STATISTICAL PROBLEMS ENCOUNTERED IN USING TRIGONOMETRIC PARALLAXES

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The importance of understanding the properties of a data sample which is to be used for calibrating luminosities can hardly be overemphasized. In an earlier paper (Lutz 1983) I pointed out the methods for dealing with two well-defined cases. In this paper I will elaborate further on the same topic.

The two cases discussed previously were samples which were both magnitude limited. There, it was shown that an additional restriction of a minimum parallax value might or might not change the sample into a volume-limited sample. In those two cases the values of absolute magnitude computed directly from the data are biased. In the case of a strictly magnitude-limited sample (see below for a definition of a "strictly magnitude-limited" sample) the computed absolute magnitude (referred to hereafter as M(mag)) IS equal to the average absolute magnitude of the sample. However, this differs from the volume-limited sample value (referred to hereafter as M(vol)) by an amount given by the Malmquist (1920, 1936) correction. In the case of a sample limited strictly by a minimum value of observed parallax, the computed absolute magnitude IS equal to the average absolute magnitude of the sample but this differs from M(vol) by an amount given by Lutz and Kelker (1973).

The following discussion refers to a data sample which is described by the following general parameters:

1. All of the stars have been chosen by some criteria, such as spectral type and luminosity class, which assure that they all are distributed closely about some particular (unknown) value of absolute magnitude.

2. The sample is strictly magnitude limited, i.e. ALL stars which meet criteria 1. above and which are brighter than some particular value of apparent magnitude have been included in the sample. In practice some stars brighter than the magnitude limit may have to be

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omitted, but it should be clear that the reason for omission is unrelated to the luminosity or distance. For example, it might be permissable to omit stars which do not have radial velocity data, provided that the omitted stars are representative of the parent population from which the sample is being drawn.

3. All of the stars have measured trigonometric parallaxes and all of the parallax measurements have the same accuracy, σ_π.

We distinguish five cases. The sample spaces for each of these cases are shown in Figures 1-5. In each figure I have assumed that the type of star which has been selected for study has a luminosity function with mean-absolute magnitude of M(vol)=0.0 and standard deviation of 0.5 mag. Thus, over 99% of the stars will lie within the dashed lines in the figures.

Case A

The sample space for case A is shown in Figure 1. Here all of the stars in the magnitude-limited sample will be used in computing the average-absolute magnitude of the sample.

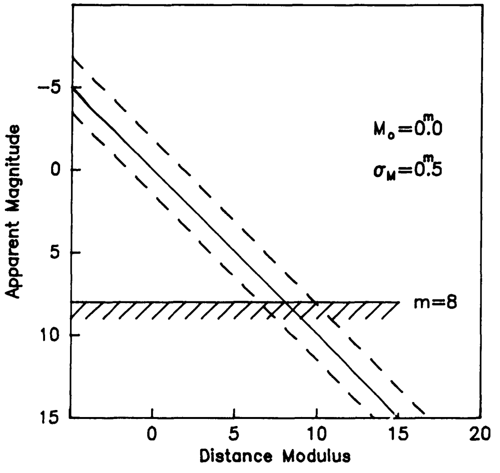

Figure 1.

Since it is likely that there will be stars with negative values for their measured trigonometric parallaxes, the method of reduced parallaxes must be used. It will be necessary to take into account the difference between the average of the log of the reduced parallaxes (the desired quantity) and the log of the average of the reduced parallaxes (the quantity which can be computed).

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The result will be an estimate of M(mag). To obtain an estimate of M(vol) it will be necessary to apply a Malmquist correction. Both the Malmquist correction and the correction from the log of the average to the average of the log require prior knowledge of the form of the luminosity function of the stars in question.

Case B

In the remaining cases to be discussed a lower limit will be placed upon the values of observed parallax which will be allowed in the sample. With σ_π fixed, this is the same as placing a limit on the ratio σ/π_o.

i.

First I will discuss samples in which, in addition to the magnitude limitation, stars are only allowed in the sample if they have a value of $\sigma/\pi_o < 0.175$. There are two such cases.

a.

In the first case, the sample space is shown in Figure 2. Here, the volume boundary is large compared with the distance of a star which has apparent magnitude equal to the magnitude limit and absolute magnitude M(vol). When the boundary is sufficiently far to the right, very few stars will be omitted, and the situation is exactly the same as in case A.

b.

In the second case, the sample space is shown in Figure 3. Here, the volume boundary is small compared with case B.i.a. This is clearly not a magnitude-limited sample. The average sample absolute magnitude is obtained by computing an absolute magnitude for each star from its observed parallax and apparent magnitude. Then, after applying the corrections of Lutz and Kelker, the average of these (corrected) absolute magnitudes is an estimate of M(vol).

The transition between cases B.i.a and B.i.b is the subject of Lutz (1979). This transition is complicated by the need to know the luminosity function of the stars in question a priori. It is further complicated by the fact that the mean absolute magnitude of the sample changes from M(mag) to M(vol) as the boundary is moved from case B.i.a to case B.i.b. These complications suggest that it will not be easy to find a method for dealing with data samples in this transition zone which is both statistically rigorous and straightforward to use.

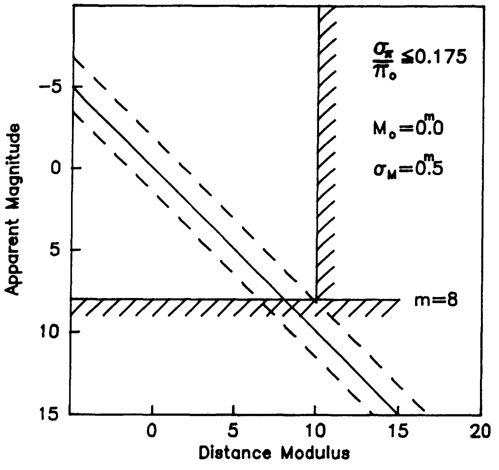

Figure 2.

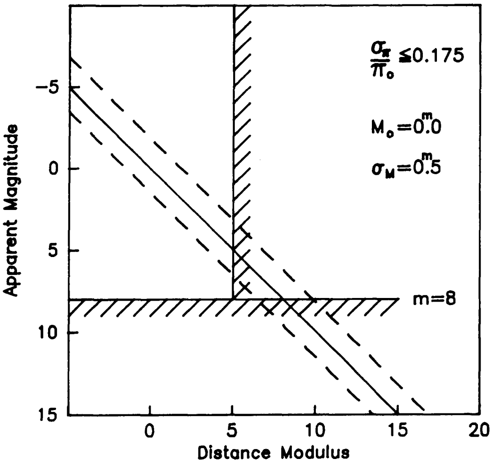

Figure 3.

ii.
Here I will discuss the cases where a lower limit on the observed parallaxes is imposed, but values of σ/π_0 >0.175 are allowed. Again, two cases are distinguished.

a.
The sample space for the first of these cases is shown in Figure 4. This case is similar to case B.i.a., but the restriction on the observed parallaxes has been relaxed. It would seem that the only thing one can do is to remove the limitation on observed parallax and follow the procedure described above for case A.

In this case then it is possible to determine a value for M(mag).

b.
The sample space for the second case in this category is shown in Figure 5. The possibilities here are the same as for case B.ii.a. above.

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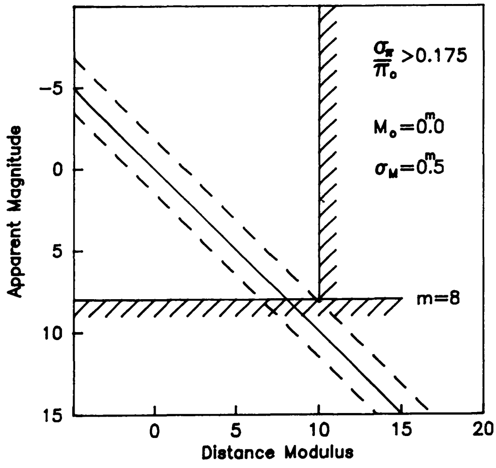

Figure 4.

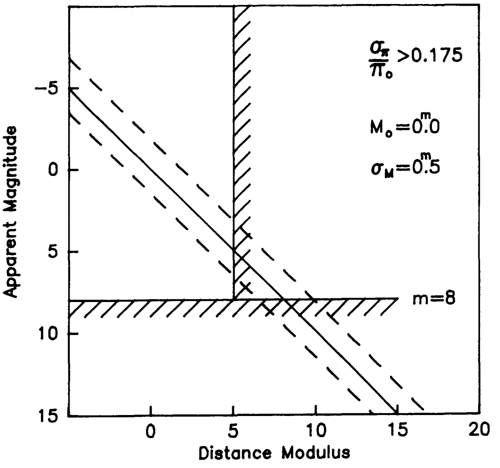

Figure 5.

In both of these last two cases it is tempting to want to use the (usually) numerous parallaxes with σ/π_0 > 0.175 to compute M(vol). Lutz (1979) developed an iterative scheme which would allow the use of these smaller parallaxes, but it depended upon a prior knowledge of the luminosity function. It has been suggested (see discussion following Lutz 1983) that trial values for the mean and standard deviation of the luminosity function can be assumed and that, by iteration, the computations will converge to the correct values. However, it is not known that such a procedure will converge. Since it will of necessity depend upon the FORM adopted for the luminosity function it is not clear that it will converge to the correct values if it does converge (i.e. the choice of the wrong luminosity function may allow convergence to the wrong mean and standard deviation giving a systematic error). Finally it is not clear that the (random) errors introduced by using less accurate parallaxes will yield improved values for the mean and standard deviation of the luminosity function. More realistically, astronomers attempting a luminosity calibration often are forced to gather their data from the published literature. It must be emphasized that taking data from the literature and omitting stars fainter than a pre-selected magnitude limit will NOT, in general, result in a magnitude-limited sample. This is because the stars chosen for observation by other astronomers may have been selected for reasons related to their distance and/or their luminosity, and some stars brighter than the magnitude limit omitted because they did not meet these criteria. In general, the burden of proof is on the investigator to show that a sample is magnitude limited. Samples which are not magnitude or volume limited, no matter how closely restricted by the selection criteria in 1. above, can have average values which differ from M(mag) or M(vol) by

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amounts greater than the corrections of Malmquist (1920, 1936) or Lutz and Kelker (1973). It is probable that some of the discrepancies between luminosity calibrations of the same types of stars are caused by effects of this type. Control of the data samples is just as important in astronomy as it is in other fields of science.

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REFERENCES

Lutz, T. E.: 1979, Monthly Notices Roy. Astron. Soc. 189, pp. 273-278.
Lutz, T. E.: 1983, In "Nearby stars and the Stellar Luminosity Function", IAU Coll. No. 76, eds. Phillip, A. G. D. and Upgren, A. R., Van Vleck Observatory, Middletown, pp. 41-49.
Lutz, T. E. and Kelker, D. H.: 1973, Pub. Astron. Soc. Pacific 85, pp. 573-578.
Malmquist, K. G.: 1920, Lund Astron. Obs. Medd., Ser. II, No. 22.
Malmquist, K. G.: 1936, Stockholm Obs. Medd., No. 26.