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Some characterisations of the ellipsoid and the Minkowski theory of reduction

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The Minkowski theory of reduction of n-ary, positive definite quadratic forms is concerned with properties of certain such forms, called reduced forms, especially when the variables take integer values. Each such form may represent an ellipsoid in n-dimensional Euclidean space, R^n, and then the theory may be formulated geometrically in terms of the relations between a fixed ellipsoid and a variable lattice. The theory of reduction of a convex body K is the natural generalisation of the Minkowski reduction theory, obtained by replacing the ellipsoid by K.

Minkowski obtained, among many other results, two finiteness theorems, namely: finitely many linear inequalities in the coefficients are sufficient to determine all reduced forms; only finitely many unimodular transformations can transform a reduced form again into a reduced form. It is natural to ask whether the geometrical version of these results can be generalised, and in fact Minkowski showed that both hold in R^2 for convex bodies other than the ellipsoid.

This thesis shows that in R^n, for $n \geq 3$, neither of the results hold for a convex body K other than the ellipsoid. The proof involves a new characterisation of the ellipsoid, namely: if any two parallel planes "sufficiently close" to a tac plane of K, intersect K in equivalent convex bodies, then K is an ellipsoid. Here, two convex bodies are equivalent if, for some Minkowski metric, both have constant width. For example, two bodies of constant width are equivalent, or two homothetic bodies.

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