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Trigonometric Survey.—The object of this note is to show how various problems commonly occurring in Elementary Trigonometry may be classified as particular cases of trigonometric survey. The general case may be stated first, and certain well-known particular cases appended.

General Case (Fig. 1).—Let $ABCD$ be a quadrilateral, where $AB(=d)$ is a measured base, and angles $BAC(=\alpha)$, $ABC(=\beta)$,

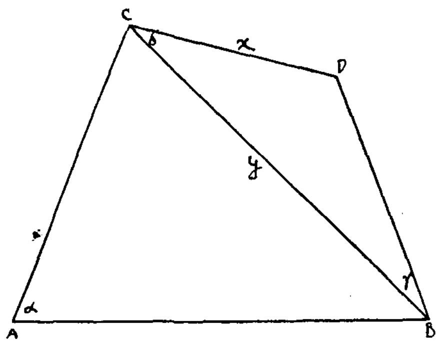

FIG. 1.

$CBD(=\gamma)$, $BCD(=\delta)$ are measured by theodolite, the first pair from the ends of the base AB, the second pair from the ends of the derived base BC; to determine the length of CD. Put $CD=x$, $BC=y$.

Then $$\frac{x}{d}=\frac{x}{y}\cdot\frac{y}{d}=\frac{\sin\gamma}{\sin(\gamma+\delta)}\cdot\frac{\sin\alpha}{\sin(\alpha+\beta)}$$

or $$x=\frac{d\sin\alpha\sin\gamma}{\sin(\alpha+\beta)\,.\,\sin(\gamma+\delta)}.$$

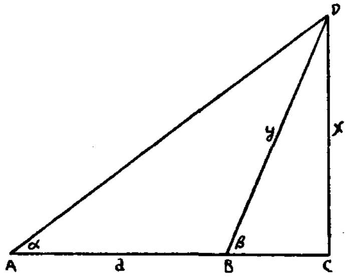

FIG. 2.

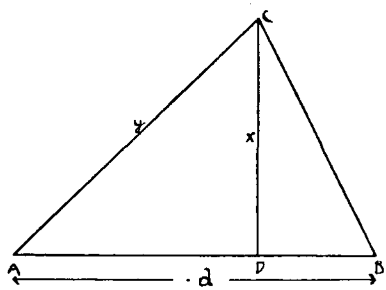

FIG. 3.

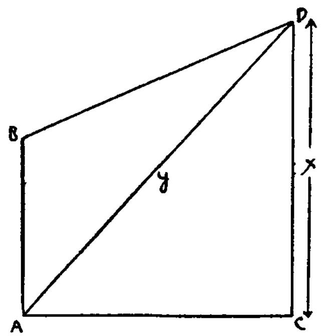

FIG. 4.

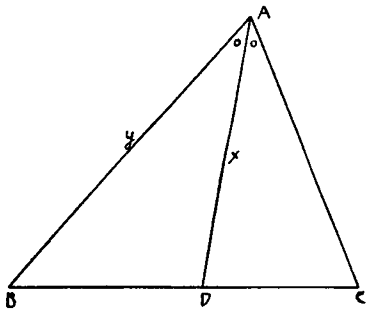

FIG. 5

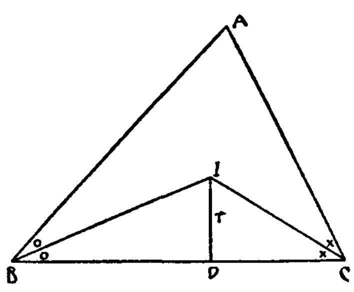

FIG. 6

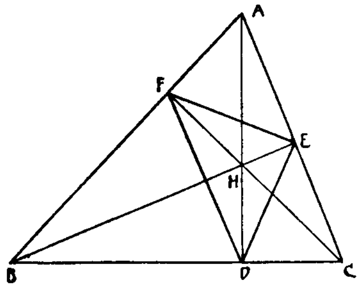

FIG. 7.

First Case (Fig. 2).—Let CD be a tower of unknown height, and let ABC be a horizontal road, where $ABCD$ is a vertical

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plane; let $AB = d$, $\widehat{CAD} = \alpha$, $\widehat{CBD} = \beta$; to determine the height CD or x, say, we regard $\triangle ABD$ as determined by trigonometric survey from the base AB, and the known angles BAD, ABD. Next we regard BD as a derived base and $\triangle BCD$ as determined from the base BD and the base angles DBC, BDC. Put $BD = y$; then

$$\frac{x}{d} = \frac{x}{y} \cdot \frac{y}{d} = \frac{\sin\beta}{\sin\frac{\pi}{2}} \cdot \frac{\sin\alpha}{\sin(\beta - \alpha)}$$

or
$$x = \frac{d\sin\alpha\sin\beta}{\sin(\beta - \alpha)}.$$

If BC or z is required, we have, similarly,

$$\frac{z}{d} = \frac{z}{y} \cdot \frac{y}{d} = \frac{\sin\left(\frac{\pi}{2} - \beta\right)}{\sin\frac{\pi}{2}} \cdot \frac{\sin\alpha}{\sin(\beta - \alpha)}$$

or
$$z = \frac{d\sin\alpha\cos\beta}{\sin(\beta - \alpha)}.$$

Second Case (Fig. 3).—In an acute-angled triangle ABC, let CD be the height; to express CD or x, say, in terms of $AB(=d)$ and angles BAC, ABC.

Regard $\triangle ABC$ as determined by trigonometric survey from base d and angles A and B, and then $\triangle ACD$ as determined from the derived base AC and angles A and DCA. Put $CD = x$, $AC = y$ Then

$$\frac{x}{d} = \frac{x}{y} \cdot \frac{y}{d} = \frac{\sin A}{\sin 90^\circ} \cdot \frac{\sin B}{\sin(A + B)}$$

or
$$x = \frac{d\sin A\sin B}{\sin(A + B)}.$$

Third Case (Fig. 4).—Consider the problem of finding the unknown height of a vertical projection CD, by deduction from the known height d of a tower AB and from the angles of elevation of D as seen from A and B. Trigonometric survey determines triangle ABD, and a repetition of the process determines $\triangle ACD$ from the derived base AD. Put $CD = x$, $AD = y$.

Then $$\frac{x}{d}=\frac{x}{y}\cdot\frac{y}{d}=\frac{\sin DAC}{\sin 90^\circ}\cdot\frac{\sin ABD}{\sin ADB}.$$

or $$x=\frac{d\sin DAC\,.\sin ABD}{\sin ADB}.$$

Fourth Case (Fig. 5).—To express AD, the length of the bisector of angle A of triangle ABC, in terms of base BC and base angles B, C.

Trigonometric survey determines $\triangle ABC$ from a, B, C, and then determines AD from the derived base AB and the known base angles ABD and DAB. Put $AD=x$; then

$$\frac{x}{a}=\frac{x}{c}\cdot\frac{c}{a}=\frac{\sin B}{\sin\left(\frac{A}{2}+B\right)}\cdot\frac{\sin C}{\sin A}$$

$$\therefore\quad x=\frac{a\sin B\sin C}{\sin A\sin\left(\frac{A}{2}+B\right)}=\frac{a\sin B\sin C}{\sin(B+C)\cos\frac{1}{2}(B-C)}.$$

Fifth Case (Fig. 6).—To express r, the in-radius of $\triangle ABC$, in terms of a, B, C.

Let I be the in-centre and ID the perpendicular from I to BC. Trigonometric survey determines $\triangle BIC$ from base BC and base angles $\frac{1}{2}B$, $\frac{1}{2}C$, and then determines ID or r from the derived base IB and the base angles IBD and BID. Put $IB=y$.

Then $$\frac{r}{a}=\frac{r}{y}\cdot\frac{y}{a}=\frac{\sin\frac{B}{2}}{\sin 90^\circ}\cdot\frac{\sin\frac{C}{2}}{\sin\left(\frac{B}{2}+\frac{C}{2}\right)}$$

or $$r=\frac{a\sin\frac{B}{2}\sin\frac{C}{2}}{\sin\left(\frac{B}{2}+\frac{C}{2}\right)}.$$

Sixth Case (Fig. 7).—If DEF be the pedal triangle of $\triangle ABC$, to express EF in terms of a, B, C.

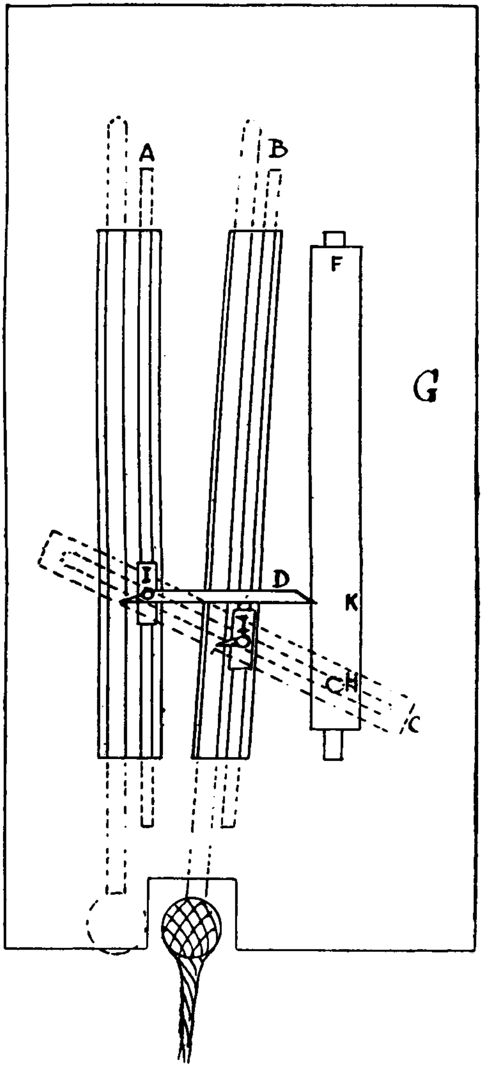
We may suppose a trigonometric survey made of $\triangle BEC$ from base BC, and then a survey made of $\triangle CEF$ from the derived base CE.

Then $\frac{EF}{a} = \frac{EF}{EC} \cdot \frac{EC}{a} = \frac{\sin ECF}{\sin EFC} \cdot \frac{\sin EBC}{\sin BEC} = \frac{\cos A}{\cos C} \cdot \frac{\cos C}{1}$.

or $EF = a\cos A = -a\cos(B+C)$.

P. PINKERTON.

A Direct Reading Hygrometer.—The instrument is an ordinary dry and wet bulb hygrometer adapted to give a direct



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