Binary Pulsar Tests of General Relativity in the Presence of Low-Frequency Noise

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Abstract. The influence of the low-frequency timing noise on the precision of measurements of the Keplerian and post-Keplerian orbital parameters in binary pulsars is studied. Fundamental limits on the accuracy of tests of alternative theories of gravity in the strong-field regime are established. The gravitational low-frequency timing noise formed by an ensemble of binary stars is briefly discussed.

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1. Background

The discovery of the first binary pulsar in 1974 by R. Hulse and J.H. Taylor had opened fascinating new opportunities for testing alternative theories of gravitation in the radiative and in the strong-gravitational-field regimes outside the boundaries of the Solar system. Presently, two distinguished binary pulsars - PSR B1913+16 and PSR B1534+12 - serve as primary extra-solar astrophysical laboratories for gravitational physisists. The radiative $\dot{\omega} - \gamma - \dot{P}_b$ test of General Relativity (GR) in the case of PSR B1913+16 has been performed with a precision about 0.4% (Taylor & Weisberg 1989; Damour & Taylor 1991). In addition, the binary pulsars PSR B1534+12 strong-field $\dot{\omega} - \gamma - s$ test of GR has been done with the precision 1% (Stairs et al. 1998). What is worth noting is that the binary pulsar tests of GR have been carried out with an accuracy being comparable to, or in some aspects, higher than the GR tests in the Solar system which have been always performed in the near, weak-field zone of the solar system gravitating bodies.

Reflecting about the future of gravitational experiments in binary pulsars one can ask about whether the accuracy of GR tests can be further improved and, if not, what physical influences can limit this accuracy. It is well known there are two main obstacles for improving precision of any experiment - systematic and random errors[1]. Systematic errors cause the bias in the mean values of estimated parameters and can lead to inconsistent conclusions on the validity

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[1]There exist a third reason which restricts the precision of GR tests in binary pulsars. It relates to the fact that some additional ("hidden") parameters appear in the form of additive linear corrections to measured values of the main relativistic effects. These parameters can not be effectively separated from the parameteres making the biggest contribution to the effect so that

of the theory of gravitation under investigation. Fortunately, continuous improvements being made in observational techniques and growing knowledge on dynamics of our galaxy can siginificantly reduce or even eliminate the influence of systematic errors (Damour & Taylor 1991). On the other hand, random errors fully determine the magnitude of the numerical value of the dispersion of measured parameters which itself crucially depends on the nature of noise in the timing measurements. Usually, on rather short time spans the white noise of measurement errors dominates. However, to improve the accuracy of the GR tests one has to observe binary pulsars over time intervals of about 10 years and longer when the white noise is strongly supressed and low-frequency (or "red") noise dominates in observed times-of-arrivals (TOA) of pulsar' pulses. The red noise worsens estimates of measured parameters and increases the errors in their mean values. What kind of problems will one meet in processing such timing observations contaminated with the presence of the red noise ? Before trying to answer this question let us briefly descibe the timing model of binary pulsars and the properties of red noise.

2.   Timing Model

The conventional timing model is based on the Damour-Deruelle analytic parameterization of the two-body problem (Damour & Deruelle 1985; see also Klioner & Kopeikin 1994). Schematically it reads as follows

$$N(T) = N_0 + \nu T + \frac{1}{2}\dot{\nu}T^2 + \epsilon_{int}(T) \ , \tag{1}$$

where N is the number of observed pulse, N_0 is a constant, ν and $\dot{\nu}$ are the pulsar's rotational frequency and its time derivative, $\epsilon_{int}(T)$ is the pulsur's intrinsic noise, T is the pulsar's proper time, which is related to the observer's proper time t by the equation

$$T = t + \Delta(T) + \Delta_\odot(t) + \epsilon_{pr}(t) \quad . \tag{2}$$

Herein, $\Delta(T)$ and $\Delta_\odot(t)$ describe the classic and relativistic time delay corrections for the binary system and for the Solar system respectively, $\epsilon_{pr}(t)$ are noises produced by a number of diverse perturbations during the time of propagation of pulses from the pulsar to observer, inaccuracies in ephemerides of the solar system bodies, imperfectness of atomic clocks used for the time metrology, and electronic equipment used for timing observations. The equipment noise is performed to be white irrespective of the longivity of the observations. On the other hand, any other kind of noise present has low-frequency components which show up when the measurements are carried out over a sufficiently long time span. It is worth noting that standard procedures of data processing were worked out only for white noise which has a gaussian distribution of timing residuals. If timing residuals are dominated by red noise, the standard statistical estimations give unrealistic (overestimated) numerical values of the measured parameters.

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one can not improve its numerical value (for more details see, for example, Damour & Taylor 1991; Kopeikin 1994, 1996; Wex & Kopeikin 1999).

Table 1. Dependence of variances of the pulsar's rotational frequency ν and some of the Keplerian and post-Keplerian parameters from the total span of observations τ in case of presence of red noise with the spectrum $S(f) = h_n/f^n$, where h_0, h_1,...,h_6 are constants characterizing the intensity of the noise

$S(f)$	ν	T_0	x	P_b	$\dot{x}$	$\dot{P_b}$
h_0	τ^{-3}	τ^{-1}	τ^{-1}	τ^{-3}	τ^{-3}	τ^{-5}
h_1/f	τ^{-2}	τ^{-1}	τ^{-1}	τ^{-3}	τ^{-3}	τ^{-5}
h_2/f^2	τ^{-1}	τ^{-1}	τ^{-1}	τ^{-3}	τ^{-3}	τ^{-5}
h_3/f^3	const.	const.	const.	τ^{-2}	τ^{-2}	τ^{-4}
h_4/f^4	τ	τ	τ	τ^{-1}	τ^{-1}	τ^{-3}
h_5/f^5	τ^2	τ^2	τ^2	const.	const.	τ^{-2}
h_6/f^6	τ^3	τ^3	τ^3	τ	τ	τ^{-1}

3. Red Noise and Limits on Estimates of Parameters

Recently, we have begun to study the problem of pulsar data processing in the presence of red noise (Kopeikin 1997; Kopeikin 1999a; Kopeikin & Potapov 1998). To model the red noise we have used a shot-noise approximation with a specific choice of step function in such a way that any rational spectrum of the red noise could be restored, including flicker noise of phase ($1/f$), random walk of phase ($1/f^2$), and so on. The noise model includes both stationary and non-stationary parts of the autocovariance function which are well separated algebraically. In addition, an exhaustive treatment of the polynomial drift of the noise was worked out in full detail. Applying the model for processing fake data of a binary pulsar in a circular orbit we set upper limits on the numerical values of the parameter's variances. These upper limits depend on the total span of observations τ and can either decrease or grow as τ increases. The time dependence some of the limits are shown in Table 1.

One can see that in the case when the spectral index $n = 0, 1, 2, ...$ of the noise is big enough variances of some, or even all, parameters grow such that one can not get improvements in testing GR, which prevents better determination of masses of neutron stars and other physical characteristics of the binary system. Special methods of observations and/or data processing should be suggested to overcome this difficulty.

This problem can also be considered from a different point of view. The fact is that some of the red noises have a specific astrophysical origin and their study would deliver extremely valuable information about physical processes generating such noises[2]. One particular example represents a low-frequency timing noise produced by the variable gravitational fields of binary stars.

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[2] A striking example which comes to mind is the case of electromagnetic cosmic background radiation (CMB) which was discovered by Pensias & Wilson as an excess noise in the equipment they used for radio survey of the sky.

4.   Gravitational timing noise from binary stars

Precise calculations of the autocovariance function of the gravitational timing noise from binary stars requires having a mathematically complete solution to the problem of propagation of electromagnetic waves in variable gravitational fields of localized self-gravitating sources. Significant progress in solving this problem has been achieved recently by Kopeikin et al. (1999) and Kopeikin & Schäfer (1999). The relativistic time delay in an arbitrary time-dependent gravitational field has been presented as a function of the relative distances between observer, source of light, and localized source of the non-stationary gravitational field as well as the intrinsic characteristics of the source. Using the exprerssion for the time delay, the gravitational timing noise from an ensemble of binaries in our galaxy has been evaluated for the case of PSR B1937+21 under some simplifying assumptions (Kopeikin 1999b).

Using the same approach the gravitational timing noise from an ensemble of binaries in a globular cluster can also be parameterized and predicted. Long-term precise timing monitoring of the bunch of millisecond pulsars in 47 TUC (or other globular cluster) will be required to test the presence of the noise and its properties.It will help to better understand spatial distribution, mass function, and other statistical properties of binaries in the globular cluster which are important for a proper estimate of black-hole merger rate (Zwart & McMillan 1999).

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