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CORRIGENDUM: EQUIVALENT WEIGHTS AND STANDARD HOMOMORPHISMS FOR CONVOLUTION ALGEBRAS ON $\mathbb{R}^+$

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Abstract There is an error in one of the major results in our original paper 'Equivalent weights and standard homomorphisms for convolution algebras on $\mathbb{R}^+$'. We describe the error and give a counterexample to the result as stated. We then give a substitute result which is in many ways stronger than the erroneous result. We will also indicate what changes need to be made in the original paper to accommodate the replacement of the erroneous result by the substitute.

Keywords: convolution algebra; weight; standard homomorphism; semigroup

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We point out an error in [**1**] and give a new result to substitute for the original erroneous result. We will then describe the changes needed in the original paper to use the new result in place of the original erroneous result. This note is written as a continuation of our original paper, and we will use the same terminology, definitions and numbering of results.

4. Corrections

Theorem 2.5 is incorrect as stated. Condition (iii) in the theorem does not follow from conditions (i) and (ii). The other parts of Theorem 2.5 are true, with their proof unchanged. The error in the proof is that the function $\omega_1(x)$ defined in the proof need not be right continuous.

We will now construct a counterexample to Theorem 2.5 as stated. For $n < x \leqslant n+1$, where n is a non-negative integer, we let $\omega(x) = \mathrm{e}^{-n^2}$. We also let $\omega(0) = 1$. It is easy to check that $\omega(x)$ is submultiplicative and left continuous. On the other hand, for each positive integer n we have $\omega(n)/\omega(n^+) = \mathrm{e}^{2n-1}$, which is unbounded. Hence, there is no right-continuous weight equivalent to $\omega(x)$. If we redefine ω at the positive integers to obtain a right-continuous weight $\omega_1(x)$, then $\omega_1(x)$ is an algebra weight essentially equivalent, but not equivalent, to $\omega(x)$.

Corollary 2.6 is also wrong. We will give a substitute for the corollary. This substitute will only require that $L^1(\omega)$ be an algebra instead of requiring the stronger condition that $M(\omega)$ is an algebra. The substitute will also allow us to use Theorem 2.4 in place of the false Theorem 2.5 in applications. After we prove the substitute, we will use it to show that Corollary 2.6 is false as stated.

To obtain the substitute result, we need to modify the definition of $C_0(1/\omega)$ given before Theorem 1.1. Suppose that ω is a weight. We let $C_0(1/\omega)$ be the space of continuous functions in $L^\infty(1/\omega)$ for which the essential limit of $h(x)/\omega(x)$ as $x \to \infty$ is 0. On $C_0(1/\omega)$ we use the norm of $L^\infty(1/\omega)$; that is

$$\|h\| = \|h\|_{1/\omega} = \text{ess sup}\left|\frac{h(x)}{\omega(x)}\right|.$$

When $\omega(x)$ is right continuous, $h(x)/\omega(x)$ is bounded and has limit 0 as $x \to \infty$. In this case the norm

$$\|h\| = \sup\left|\frac{h(x)}{\omega(x)}\right|.$$

Thus, our definition reduces to the original definition for right-continuous weights. With this extended definition, we have that $C_0(1/\omega) = C_0(1/\omega')$, with equivalent norms, whenever ω and ω' are essentially equivalent.

We can now give the replacement for Corollary 2.6. Recall that if $L^1(\omega)$ is an algebra (equivalently, if ω is essentially K-submultiplicative for some K), then it follows from Theorem 2.4 that there is an algebra weight ω' essentially equivalent to ω.

Theorem 4.1. *Suppose that $L^1(\omega)$ is an algebra, and let $\omega'(x)$ be an algebra weight essentially equivalent to ω. Then we have the following.*

(i) $M(\omega') = C_0(1/\omega)^*$, *with equivalent norms, when we identify the measure μ with the linear functional*

$$\langle \mu, h\rangle = \int_{\mathbb{R}^+} h(x)\,\mathrm{d}\mu \quad \text{on } C_0(1/\omega).$$

(ii) $M(\omega') = \mathrm{Mult}(L^1(\omega))$, *with equivalent norms, when we identify the measure μ with the operator $f \mapsto \mu * f$ on $L^1(\omega)$.*

Proof. Since ω' is essentially equivalent to ω, we have $L^1(\omega) = L^1(\omega')$ and $C_0(1/\omega) = C_0(1/\omega')$, with equivalent norms. But it follows from Theorem 1.1 that $M(\omega') = C_0(1/\omega')^*$ and $M(\omega') = \mathrm{Mult}(L^1(\omega'))$. Combining these equalities gives the theorem. □

It is important to notice that the dual space and multiplier algebra given in Theorem 4.1 do not depend on which algebra weight is chosen. For, if ω_1 and ω_2 are right-continuous weights that are each essentially equivalent to ω and hence to each other, it

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follows from Lemma 2.3 that ω_1 and ω_2 are actually equivalent to each other, so that $M(\omega_1) = M(\omega_2)$.

When $M(\omega)$ is an algebra but ω is not equivalent to any right-continuous weight (as in the example above) we have $M(\omega) \neq M(\omega')$, where ω' is an algebra weight essentially equivalent to ω. Hence Corollary 2.6 is false in this case.

Since $L^1(\omega)$ is an algebra in Theorem 4.1, we have that $L^1(\omega) \subseteq \mathrm{Mult}(L^1(\omega)) = M(\omega')$. Thus, $M(\omega') = C_0(1/\omega)^*$ defines a (relative) weak* topology on $L^1(\omega)$. Similarly, if $M(\omega)$ is an algebra, then we have $M(\omega) \subseteq M(\omega')$. But, as we pointed out, $M(\omega) = M(\omega')$ if and only if ω is equivalent to a right-continuous weight.

The only numbered results in the original paper that are incorrect are Corollary 2.6 and condition (iii) in Theorem 2.5. We need to replace Corollary 2.6 with Theorem 4.1 and eliminate condition (iii) in Theorem 2.5. In applications, we now use Theorem 2.4 together with Theorem 4.1 where we previously used Theorem 2.5 and Corollary 2.6.

The applications have the following form, as indicated in the paragraph after Theorem 2.5. One proves results for what we call algebra weights and then extends the results to more general weights ω for which $L^1(\omega)$ is an algebra, by replacing ω with an essentially equivalent algebra weight ω'. In the original formulation, which we now know is false, we needed to also have that $M(\omega)$ was an algebra. Now we know that it is enough for $L^1(\omega)$ to be an algebra, but we have to replace $M(\omega)$ with $M(\omega')$.

Here are some other places where statements in the original paper are not correct: the sentence in the abstract of the original paper containing $M(\omega)$ is false and should be eliminated; the statement following that sentence would need to be rewritten along the lines just indicated. The definition given for $C_0(1/\omega)$ in § 1 only works when ω is right continuous. In the general case, we need to substitute the definition given in the present section. As far as the author is aware, all the changes needed in § 2 have already been indicated. In § 3, the results are given for algebra weights and are therefore unchanged. Only the discussion (in the first paragraph of § 3) on extending the results to more general weights needs to be changed as indicated in the present section. In particular, the results remain true if $L^1(\omega_1)$ and $L^1(\omega_2)$ are algebras even if $M(\omega_1)$ and/or $M(\omega_2)$ are not algebras.

References

1. S. Grabiner, Equivalent weights and standard homomorphisms for convolution algebras on $\mathbb{R}^+$, *Proc. Edinb. Math. Soc.* **52(2)** (2009), 409–418.