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Coupled Alfvén and kink oscillations in an inhomogeneous corona

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Abstract. We perform 3D numerical simulations of footpoint-driven transverse waves propagating in a low β plasma. The presence of inhomogeneities in the density profile leads to the coupling of the driven kink mode to Alfvén modes by resonant absorption. The decay of the propagating kink wave as energy is transferred to the local Alfvén mode is in good agreement with a modified interpretation of the analytical expression derived for standing kink modes. This coupling may account for the damping of transverse velocity perturbation waves which have recently been observed to be ubiquitous in the solar corona.

Keywords. MHD, Sun: atmosphere, Sun: corona, Sun: magnetic fields, Sun: oscillations, waves

1. Introduction

Recent observations of the solar corona by Tomczyk *et al.* (2007) and Tomczyk & McIntosh (2009) with the ground-based coronagraph *CoMP* show transverse oscillations propagating outwards everywhere. The dominance of outward-propagating wave power over inward-propagating wave power along closed loops suggests significant attenuation *in situ*. The propagating waves were initially interpreted as Alfvén waves but this was disputed by Van Doorsselaere *et al.* (2008a,b) who interpret them as kink waves. These different interpretations have consequences for the inferred coronal magnetic field strength and the energy budget calculations for the coronal heating problem.

Ruderman & Roberts (2002) show how resonant absorption can damp coronal loop oscillations. Their work was motivated by the global standing kink modes as seen by *TRACE* and is similar to the Edwin & Roberts (1983) configuration except for the inclusion of an inhomogeneous layer of thickness l. Resonant absorption takes place in this inhomogeneous layer, transferring energy from the global kink mode to the Alfvén mode.

Our studies (Pascoe *et al.* 2010) show that the transverse waves we launch from the boundary couple efficiently to Alfvén waves when the medium is non-uniform. This is seen as a decay of the driven wave fields. Once the energy is in the form of Alfvén waves, it is well known that these fields will phase-mix (Heyvaerts & Priest 1983; Mann *et al.* 1995) and lead to the development of small transverse scales.

2. Model

Considering the rapid damping of the global kink standing mode in a zero β cylindrical flux tube with an inhomogeneous layer, Ruderman & Roberts (2002) derived the relationship

$$\frac{\tau}{P} = C\frac{a}{l}\frac{\rho_0 + \rho_e}{\rho_0 - \rho_e} \tag{2.1}$$

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where τ is the damping time, P is the period of oscillation, a is the loop radius, l is the inhomogeneous layer thickness, and ρ_0 and ρ_e are the internal and external mass densities, respectively. The constant C depends upon the density profile in the inhomogeneous layer, e.g. $C = 2/\pi$ for a sinusoidal profile, whereas for a linear density profile $C = (2/\pi)^2$ (see e.g., Hollweg & Yang 1988; Goossens *et al.* 1992; Roberts 2008).

In our model we consider a straight, uniform magnetic field in the z direction. We use a zero plasma β approximation. Our density profile describes a cylindrical tube aligned with the z axis and defines three regions; the core region with an internal density ρ_0, the external or environment region with density ρ_e, and the inhomogeneous shell region in between, where the density varies linearly from ρ_0 to ρ_e.

The driving condition is applied to the lower z boundary to simulate excitation by footpoint motions (e.g., De Groof *et al.* 2002) and prescribes the x and y components of velocity to have a time dependence based on a single period displacement of the loop axis in the x direction. The spatial dependence of the driver is based on a 2D dipole. In the core region the velocity is constant $\mathbf{u} = (u_0, 0, 0)$ and only in the x-direction, where $u_0 = 0.002$ is chosen to be small to avoid non-linear effects. In the surrounding environment the driver has a 2D dipole form. This flow corresponds to 2D incompressible dipole flow around a circular tube that moves with velocity $(u_0, 0, 0)$. In cylindrical coordinates, this would be described as the $m = 1$ mode, in which u_r is continuous and u_ϕ discontinuous at the tube boundary. We avoid numerical problems arising from such a discontinuity by smoothly changing in the inhomogeneous layer from the core velocity profile to that in the environment.

The simulations are performed using the MHD code Lare3D (Arber *et al.* 2001). The numerical domain is much larger in the z direction than in x or y, but the resolution is higher in the x and y directions in order to resolve the activity in the inhomogeneous layer, particularly when phase-mixing occurs.

3. Results

In the case of a uniform medium the Alfvén speed is constant everywhere. The perturbation applied at the lower boundary propagates upwards uniformly with no signs of any significant attenuation or dispersion. Although we do not drive the boundary for many cycles, and hence do not have a quasi-monochromatic source, the driver does have a well-defined nodal structure and is dominated by a time scale of $P \approx \frac{2}{3} P_0$.

In the case of an inhomogeneous layer ($\rho_0/\rho_e > 1$) the Alfvén speed now varies across the driven region. The Alfvén speed varies continuously in the inhomogeneous layer and resonant absorption occurs where the phase speed of the kink mode matches the Alfvén speed.

This coupling of the wavetrain to a local Alfvén mode causes a decrease in wave energy in the core (and the environment) and hence appears to damp the tube oscillation. Also, since the Alfvén mode is in an inhomogeneous layer, it is subject to phase mixing. The corresponding characteristic spatial scale becomes smaller as a function of time. The time dependence of the phase-mixing length scale agrees well with the analytical relationship of Mann *et al.* (1995). These effects can be seen in Fig. 1, which shows v_x at the axis of the cylinder at time $t = 1.5 P_0$. The density profile, outlined by the vertical lines, is defined by $\rho_0/\rho_e = 2$ and $l/a = 0.5$.

In order to quantify the behaviour in our model we calculate the wave energy density, as in Terradas *et al.* (2006, 2008). Mode coupling in the inhomogeneous shell causes the wave energy to become localised there with a corresponding decrease in energy in the core and environment regions, which was used to calculate a decay time τ.

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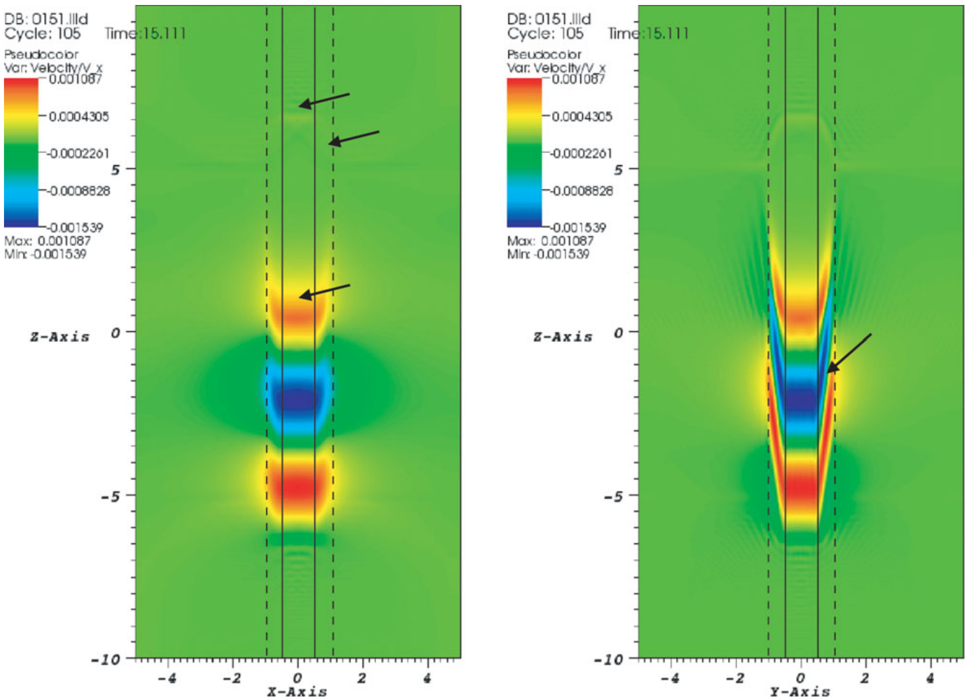

Figure 1. v_x at $y = 0$ (left) and $x = 0$ (right) and at time $t = 1.5P_0$ for $\rho_0/\rho_e = 2$ and $l/a = 0.5$. Energy is transferred from the kink mode in the core region to the Alfvén mode in the inhomogeneous shell region by resonant absorption.

Note that we have a decaying propagating wavepacket, not a decaying standing mode. However, Hood *et al.* (2005) show that standing normal mode calculations can be a good indicator of wavepacket behaviour, when the spatial and temporal scales are similar. With this in mind we compare our wavepacket decay rates with the Ruderman & Roberts (2002) formula for standing modes in eq. [2.1].

For our linear density profile in the inhomogeneous layer, the analysis predicts $C = (2/\pi)^2$ for standing modes. We find for our simulations $C = 0.9$ for all values of ρ and l considered. Choosing this value of C, the Ruderman & Roberts (2002) result in eq. [2.1] can be regarded as providing an empirical fit to our simulation results. That the value of C differs from that in the Ruderman & Roberts (2002) calculation can be attributed to the fact that our model is not accurately described as a thin flux tube with a thin boundary layer.

4. Implications

Our non-monochromatic driver on the lower boundary produced an upward propagating wavetrain. In the case of an inhomogeneous medium the wavetrain is subject to mode coupling through resonant absorption. Here we will discuss these results in the context of the observations of Tomczyk *et al.* (2007) and the interpretation of Van Doorsselaere *et al.* (2008a,b) as kink waves. In this case our driver corresponds to some general photospheric motion and our wavetrain corresponds to the Doppler shift in coronal emission observed by Tomczyk *et al.* (2007).

In the most general case of a continuously non-uniform corona, the behaviour demonstrated by our numerical simulations is that of a quasi-mode composed of the kink mode

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coupled to the Alfvén mode. Due to resonant absorption and the introduction of a characteristic damping time (τ) the time-dependent nature of the mode must be considered. For $t > \tau$ the wave energy is concentrated in the Alfvén mode, whereas for $t < \tau$ the behaviour is described by the kink mode of Edwin & Roberts (1983). The coupled nature of MHD waves in a non-uniform plasma is also discussed by Goossens *et al.* (2009).

According to this interpretation, the transverse waves observed by Tomczyk *et al.* (2007) are in the regime $t \approx \tau$ and so are an intrinsically coupled mode. This will be the case unless the corona is uniform or contains a flux tube whose density profile is discontinuous. The properties of the observed Doppler shifts will resemble a damped kink mode since the coupled Alfvén mode component is generally unresolved by modern solar instruments. However, its presence is necessary to explain the rapid damping and leads to the concentration of wave energy to smaller scales and so may be an important process contributing to coronal heating.

For the typical parameters of our numerical simulations, we estimate that for the observed waves of period $P \approx 300$ seconds (Tomczyk & McIntosh 2009) we expect a damping length of $L_d \approx 750$ Mm. This would require a footpoint separation of $2L_d/\pi \approx 500$ Mm for closed loop structures to give an outward propagating wave signature. This simple estimate of damping length scales is consistent with Tomczyk & McIntosh (2009), who show that the outward directed wave velocity power dominates the inward directed power for loop structures with a footpoint separation greater than 300 Mm.

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