SURFACE MOVEMENT AND ITS RELATIONSHIP TO THE AVERAGE ANNUAL HYDROLOGICAL BUDGET OF LEMON CREEK GLACIER, ALASKA*

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Abstract. The object of the 1957 program of the Juneau Ice Field Research Project on Lemon Creek Glacier was (1) to obtain data on surface movement in the accumulation region, and (2) to relate the movement to the average annual hydrological budget. Results show a net loss of ice over the past four years, although this deficit does not appear to be reflected by the movement of the ice. Lemon Creek Glacier appears to be in approximate equilibrium.

Zusammenfassung. Der Zweck des 1957 Programms des Juneau Ice Field Research Project über den Lemon Creek Glacier war (1) Data über die Oberflächenbewegung in dem Akkumulationsgebiet zu bekommen und (2) die Bewegung mit dem durchschnittlichen jährlichen hydrologischen Budget in Beziehung zu bringen. Die Resultate zeigen für die vergangenen vier Jahre einen Netto-Verlust von Eis an, obwohl dieses Defizit nicht durch die Bewegung des Eises wiedergegeben zu werden scheint. Der Lemon Creek Glacier scheint in annäherndem Gleichgewicht zu stehn.

Introduction

Lemon Creek Glacier on the western margin of the Juneau Ice Field in south-eastern Alaska has been studied annually by the American Geographical Society since 1953 as part of the Juneau Ice Field Research Project. During the summer of 1957 a program of movement studies was carried out in the accumulation region of the glacier. In early June 1957 a small field party set up and surveyed two transverse profiles of movement stakes where the cross-section of the glacier was known. The upper movement profile was at an elevation of approximately 3980 ft. (1214 m.), 3·2 miles (5·3 km.) above the terminus and a mile (1·6 km.) from the head of the glacier. The lower profile, at approximately 3700 ft. (1130 m.) was 2·5 miles (4·0 km.) from the terminus and only a few hundred meters above the firn limit. The stakes were resurveyed near the close of the ablation season after a period of 88 days.

The purpose of this study was (1) to get a detailed picture of the surface movement of a glacier in the accumulation region and (2) to relate the movement to the average annual hydrological budget. Data were available on the glacier from the previous work of the project. Extensive snow surveys and ablation measurements had been made so that the hydrological budget of the Lemon Creek Glacier was known since the 1953-54 budget year. Because of the elaborate snow surveys that had been made on the accumulation region, it was only necessary for us to make sample measurements of snow depth and ablation rate in order to ascertain the budget for 1957.

An outline map of Lemon Creek Glacier made by the A.G.S. in 1955 provided the dimensions of the glacier and a network of base lines from which surveying of the movement stakes could be done. The depth cross-sections on the surface of which the movement profiles were set up were measured by a gravity method in 1955.[1]

Movement Survey

A 3521 ft. (1073 m.) base line was chosen on a ridge of rock on the western side of the glacier for surveying the two movement profiles. The position of the base line was such that the triangle formed by the ends of the base line and the middle stake of each movement profile was approximately equilateral. This assured us of the maximum accuracy in the determination of the position of the stakes for a given random error in the angles measured from the ends of the base line. The angles were measured with a Wild T-2 theodolite to an

* Carried out by the Juneau Ice Field Research Project under contract with the Office of Naval Research.

accuracy of 1 second of arc. Each angle was measured six times on 7 June and on 9 September at the end of the 88-day movement period.

The south end of the base line was chosen as the origin of a three-dimensional rectangular coordinate system. The positive y axis, the base line, was parallel to the flow axis of the glacier. The x axis was positive eastward across the glacier. The z axis was negative downward. From the triangle formed by the base line and each stake, the x, y, and z coordinates of the position of each movement stake were calculated at the beginning and the end of the movement period. In this way the absolute value of the displacement of each stake was found for each of the three directions x, y, and z.

A random error of ± 15 seconds of arc was found in averaging the six readings of each angle. To find the error in position due to the inaccuracy of the angles, a standard error of 30 seconds of arc was added to each measured angle and a new position was recalculated for each stake. These differences of position were combined in a way that would give the maximum error for the displacement of the stake.

It was necessary to measure accurately the vertical displacement of the stakes due to movement of the ice to know the change in the elevation of the snow surface due to ablation and compaction. In order to accomplish this, a ½ in. (1·3 cm.) hole was rammed 15 ft. (4·6 m.) into the snow at each stake at the beginning of the movement period. When the stakes were resurveyed in September these holes were remeasured to find the change, relative to the bottom of the hole, in the height of the snow surface. A type of movement stake that would settle evenly on the snow without causing any differential ablation was used. The stakes were made in the form of an equilateral triangular pyramid, of 6 ft. 0 in. × 1 in. × 1 in. (183 × 2·5 × 2·5 cm.) lumber wired, through holes, at the vertexes. A 20 lb. (9 kg.) rock was suspended from the top vertex to just above the level of the snow to prevent the wind from toppling the stakes. This form of stake was very stable and did not increase or inhibit the ablation of snow beneath it.

The change in elevation of the stakes due to ablation was subtracted from the change in z as determined by the survey to give the actual movement of the ice. Movement data for the two profiles appear in Table I.

TABLE I. MOVEMENT DATA FOR PROFILES I AND II ON LEMON CREEK GLACIER

	d		T		V		φ	β
Stake No.	ft.	m.	ft.	m.	ft./yr.	m./yr.	degrees	degrees
				Profile I				
1	1460	445	473	144	51·1± 6	15·6±2	43·8	−41·5
2	2085	635	656	200	69·5± 8	21·2±2	19·3	−32·1
3	2610	796	742	226	75·6±10	23·0±3	11·0	−20·5
4	3215	980	683	208	94·5±12	28·8±4	1·5	−24·8
5	3920	1195	512	156	58·0±14	17·7±4	−45·6	−40·1
				Profile II				
1	740	226	296	90	47·2±8	14·4±2	19·8	−21·7
2	1690	515	565	172	144 ±8	44·0±2	9·2	− 8·6
3	2475	754	656	200	141 ±8	43·0±2	−11·8	− 8·8
4	3180	969	591	180	104 ±9	31·7±3	− 9·8	− 7·6
5	4000	1219	434	132	73·1±9	22·3±3	−10·6	− 6·1

d = distance of stake from west edge of glacier.
T = thickness of ice.
$\mathbf{V}$ = magnitude of velocity vector $= \sqrt{(V_x^2 + V_y^2 + V_z^2)}$.
ϕ = angle between $\mathbf{V}$ and positive y axis measured positive clockwise.
β = angle between $\mathbf{V}$ and glacier surface.

HYDROLOGICAL BUDGET

An average ablation rate of 3·5 cm./day for snow was found for the stakes on Profile I over the 88-day period starting on 7 June. The ablation rate for ice on the lower part of the glacier was found to be 5·7 cm./day. This rate was measured during an 11-day period in September. A 10-ft. (3 m.) hole was drilled into the ice with a 1-in. (2·5 cm.) coring auger. The bottom of the hole was used as the reference level for measuring the rate of ablation of the ice. On 11 September a snow survey was made in the accumulation region above Profile I. The depth of the 1956-57 snow cover was found to be very uniform over this area with an average thickness of 2·4 m. From the above data and the photographs of the firn line taken during the 1957 ablation season it was possible to show that the 1957 hydrological budget for the Lemon Creek Glacier was the same as that for 1954.

The following list of comparative data will illustrate the similarity:

(1) On 14 September 1954 the area below the firn line was $3\cdot27\times10^6$m.^{2}. On 15 September 1957 this area was $3\cdot21\times10^6$m.^{2}.

(2) The snow ablation rate near Profile I in 1954 was 3·5 cm./day, exactly the same as that for 1957.

(3) The total ice ablation for 1954 was $6\cdot04\times10^6$m.^{3} of water. In 1957 there were $6\cdot7\times10^6$m.^{3} of water of ablation.

(4) The ice ablation rate below the ice fall averaged over the entire summer in 1954 was 5·98 cm./day. The rate for an 11-day period late in the ablation season in the same place in 1957 was 5·7 cm./day. This lower rate could be accounted for by the cooler temperatures in September.

Significance of Movement

The problem of describing the flow of ice in a valley glacier is a unique one in that both the "source" and the "sink" of the flowing material are at its surface. Thus, ice formed near the surface in the accumulation region flows down into the interior of the glacier, moves downstream and eventually rises to the surface in the ablation region where it is lost largely as melt water.

By inspecting a longitudinal cross-section and the transverse velocity distribution of a valley glacier, it is possible to predict in a general way the direction and relative magnitude of the ice flow at the surface. Consider in Fig. 1 a longitudinal cross section of unit width near the axis of a valley glacier with the vertical velocity distribution as shown. The area *a* through *f*, represents the volume of ice transported past the firn limit per year. Each year a layer of new ice, *a′* through *f′*, is formed at the surface in the accumulation region. Because ice is flowing down-valley within the body of the glacier, the new surface layer or "source" ice must pass into the glacier in order to supply this sub-surface flow. In Fig. 1 the yearly volume of ice *a* flowing past the firn limit is supplied by the ice formed in the region *a′*; the volume *b* is supplied by *b′* and so on until the ice *f*, moving at the bottom of the glacier, is supplied by ice formed at *f′* many years ago. Thus, at the firn limit, the surface ice is young and at depth the ice is very old because its source was the head of the glacier. From this idealized picture it follows that ice in the region *b′*, for example, must have a larger down-glacier velocity than ice at *c′*. These facts are well illustrated by the Lemon Creek Glacier movement profiles.

The average down-glacier component of velocity was 44 ft./year (13 m./yr.) for Profile I and 82 ft./year (25 m./yr.) for Profile II nearer the firn limit. The angles of inclination of the velocity vectors for Profiles I and II are shown in Fig. 3. It is clear from this illustration that the average angle of inclination of the flow of the ice is greater for Profile I near the head of the glacier than for Profile II.

It is well known that in the transverse velocity distribution for the surface movement of a valley glacier, the maximum velocity occurs near the middle on the axis of the glacier. Thus, in unit time a greater volume of ice moves down the axis of a glacier than near the edges. Since the source of the moving ice is an even layer across the surface in the accumulation region of the glacier, the ice above the firn limit must have a component of velocity toward

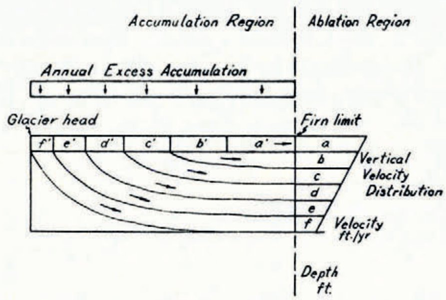

Fig. 1. *Longitudinal section of a glacier in the accumulation region*

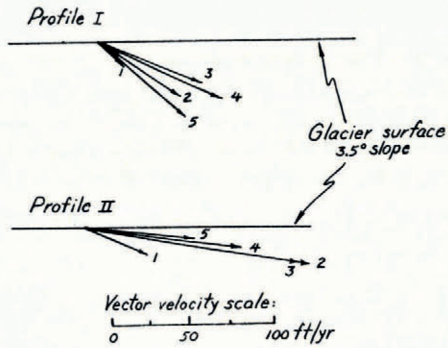

Fig. 3. *Angles and velocities of vectors for stakes in Profiles I and II on Lemon Creek Glacier*

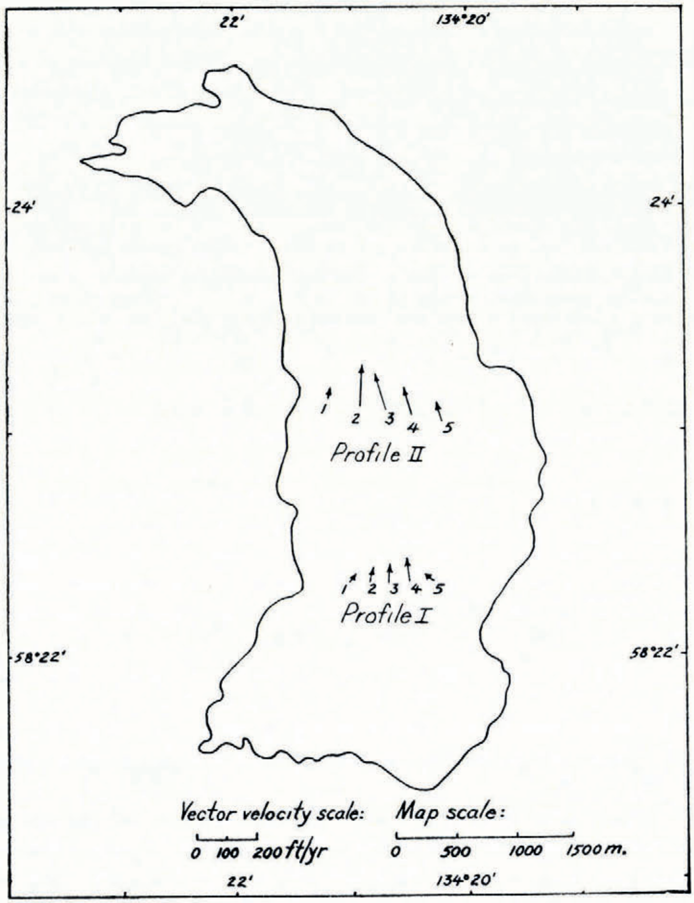

Fig. 2. *Sketch map of Lemon Creek Glacier, showing directions and velocity vectors for stakes in Profiles I and II; map and velocity scales are not uniform*

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the axis of the glacier to supply the greater axial volume of flow. As the firn limit, and thus the end of the accumulation region, is approached, this convergence of ice toward the glacier axis becomes less and less until, at the firn limit, the ice is all moving parallel to the glacier axis. This convergence of ice in the accumulation region is well shown by the Lemon Creek Glacier profiles. In Fig. 2 the profiles appear in plan view on an outline map of the Lemon Creek Glacier. The convergence on Profile I can be seen to be much greater than that for Profile II. The gradient of x component of velocity taken in the x direction is 0·016 for Profile I and 0·010 for Profile II. This numerically illustrates that the convergence is greater on Profile I than on Profile II. The convergence in the vertical plane is illustrated in Fig. 4, in which the velocities of the movement stakes are plotted on the cross-section of the glacier underlying the surface profile. In Figs. 2 and 4, the tails of the arrows represent the position of the stake on the glacier, while the arrows give the magnitude and direction of the velocities.

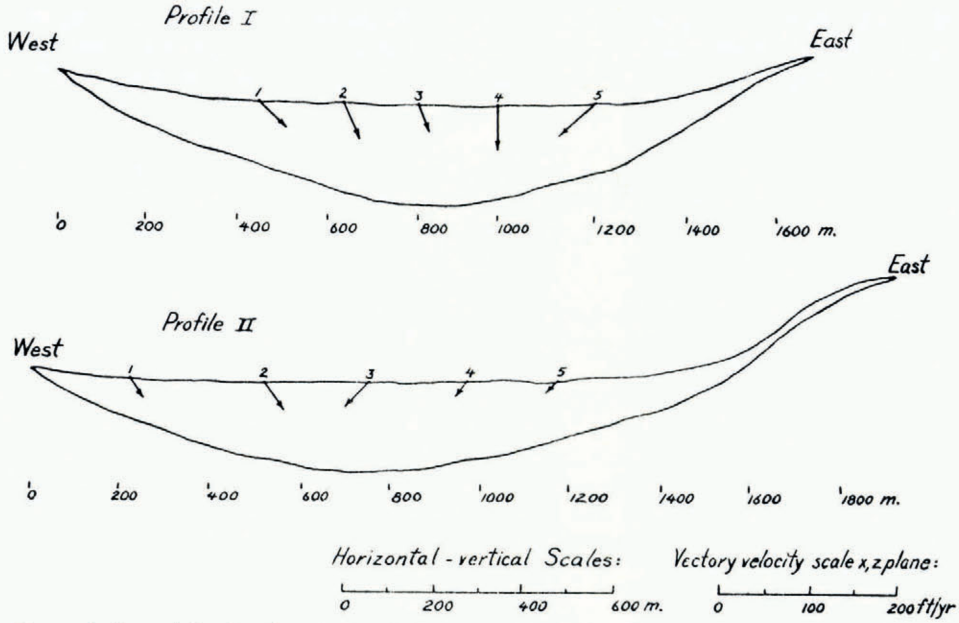

Fig. 4. Vector velocities and directions for stakes in relation to cross-sections of Lemon Creek Glacier; vector velocity scale is not uniform with horizontal and vertical scales

In the ablation region the situation is reversed. There is a divergence of ice away from the glacier axis, and the ice rises to the surface to supply the ablation layer from below.

Discussion

A glacier will maintain constant physical dimensions over a period of several years, if the masses of its average annual net accumulation of snow and its average annual ablation of ice are equal. Such a glacier is said to be in equilibrium. The most obvious method of discovering whether or not a glacier is in equilibrium is to see if its dimensions remain the same from year to year. However, this would require very elaborate annual surveys of the entire mass. The equilibrium theory of glaciers [2] presents a new approach to the problem and requires less field work. This method correlates the average annual hydrological budget of a glacier with the surface expression of the movement of the ice through cross-sections of known depth.

When glacier ice is formed by firnification near the surface in the accumulation region of an equilibrium glacier, the new ice supply is transported by internal flow to the region of ablation in just sufficient quantities to maintain the equilibrium slope of the glacier. This process of replacement of ice is analogous to isostatic readjustment or to a fluid seeking its own level. According to the theory of equilibrium glaciers, the mass of ice M_i flowing in a year through any cross-section i, in the accumulation region of a glacier, is directly proportional to the area A_i of the surface of the glacier above the position of the cross-section. This is true if, and only if, the average annual net accumulation per unit area K is constant over the area A_i. Thus

$$M_i = KA_i. \qquad (1)$$

The mass of ice M_i of density ρ flowing per year through a cross-section for which area S_i is known may be calculated from the equation

$$M_i = \rho\bar{v}_i S_i, \qquad (2)$$

where $\bar{v}_i$ is the bulk velocity of the ice normal to the cross-section. Nielsen has shown that for glaciers that are wide relative to their depth, the bulk velocity, $\bar{v}_i$, and the average of the down-glacier component of the surface velocity $\bar{V}_i$ are directly proportional to each other. Thus

$$\bar{v}_i = k\bar{V}_i, \qquad (3)$$

where k is a constant. Now for two cross-sections i and j both in the accumulation region, with one up glacier from the other, we can combine equations (1), (2), and (3) to get

$$\frac{\bar{V}_i}{\bar{V}_j} = \frac{A_i S_j}{A_j S_i} \qquad (4)$$

Equation (4) gives the ratio of the average surface velocities $\bar{V}_i$ and $\bar{V}_j$ for two cross-sections i and j of cross-sectional areas S_i and S_j with surface areas of accumulation A_i and A_j up glacier from the cross-sections.

All the quantities in equation (4) can be measured. The values of $\bar{V}_i$, A_i, and S_i for the two movement profiles that were measured on the Lemon Creek Glacier are given in Table II. The values of $\bar{V}_i$ and $\bar{V}_j$ were obtained by a numerical integration of the y components of velocity for each profile. The S_i and S_j values were measured by a gravity method.[1] The areas A_i and A_j were taken from the American Geographical Society outline map of the Lemon Creek Glacier, Fig. 2. Upon substitution of the values from Table II we get

$$\frac{\bar{V}_{I}}{\bar{V}_{II}} = 0{\cdot}488; \; \frac{A_{I}S_{II}}{A_{II}S_{I}} = 0{\cdot}472.$$

Table II. Values for $\bar{V}_i$, A_i, and S_i for Profiles I and II on Lemon Creek Glacier

	$\bar{V}_i$		A_i	S_i	*Bulk velocity*	
	ft./yr.	m./yr.	m.²	m.²	ft./yr.	m./yr.
Profile I	44	13	$2{\cdot}7\times10^6$	$2{\cdot}10\times10^5$	38	12
Profile II	82	25	$5{\cdot}5\times10^6$	$2{\cdot}04\times10^5$	70	21

For the above very close agreement of equation (4) to be spurious, the error induced from the assumption of a constant K would have to cancel the error due to the assumption that the glacier is in equilibrium. This is considered very unlikely. The difference in the average elevation of the two accumulation areas A_{I}, A_{II}, is only 200 ft. (60 m.) over a distance of 2·5 miles (4 km.). Thus, there should not be much variation in the values of K for these areas.

Equations (1) and (2) can be solved directly for Profile I. The data on annual net accumulation, taken over the period from 1954 to 1957, and the movement work from 1957 give:

M_{I} from equation (1) $= 2{\cdot}2\times10^6$ m.³ water

M_{I} from equation (2) $= 2{\cdot}43\times10^6$ m.³ water

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The two values of the mass of ice transported through the cross-section under Profile I per year indicate that slightly more ice is moving away from the head of the Lemon Creek Glacier than has been formed by firnification over the past four years.

According to the equilibrium theory of glaciers the amount of ice flowing through the cross-section at the average firn limit should be equal to the average annual ablation of ice below the firn limit. Over the period from 1954 to 1957 the amount of ice ablation has been calculated for each summer. This was done by combining measurements of the rate of ice ablation as a function of elevation with photographs of the position of the firn line taken periodically throughout the ablation season. The average water content of the ice ablated during this period was:

$$M_{\text{II}} = 5 \cdot 05 \times 10^6 \text{m.}^3 \text{ water}$$

The ratio of the surface area of the Lemon Creek Glacier above the firn limit to the area above Profile II is 1·20. Thus, there is very little additional area of accumulation between Profile II and the firn limit. If it is assumed that the same amount of ice per unit area is added to the glacier between Profile II and the firn limit, then an approximate value for the amount of ice flowing through the cross-section at the firn limit can be obtained. This is done by multiplying the value M_{II} calculated by equation (2) for Profile II by the ratio of the accumulation areas, 1·20:

$$M'_{\text{II}} \times 1 \cdot 20 = 5 \cdot 12 \times 10^6 \text{m.}^3 \text{ water}$$

The close agreement between M_{II}, the mass of ice ablated annually and M'_{II}, the mass of ice moving through the firn limit cross section per year, can only serve as an indication and should not be taken as proof of the equilibrium condition of the glacier because of the approximations involved.

The average of the hydrological budgets for the Lemon Creek Glacier from 1954 to 1957 shows a net loss of $1 \cdot 0 \times 10^6$m.^{3} of water per year. This deficit is 20 per cent. of the average annual ablation. Because of the errors involved in ascertaining the net accumulation and ablation, such measurements are probably only accurate to within 20 per cent. of the correct value.

The last piece of evidence to be inspected pertaining to the regime of Lemon Creek Glacier concerns the dimensions and terminus position of the glacier. From the inspection of aerial photographs of the terminus of the glacier, only a slight recession and thinning of the glacier front are apparent. A comparison of the elevation of a movement stake in the middle of the glacier on Profile I in 1957 with the elevation of the location in 1955 shows that the difference is only −3 ft. (−1 m.). Thus, there has not been any large scale thinning of the glacier in the accumulation region.

Conclusions

Although the hydrological budget of Lemon Creek Glacier seems to indicate a net loss of ice over the past four years, this deficit does not seem to be reflected by the movement of the ice. It is possible that it may take several years for the movement of the ice to readjust itself to any change in the glacier's regime.

From the considerations and calculations given above, Lemon Creek Glacier appears in approximate equilibrium. Any deviation from equilibrium does not seem to be great enough to affect the equilibrium flow of the ice.

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