = (M + D)^2$. Bulmer's correction is $0.36M^2 - 0.53D^2$, which is negative if $D \geq 0.82M$. With the exception of the Asian populations, the DZ rate D is almost always greater than the MZ rate M and, consequently, a deficit in the triplet rate compared with Hellin's law was obtained.

In our earlier study (Fellman & Eriksson, 2004), we had information about the sex composition of twin pairs and the triplet sets in Sweden for the period 1869–1878 (Berg, 1880) and for the decades 1901–1960. Following Bulmer's attempt, we obtained the estimated model $R = 1.39M^2 + 2MD + 0.44D^2$. The parameter estimates differ only slightly from Bulmer's. According to this formula, a deficit of triplet sets compared with Hellin's law is obtained if $D \geq 0.84M$. In fact, according to the data analysed, $D \approx 2.89M$ and the deficit is marked. This deficit can be clearly seen in Figure 2.

Fellman and Eriksson (1993) gave a mathematical proof that Hellin's law cannot hold in general. If one aggregates heterogeneous data, the fluctuations are smoothed out, but according to Hellin's law the relation between the TWR and the TRR is not linear, and consequently, the aggregated and disaggregated data cannot simultaneously satisfy Hellin's law. Jenkins (1927) noted that Hellin's law can be assumed only for age-specific data and that the discrepancy between Hellin's law and the observed aggregated data demands the use of formula (1). In fact, Jenkins's formula coincides mathematically with the integral proposed by Fellman and Eriksson (1993). However, Jenkins did not explain the disagreement in Hellin's law for aggregated and disaggregated data.

Discussion

The first advanced studies of twinning rates can be found in Bertillon (1874), Neefe (1877) and Berg (1880) and to some extent in Veit (1855). In these publications, the twinning phenomenon was the central topic. Our presentation of the results given by the forerunners has concentrated on their contribution to twinning research. A closer look at their publications shows that they have had very different interests. Wappäus (1859) wrote a whole book concerning demographic statistics at large. Strassmann (1889), Drejer (1895) and Hellin (1895) were mainly interested in clinical problems. In all of these studies, the statistical analyses of twinning formed only a small part of the research.

Jenkins (1927) stressed that Hellin's law is a first approximation. It is commonly accepted that the main argument for Hellin's law is that the probabilities of additional ovulations and the fissions of fertilized eggs can be explained by stochastic models. Consequently, in large data sets, the averages could be stable and formulated by a mathematical relation (Hellin's law). Common arguments for the observed discrepancies are that after the fertilizations there is a long process influenced by disturbing factors. The final result thus often shows only a weak resemblance to the outcome of a simple stochastic process. This fact seems to be the main cause of the discrepancies between Hellin's law and empirical findings. Jenkins (1927) and Komai and Fukuoka (1936), for instance, assumed that differential mortality of twins and triplets in utero could be one such factor.

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