REFERENCE SYSTEMS

TOSHIO FUKUSHIMA
National Astronomical Observatory
Mitaka, Tokyo, Japan

Abstract. A reference system is a relation connecting observables and their mathematical representententions. The principle of general relativity assures that any sort of coordinate system can be used to describe physical phenomena. Thus, any reference system is only a convention. There is no absolutely true reference system. Instead, people seek for a *best* reference system, whose meaning may differ thus need to clarify. Taking an example from Earth rotation, we discuss how to find such a best reference system. The definition of the best system will change as scientific understandings deepen and computational environments develop. Therefore, we can not stop improving reference systems. However, when replacing an existing widely-spread system, one must take great care to minimize the inconvenience caused by its transition, especially the inconvenience which users might endure. The Standards Of Fundamental Astronomy (SOFA) project being conducted by the IAU WG on Astronomical Standards has the opportunity to ease this troublesome task. The World Wide Web (WWW) will be a main device to realize the project, namely to provide working standards including reference systems to the world.

1. What are Reference Systems?

It was in 1989 when "Reference Frames in Astronomy and Geophysics", or "Reference Frames" in short, was published (Kovalevsky *et al.* (eds), 1989). Until then, the bible on this subject had been the so-called "Explanatory Supplement" (HMNAO and USNAO, 1961). Now that its second version (Seidelmann (ed.), 1992) is available, these two have been regarded as the standard textbooks on the difficult field of *reference systems.* Let us quote a phrase defining the reference frame/system from "Reference Frames":

> *Motion and posisiton are not abolute concepts and can be described only with respect to some reference. $\cdots$ there should exist an observational*

Celestial Mechanics and Dynamical Astronomy **66**: 107–114, 1997.

relationship between the point and the physical objects that are used to obtain its coordinates. We shall call 'reference frame' the physical realization of such a reference system.

Here, the word *frame* is defined as a realization while *system* refers an idealization of the same concept. These two terms should be differentiated in their usage. For example, a TT second, the idealization of TAI second, is defined as an SI second on geoid while its realization, a TAI second, is accessible via GPS satellite. In this case, TT second is a reference system and TAI second is a reference frame. Within fundamental astronomy and space geodesy, typical examples of such realizations are seen in a series of IERS Standards and Convention (McCarthy (ed.), 1989, 1992, 1996), while some examples of *systems* are found in the resolutions of IAU covering fundamental astronomy. One good illustration is a set of resolutions adopted at the 21st General Assembly in 1991. For example, the coordinate systems recommended there were defined by specifying the functional form of the metric of spacetime. See their details in the proceedings of IAU Colloquium No.127 held at Virginia Beach, USA, in 1990 (Hughes *et al.* (eds), 1991). Then, reference systems/frames exist to define the relation between observed objects and their mathematical representations. Note, that reference systems are defined here in a wider sense. It is not limited to the coordinate system. An example of other kinds of reference systems are astronomical units like the AU or day or astronomical constants such as GM of the Earth. Also important are time derivatives of these quantities such as proper motion of stars.

2. Reference System is Convention

Now the definition of reference systems has become clear. However, before proceeding further, we will present a remark which seems to have been ignored so far. We stress that **any reference system is merely a convention which we, scientists, adopt in order to make our work easy**. Does this sound too eccentric? McCarthy at least seems to have the same opinion since he has changed the name of working standards of IERS from IERS Standards to IERS Conventions (McCarthy, 1996). We remark that the above assertion is compatible with physical laws. In fact, the general theory of relativity assures the freedom to choose coordinate systems to describe observables. For example, there exist multiple geocentric coordinate systems even if we restrict them to be rectangular (Brumberg, 1996). Any physically meaningful phenomenon can be described in any of these coordinate systems, *and* its numerical values should be the same. Thus, the adopted coordinate system does not affect the magnitude of observables. Therefore, the most *correct* reference system cannot be defined. The only possibility is the best reference system. Here, the word 'best' sometimes means 'the most compact to express', sometimes 'the fastest to evaluate', sometimes 'the most precise in simulating observations', sometimes 'the easiest to understand physically', and so on. We should acknowledge that the definition of the best reference system depends on one's preference. Also, it will be altered according to the change of scientific environments such as the precision of observation or the level of available computational power. Thus, it is no wonder that we always disagree on reference systems. However, we frequently talk about the best reference system as if there were only one aboslute answer, and sometimes we tend to continue a fertile debate on issues, which are no more than a matter of taste. To avoid this, we should keep the above fact firmly in mind.

3. Case Study — Precession/Nutation

Let us study a test case of the best reference system selection using an example from the predictable part of Earth orientation: precession and nutation. Frequently, the present IAU system of precession and nutation has been argued to be far from the best solution to explain VLBI and other high-precision observations. Sometimes it is even claimed that the conventional separation into precession and nutation is artificial itself, and therefore meaningless. However, such claim is nonsense from the viewpoint that any reference system is an artifact. Rather, the questions are

1. In what sense this system is said to be the best?
2. In that sense, is it really the best solution among available systems?
3. Should such sense be sought? And if not,
4. what sense should be the determining one? And,
5. what is the best available system in this new sense?

In order to illustrate how to develop a constructive discussion, we present below the author's personal comments on these questions.

1. The present convention to separate into precession and nutation had been the best in an economic sense. More specifically, it was designed to minimize the total labor and resources to calculate the Earth orientation for various levels of accuracy. In fact, this system had satisfied the requirements to explain the Earth rotation at a level compatible with the precision of classical ground-based optical measurements such as that achieved by meridian circles. Experiences with non-experts make us doubt whether the present system is the easiest to understand. This may come from the complexity that arises from three (the ecliptic, the mean equatorial, and the true equatorial) moving planes being dealt with together. Such complexity was permitted since the best availa-

https://doi.org/10.1017/S025292110004642X Published online by Cambridge University Press

ble theory of planetary motions until 1950s was that referred to the ecliptic plane.

2. The present system is no longer the best in the above economic sense. This is because

   (a) computational power has dramatically increased due to a wide availability of cheap and powerful PCs, and because

   (b) the present formulation is mathematically insufficient to express the total rotation from the true equatorial coordinate system of one date to that of another.

   As for the latter reason, a problem lies in the introduction of an intermediary plane, the ecliptic. Precession is OK. Although its theory is based on the ecliptic, its practical expressions are in the form that by-pass the effects. However, the situation is different in nutation. See a similar discussion in Capitaine (1996). Also the recent development of highly accurate observations has pushed up the required precision of future systems considerably. If we try to increase the precision of precession/nutation while keeping the present Fourier series form, the number of terms will grow rapidly and the resulting expressions will become very time-consuming.
3. It is quite questionable to consider only the speed and convenience of computations. Even so, end-users only need black-box routines like those used to calculate planetary ephemerides. From the viewpoint of present standards of computing environments, to express precession/nutation as numerical tables is much better than to evaluate hundreds of trigonometric functions. Probably, for any kind of time series including the case of precession/nutation, to express them as coefficient tables of piecewise Chebyshev polynomials would be the most compact to store and the fastest to evaluate. On the other hand, everybody seems to feel a strong frustration about the magnitude of difference in precession/nutation angles between observations and the present convention. Then, it would be natural to introduce a sort of correction formula, whether it is theoretical or experimental, in order to *improve* the precision of the present system to match that of observations. However, this is a matter of necessity. In pointing a telescope to targets, should the operators really count for the argued shift of $\approx 0.3''$/century in the precession constant?
4. One way in which the best reference system is constructed is the easiness to understand. It is often argued that VLBI observations are sensible only to the luni-solar precession/nutation. The planetary precession/nutation contributes to the motion of ecliptic. However, this is only an approximate explanation. As the level of approximation in-

creases, the discrimination of these has become uncertain. If we find a suitable platform to express Earth rotation in an easier-to-understand form, it will lead to a deeper understanding of the Earth interior. Another direction to be sought for is the compatibility with other existing reference systems. In fact, the present system has a weak point in this sense. Until now, all theories on precession/nutation were analytical, and as such analytical theories required analytical theories of planetary and lunar motions. However, the current theories on planetary and lunar motions are numerical. Why not precession/nutation? Another similar example is the so-called physical libration of the Moon. Although its rough motion is described compactly as Cassini's laws, the present computations are fully based on the numerical integration of the Moon's rotation.

5. Let us dare present a proposal which may be controversal.
   (a) Abandon the present system to express the predictable part of Earth rotation as a combination of precession and nutation.
   (b) Instead, create a purely numerical theory of Earth rotation by integrating dynamical equations of rotation numerically and by fitting parameters such as $(C - A)/C$ from the observations.
   (c) Express the integrated rotation angles in numerical tables of piecewise Chebyshev coefficients. It would be nice to incorporate such tables as a part of planetary/lunar ephemerides just as the Moon's physical libration is given in recent versions of JPL DE series.
   (d) Provide abridged versions of these tables corresponding to a few approximation levels. It may be more appropriate to give simple formulas of such angles as function of time. These are to give multiple levels of low-precision reference systems for novice users.

4. SOFA Project

The comments given in the previous section seem to contain some contradictions. In one paragraph, they warn of the possible danger of a drastic change. While, in another place, they propose such a drastic change. However, we think that this is a matter of realization. Imagine to change a reference system. If we adhere to the old custom of presenting a new system to the astronomical community only through the IAU transactions and other paper-based literature, it will bring much confusion and cause a lot of trouble. As for the astronomical constants, we have introduced a new mechanism to avoid such confusion and trouble. In 1991, the IAU had set up a WG to consider such a mechanism as well as other missions: the IAU

WG on Astronomical Standards (IAU/WGAS). After 3 years discussion, the WG proposed a *two-tier* mechanism. The essence of the mechanism is to follow the IAG style of presenting the list of recommended constants in two ways:

1. the System of Astronomical Constants, which is to be used for creating long-time standards such as nautical almanacs and star catalogs and will not be changed frequently, and
2. the Current Best Estimates of Astronomical Quantities, which are to be used for research and will be updated periodically (every 3 years?).

Refer to the proceedings of JD14 of 22nd IAU General Assembly in Vol. 10 of *Highlights of Astronomy.*
Now, to return to general issues. Examples of expected problems are:

- Defining a new system may not mean establishing a practical computational scheme. A good example is the precession. The specification of the precession constant, its numerical value at some epochs, alone does not help us to calculate the new precession at any specified time. We need the so-called precession formula, which is a product based on not only the precession constant but also on planetary masses and planetary/lunar theories. In fact, the present IAU precession system was not available when the IAU adopted the current precession constant in 1976. It was completed when Lieske *et al.* published the present analytical formula in 1977.
- Even if practical schemes are presented, their implementations into actual computational codes may differ from person to person. This tendency does increase with the complexity of schemes. Obviously, the resulting differences in codes lead to spurious differences in analyzed results. This will introduce an error which is quite difficult to find.
- If the relation between the old and new system is not given in a practical manner at the time of transition, it will lead to another confusion. Especially, this becomes evident when time derivatives come into. An example was what we faced in the transition from the B1950.0 system to the J2000.0 system, where problems were encountered in the transformation of the proper motions of stars rather than in mean places.
- Even if the above practical solutions are intended to be prepared together, time lags among their announcements cause another kind of trouble.

In order to overcome some of these situations, the IAU WGAS has initiated a project to prepare computational codes needed for the basic calculations and to provide them electronically. The name of the project is SOFA (Standards Of Fundamental Astronomy). In 1994, the IAU permitted the WGAS to be continued for 3 more years and selected the author to continue to chair it at the beginning of 1995. The WGAS has reorganized its composition to form three major functions:

- Maintenance Committee, chaired by D.D. McCarthy of USNO, USA, to maintain the system of astronomical constants and to compile the best estimates of astronomical quantities,
- Relativity Sub-Group, chaired by V.A. Brumberg of IAA, Russia, to prepare a report to clarify the general relativistic definitions of astronomical constants and units, and
- SOFA Board of Review, chaired by P.T. Walace of STARLINK Project, UK, to organize the SOFA and to supervise its operation.

As for the first two functions, the latest information on their activities is given in the proceedings of Journées 1995. As for the SOFA Board of Review (Board in short), its internal discussion has just begun. The membership of the Board except the chair is (in alphabetical order);

W. Brouw of ATNF, Australia; C. Hohenkerk of RGO, UK; Jin Wenjing of Shanghai Obs., China; G.H. Kaplan of USNO, USA; Z. Malkin of IAA, Russia; Skip Newhall of JPL, USA; J. Percival of Univ. Wisconsin, USA; and D.D. McCarthy as *ex officio.*

5. Utilization of World Wide Web (WWW) System

Since it is too early to introduce the on-going discussion within Board, we just present the author's personal viewpoints. Note, that this is just a view of an outsider and the final conclusion from the Board may be different. The computational environment has significantly changed since when the original view of SOFA was presented by the IAU WGAS Subgroup of Standard Procedure (Fukushima, 1995). At the time of its preparation in 1992–1994, media suitable to communicate computational codes were CD-ROM, floppy disk, and anonymous FTP via Internet. However, now the World Wide Web (WWW) seems to be the most appropriate way to do it. This owes much to the wide spread of WWW browsers like Netscape, Mosaic, and Microsoft Internet Explorer together with the flood of information through it. These enable end-users to find necessary information promptly and to extract their copies easily. Usually, this is inexpensive, just the cost to use communication lines and Internet in many cases. Another important point is that the cost to the information provider is quite low, just that of maintaining a WS connecting to the Internet to operate WWW homepages. Let us list new trends brought by the WWW:

- It has become a common practice to use the WWW in providing information on scientific symposia. This Colloquium itself is an example!

- Many companies advertise their products through the WWW. Among them, software companies tend to allow registered customers to obtain update information through the WWW.
- Many scientists have started to open a showcase of their personal scientific products such as papers and computational codes.
- Some scientific journals are presenting the accepted papers (usually letters and other quick reports) via WWW. Further, a few journals have started to use WWW as their only way to *publish* articles.

Doesn't this situation fit the purpose of the SOFA project quite well? If so, the SOFA may not need any real center to operate it. Instead, it is feasible to set up a sort of *virtual* center by connecting several WWW homepages of WSs of the Board members and some volunteers to each other. In the actual implementation, a systematic and periodic mirroring of these homepages is much better than just linking them to each other in order to decrease unnecessary information traffic. Another idea is to extend the SOFA project to establish an electronic journal on *Reference Systems*. Not only the astronomical community but also the IERS and the geodetic community may jointly operate such a journal. Would it not be exciting to see the latest set of constants, IERS Conventions, and SOFA routines on WWW as well as proceedings of all relevant symposia like this volume itself? Anyway, there is no doubt that this approach to maximize the use of the WWW not only enhances the convenience of end-users but also removes difficulty of finding a financial support to realize the SOFA project.

References

Capitaine, N.: 1996, "Precession-Nutation: Theory, determinations and analysis", *Earth Rotation, Reference Systems in Geodynamics and Solar System, Journées 1995* (N. Capitaine, B. Kolaczek, S. Debarbat, eds), Warsaw, 39–44.

Fukushima, T.: 1995, "Report of the IAU WGAS subgroup on standard procedures", *Highlights of Astronomy* **10**.

HMNAO and USNAO: 1961, *Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac*, HM Stationery Office, London.

Hughes, J.A., Smith, C.A., and Kaplan, G.H. (eds): 1991, *Reference Systems, IAU Colloquium 127*, U.S. Naval Observatory, Washington, DC.

Kovalevsky, J., Mueller, I.I., and Kolaczek, B. (eds): 1989, *Reference Frames in Astronomy and Geophysics*, Kluwer, Dordrecht.

McCarthy, D.D. (ed.): 1989, "IERS Standards (1989)" *IERS Tech. Note 3*, Obs. de Paris.

McCarthy, D.D. (ed.): 1992, "IERS Standards (1992)" *IERS Tech. Note 13*, Obs. de Paris.

McCarthy, D.D. (ed.): 1996, "IERS Conventions (1996)" *IERS Tech. Note 21*, Obs. de Paris.

Seidelmann, P.K. (ed.): 1992, *Explanatory Supplement to the Astronomical Almanac*, Univ. Science Books, Mill Valley, CA.