1 Fluids and Fluid Mechanics

The wind blows into the sails, the ship sails through the huge waves, and the seabirds fly in the sky. The forces exerted by fluids are everywhere in our daily lives.

1.1 Fluids: Basic Concepts

Fluids are substances that deform more easily than solids, and includes liquids and gases. However, an object deforms only under forces, and whether it deforms easily depends on the intensity and type of forces acting on it. As a solid, nylon rope has high tensile strength but offers practically no resistance when pressing on it, and can also be easily cut by scissors. Therefore, we need a more rigorous definition to distinguish between fluids and solids.

The key difference between solids, liquids, and gases lies in the microstructure of the material. Figure 1.1 shows the microstructure in the three states of water, with oxygen and hydrogen atoms represented by small balls. The close-packed molecules of a solid strive to retain a fixed arrangement, while those of a liquid have no such
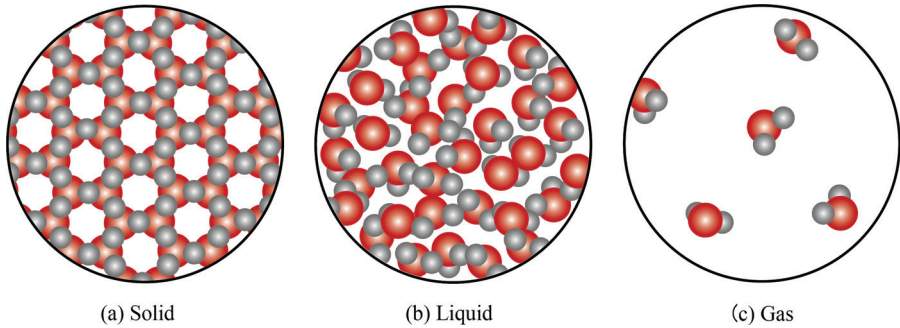
(a) Solid   (b) Liquid   (c) Gas

Figure 1.1 Microstructure in the three states of water.

intention. The wide-spaced molecules of a gas neither huddle together nor aim to retain a fixed arrangement.

Now we can try to understand the features of the three states of matter. A solid or liquid, being composed of closely packed molecules with strong cohesive forces, tends to retain its volume unless it is subjected to enormous pressure. On the other hand, under the action of external pressure the distance between the molecules of a gas will decrease, producing a change in overall volume.

Liquids share with gases the ability to flow because the atoms and molecules in both states have no tendency to be rigidly packed in a fixed pattern. Although tightly packed, the molecules in a liquid can be arranged randomly, causing the liquid to have a definite volume but no definite shape. Since the molecules in a gas move freely and independently of each other, the forces between them can be ignored, except when they collide with each other.

We understand that if the resultant force-torque acting on a rigid body equals zero, the object remains stationary or in uniform motion. However, this is not true for a real solid or fluid. Any solid material has a strength limit. Even if the resultant force-torque acting on it equals zero, the solid material inside may experience a tensile stress, a compressive stress, or a shear stress. A solid deforms under such stresses. Furthermore, stresses beyond the strength limit cause the solid material to yield and to fracture. At the microscopic level, the intermolecular (or interatomic) chemical bonds at the fracture point are broken, losing their interactive forces; this means the material cannot retain its original size and shape.

A fluid differs substantially from a solid in this respect. If a fluid is subjected to compressive stresses only, it will deform like a solid and remain stationary. When subjected to shear stresses, a fluid will deform continually and be unable to resist the external shear stresses in static equilibrium. This is somewhat similar to the case in which the forces acting on a solid far exceed its strength limit. But a fluid at rest has zero "shear strength" and will continually deform under the action of any small amount of shear force. Therefore, the essential difference between fluids and solids is that *a fluid cannot generate internal shear stresses by static deformation alone*.

To better understand the difference between fluids and solids, let us discuss the static friction between two solid surfaces. Place a solid block on a plane and gradually
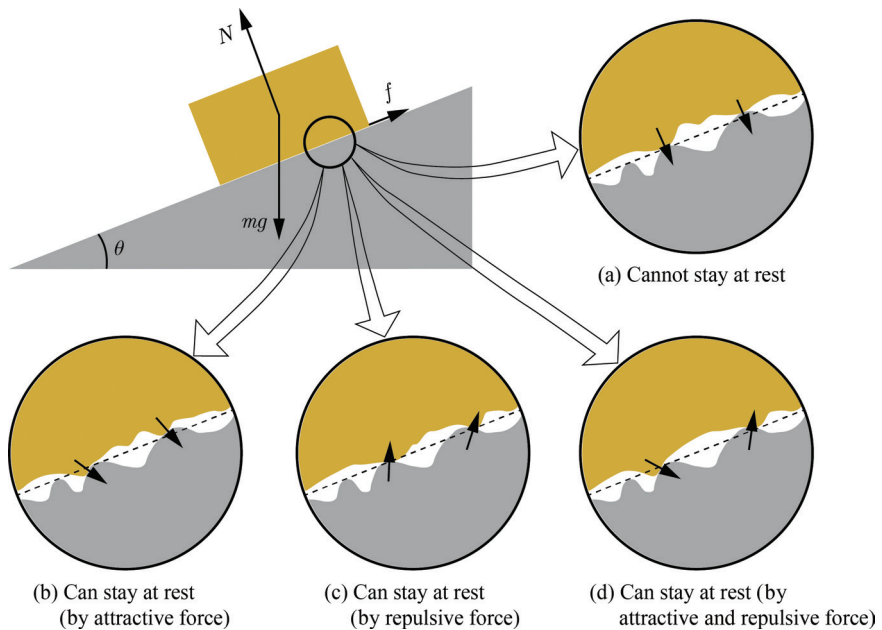
Figure 1.2 The forces exerted on a solid block on an incline and corresponding microscopic explanation of the static friction between two solid surfaces.

lift one end of the surface. Within a certain range of inclination angle the block will not move because the component of gravity acting along the inclined plane equals the static friction. Figure 1.2 shows the forces acting on the block (represented by arrows) and the corresponding microscopic explanation of the static friction between the two surfaces. As is well known, static friction results from the electromagnetic forces between molecules (or, similarly, atoms) that are in contact. In order to generate shear forces, the electromagnetic attractive and repulsive forces should not be perpendicular to the contact surface but have one component acting along it. This component may be caused by one of two factors, corresponding to rough and flat surfaces, respectively.

In general, as shown in Figure 1.2, neither the bottom of the block nor the inclined plane is a perfectly flat surface. There are some attractive and repulsive forces only between the molecules on the very small contact area between the two surfaces. When the plane starts to tilt from horizontal, the block will slide down an imperceptible distance. At the microscopic level, when the block slides down, as shown in Figure 1.2(b)–(d), the attractive and repulsive forces between the bulges of the block and the plane offer an upward component acting along the contact surface. This generates a shear force that keeps the block in static equilibrium.

If two perfectly flat planes at the molecular level are placed in contact with each other, is there any friction between them? Not only friction is present in this case, but it can be very high. This is because the contact area is quite large, and a certain form of molecular bond forms between the molecules of the two objects, both of which are similar to the situation inside a single object, resulting in a remarkable shear stress.
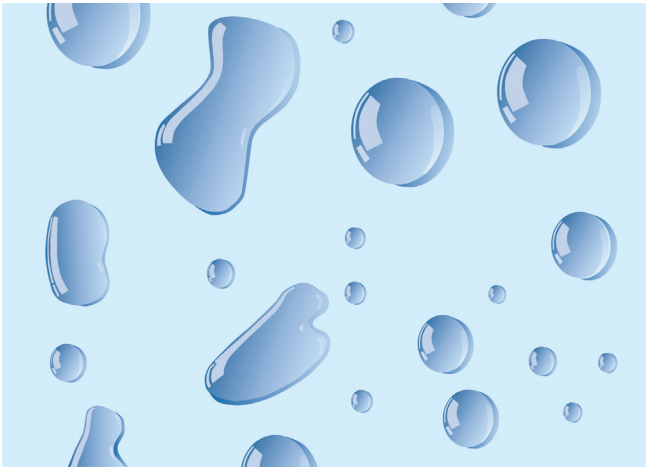
Figure 1.3 Water droplets at rest on a glass pane.

The contact between a liquid and solid interface is somewhat like that of two perfectly flat solid surfaces – that is to say, a full intermolecular contact. However, unlike the static frictional force occurring between two solid surfaces, there is no such force between a liquid and a solid interface. A typical example is a boat floating on water, which can be moved by an arbitrarily small amount of thrust. The liquid molecules adjacent to the solid surface will stick to it through the intermolecular attractive forces, forming a relatively fixed structure. However, these stuck molecules cannot apply a shear force to the adjacent molecules further inside the liquid. As a result, the solid has no way of exerting a shear force on the fluid through static displacement alone.

Even so, we notice water may flow at a uniform velocity along an inclined riverbed, which means there is a frictional force between the water and the inclined ground to balance the component of gravity along the riverbed. That is to say, *a frictional force does indeed occur in a moving fluid*. This property of a fluid is known as *viscosity*, and will be discussed in detail in the next section.

Here's an interesting counterexample: Water can remain stationary on inclines, even on vertical walls or glass. Consider water droplets clinging to window panes, for example, as shown in Figure 1.3. The water droplet is like a type of solid cantilever structure inside which internal shear stress occurs. Consequently, shouldn't there be shear stress in these water droplets?

Conclusion first: In this case there is still no shear stress. To further analyze this situation, we need to take into account the surface tension of a liquid, a concept that will be discussed later.

1.2   Some Properties of Fluids

The definitions of many properties of fluids, including density, pressure, and temperature, are consistent with those that apply to solids. However, liquids possess

other properties as well, such as viscosity and surface tension. Viscosity is an important characteristic of fluids from the viewpoint of fluid mechanics. The surface tension arises due to cohesive interactions between the molecules on the liquid surface.

1.2.1   Viscosity of Fluids

Any shear force externally applied to a fluid, no matter how small, will cause a continuous deformation of that fluid. Viscosity is a property of a fluid that describes the shear stress caused by such a continuous shear deformation. Common liquids and gases are viscous, and only superfluids (such as liquid helium near absolute zero) can be assumed to be inviscid.

Before discussing the viscosity of a liquid, we need to clarify the difference between viscous and adhesive forces: these are in fact completely different concepts. The viscous force is the shear force exerted in a moving fluid, while the adhesive force is the attractive force between molecules of two different materials. It is the adhesive force between the molecules of the glue and those on the solid surface that holds them together, rather than the viscous force discussed here. They cannot be held together by the viscous force since it does not exist in a static fluid. When a solid object moves through a fluid it is subjected to the frictional force on its sides. This force results from the combined action of the adhesive force between the solid surface and its adjacent fluid molecules, and the viscous force within the fluid.

We know that honey has a much higher viscosity than water. Let us discuss the intrinsic factors that determine the magnitude of viscous forces in a fluid by analogy with the frictional force between two solid surfaces. For a block sliding down an incline, its dynamic friction is equal to the product of the coefficient of friction and the normal force between the two solids in contact. The coefficient of friction is a numerical representation of the magnitude of the intermolecular force between neighboring molecules of the solid and the contact surface roughness. This is easy to understand. But why is the dynamic friction proportional to the normal force between the two solids in contact? After all, the normal force is perpendicular to the motion and should not affect the magnitude of the friction force.

The reason is that the greater the normal force, the larger the contact area between the two solid surfaces. Since there is essentially a linear relationship between contact area and normal force, the frictional force is proportional to the normal force. If two solids come into gapless contact with each other at the molecular level, the friction force should be independent of the normal force.

For solids, the coefficient of dynamic friction does not in general equal that of static friction due to different mechanisms behind frictional force. Static friction is a consequence of the balance of forces exerted on a stationary solid, while dynamic friction also involves the conversion of kinetic energy into internal energy. When two adjacent solid surfaces slide relative to each other, not only do forces similar to static friction occur on the contact area, as in Figure 1.2, but also the bonds between the molecules of both objects are constantly broken and reformed, with molecules and molecular groups falling from the objects. Accompanied by the change in molecular vibrational
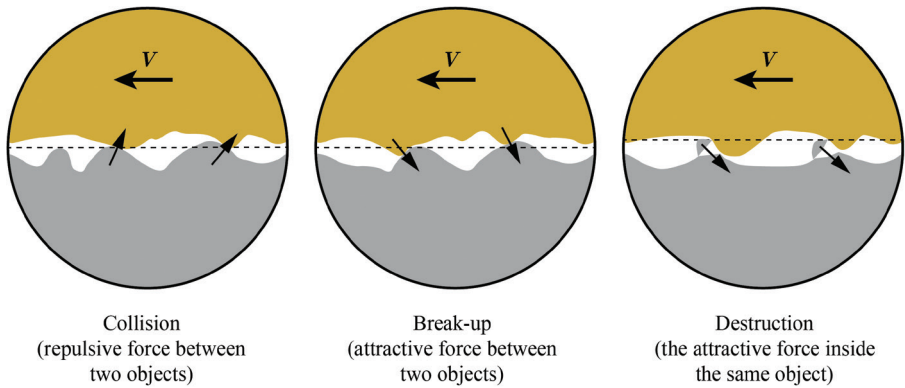
Figure 1.4 Microscopic explanation of the dynamic friction between two solid surfaces.

energy and the directional disorder of molecular motion, friction inevitably produces heat. Figure 1.4 shows three typical phenomena occurring at the contact area.

Whenever a fluid and a solid come into contact with each other, the molecules of the fluid right next to the solid surface will stick to and move with it. Thus, *the friction between a fluid and a solid surface is actually that between two fluid layers*. In other words, the friction between solid and fluid surfaces relates only to properties of fluids, independent of the properties of solid materials.

Since the liquid and the solid surface come into full contact at the molecular level, the frictional force should be independent of the normal force (i.e., the pressure). This explains why the magnitude of the viscous force in a liquid is basically independent of pressure. On the other hand, viscous force is closely related to the temperature of the liquid. *In general, for liquids, the higher the temperature, the lower the viscosity*. For example, a cold, thick syrup becomes less sticky when heated. Contrary to liquids, *the viscosity of gases increases with temperature*. The difference in viscosity between liquids and gases is directly related to the physical mechanism generating it.

If the velocities between two adjacent layers of liquid are different, the slower-moving layer is dragged along by the faster-moving one by intermolecular force. As the momentum of the molecules in the faster-moving layer decreases, that of the molecules in the slower-moving one increases. From a macroscopic point of view, the momentum transfer is caused by the frictional forces between the two fluid layers. Since friction is an internal force when we look at the two layers of liquid as a single system, the total momentum of the system is conserved. Figure 1.5 shows how the slower-moving liquid layer is dragged along by the adjacent faster-moving layer at the molecular scale.

The friction caused by the intermolecular forces in a liquid is somewhat similar to that acting between two solid surfaces in contact with one another. Unlike solid molecules, liquid molecules do not always flow in their respective layers, but instead move freely among different layers. In other words, the liquid molecules in each layer have random diffusion perpendicular to the direction of motion. Some molecules
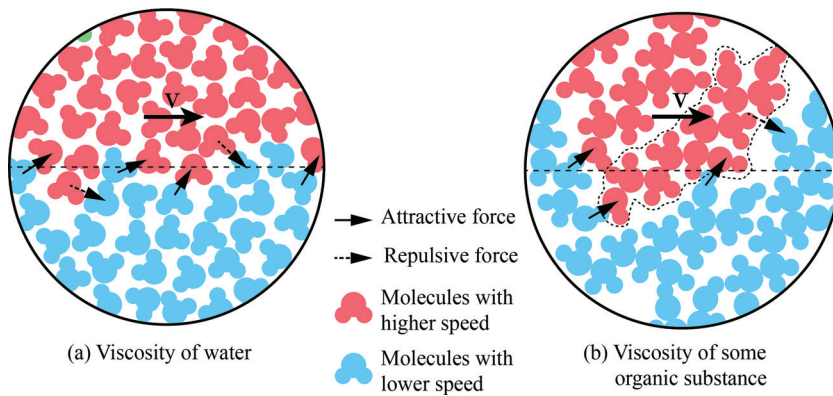
Figure 1.5 Microscopic explanation of the viscous forces in a liquid.

from the faster-moving layer may intrude into the slower-moving one, where after one or several collisions they are decelerated to move along with their neighbors. At the same time, some molecules from the slower-moving layer may intrude into the faster-moving one, where they are accelerated to move along with their neighbors after undergoing one or multiple collisions. The transfer of momentum taking place through the interaction between two adjacent layers is reflected as frictional forces along the direction of motion. For liquid molecules in laminar flow, the frictional forces generated by momentum transfer, being much smaller than their intermolecular attractive and repulsive forces, are usually ignored. Consequently, the viscosity of a liquid is considered to be caused by intermolecular forces, among which the attractive forces dominate.

Daily life experiences show that liquids of organic substances, such as paint, honey, and plasma, generally exhibit high viscosity. The large, irregular, and polar molecules of these substances are difficult to move but get easily tangled with each other. For example, glycerin molecules are not very large, but they are noncircular and polar. Glycerol molecules are more likely to get stuck when they pass over rough surfaces, and the molecules get tangled easily, so glycerol is highly viscous. On the other hand, mercury molecules are larger than glycerol molecules, but they are spherical and nonpolar. Therefore, mercury, though much more viscous than water, is less viscous than liquid organic substances of the same molecular weight. Figure 1.5(b) shows the shearing motion in a liquid organic substance, where the region enclosed by dashed lines represents a large organic molecule. As can be intuitively seen, the molecules of this organic substance, compared to the water molecules in Figure 1.5(a), are large and irregular in shape, inevitably experiencing substantial tangle and therefore having higher viscosity.

As the temperature of a liquid increases, the vibrations of individual molecules intensify and the tangle between them becomes more easily loosened, leading to a decrease in viscosity. This can be explained by analogy to the following example: When carefully positioned on a wet and muddy surface, a cellphone can be kept from
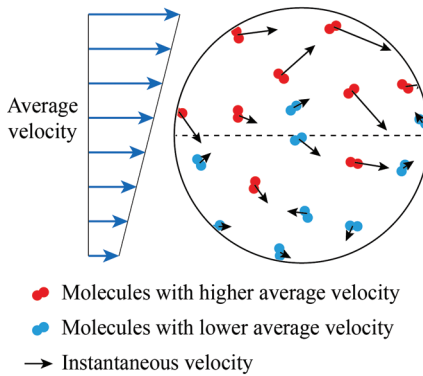
Figure 1.6 Microscopic explanation of the viscous force in a gas.

sinking for quite a long time; if someone calls, the cellphone will vibrate and quickly sink. Generally speaking, the more intense the tangle between molecules, the more sensitive viscosity is to temperature. Syrup, with elongated molecules, shows a rapid reduction in viscosity as temperature rises, but temperature has a much smaller impact on the viscosity of mercury, which has spherical molecules.

However, not all liquids show a reduction in viscosity with increasing temperature. For example, the viscosity of a single-component lubricant decreases rapidly with increases in temperature. This causes huge problems for the lubrication of mechanical parts, such as those in engines used in a wide operating temperature range. To overcome this shortcoming, a substance with large molecules whose shape changes from spherical to elongated as the temperature increases is added to modern lubricants in order to make their viscosity less sensitive to temperature.

Attractive and repulsive forces between gas molecules are negligible. The momentum transfer due to frequent collisions between molecules produces pressure force in a gas. The collisions between molecules can produce not only a normal stress such as pressure, but also a shear stress, such as viscous force. The latter may be considered as a frictional force acting between adjacent fluid layers traveling at different velocities. For instance, the molecules in the faster-moving layer constantly fly into the slower one due to random thermal molecular motion, dragging the slower neighboring molecules to accelerate; the molecules in the slower-moving layer also constantly fly into the faster one, pulling back the faster neighbors to decelerate. From a macroscopic point of view, the viscosity of a gas represents the drag forces caused by the momentum transfer between two adjacent layers of the gas. As remarked earlier, this kind of momentum transfer also exists between two adjacent layers of a liquid, but it is negligible compared to intermolecular forces. In a gas, however, molecules exert almost no forces on each other, but are themselves very active. It is the momentum transfer between the molecules that creates the viscosity of the gas. Figure 1.6 shows the mechanism responsible for gas viscosity. As slower molecules frequently collide with faster ones, the former accelerate while the latter decelerate, thereby resulting in viscosity.
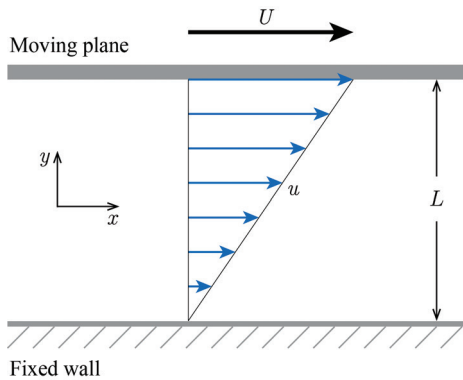
Figure 1.7 Newton's experiment for the viscous force in a liquid.

As the temperature of a gas increases, the thermal motion of the molecules intensifies, and the frequency of their collisions increases, generating more exchange of momentum between the individual gas layers. This is why the viscosity of gases increases with temperature. Just as in liquids, viscosity in gases is essentially pressure-independent. How can this phenomenon be explained?

For an ideal gas, pressure is determined by temperature and density. When the temperature is constant, a change in pressure results in a change in density and vice-versa. Therefore, we reach a conclusion that the viscosity of an ideal gas is independent of its density at constant temperature. It may seem odd that a denser gas does not generate a larger viscous force. When density increases, the collision frequency between molecules must also increase. Why is the viscosity of a gas independent of its density?

As shown in Figure 1.6. If the density of a gas is increased, more molecules in the faster-moving layer indeed fly into the slower layer in unit time. However, due to the decrease in mean free path, the two molecules in collision would have less velocity difference and therefore the momentum transfer caused by a single collision decreases. The percentage of decrease in the momentum transfer due to a single collision is equal to the percentage of increase in density, resulting in a constant momentum transfer. As a result, the change in density does not affect the magnitude of viscous force.

The above analysis, based on kinetic molecular theory, is qualitative. Quantitative derivation of viscosity can be found in some textbooks and will not be discussed here.

Classical mechanics is mainly concerned with the motion of macroscopic objects. Isaac Newton (1642–1727) was the first to attempt a quantitative study of the viscosity of liquids. In 1686, he measured the viscosity of liquids experimentally and established Newton's friction law to describe the frictional force in a fluid.

To make the flow model as simple as possible, Newton attempted an experiment in which the liquid in question was sandwiched between two large parallel horizontal plates, as shown in Figure 1.7. The bottom plate is fixed, and the top plate moves at a steady horizontal speed. When a fluid flows over a solid surface, the layer right next to the surface will attach to it. The fluid attached to the bottom plate remains stationary, while that attached to the top plate moves with a constant velocity of U. Newton's

law of viscosity states that the force required to pull the upper plate is proportional to the velocity with which the top plate moves and inversely proportional to the distance between the plates – that is,

$$\frac{F}{A} \propto \frac{U}{L},$$

where F is the force required to pull the upper plate; A is the area of the contact surface between the fluid and the plate; U is the top plate velocity; and L is the distance between the two plates.

Each successive fluid layer from the top down exerts a force to the right on the one below it. Since the pressure is constant in the fluid, the shear forces between any adjacent fluid layers are the same, and relate to the pulling force to the upper plate. Newton found that the fluid velocity varies linearly with y. He stated that the shear stress between adjacent fluid layers is proportional to the velocity gradient between two layers, written as:

$$\tau = \mu \frac{\partial u}{\partial y}, \tag{1.1}$$

where τ is the shear stress; u is the horizontal velocity in the fluid; μ is a measure of the viscosity of a fluid, known as the coefficient of viscosity, or coefficient of dynamic viscosity. The coefficient of viscosity varies greatly from one fluid to another, while that of the same fluid changes with temperature. There exist tables listing the coefficients of viscosity that have been experimentally determined for various liquids and gases.

The fluids used in Newton's experiments satisfy Equation (1.1). All gases and many common liquids with small viscosity obey Newton's law of viscosity and are accordingly called Newtonian fluids. *A Newtonian fluid is a fluid in which the viscous stresses are linearly correlated to the local strain rate.*

But there are many fluids in nature that do not satisfy this linear relationship; these are called non-Newtonian fluids. Generally speaking, Newtonian fluids are less viscous, while most more viscous fluids, such as paint, honey, and plasma, are non-Newtonian. Unlike Newtonian fluids, most non-Newtonian fluids are composed of molecules or particles that are much larger than water molecules. Since these molecules are tangled up in the flow field where the velocity gradients exist, non-Newtonian fluids exhibit a more complex relationship between shear stress and velocity gradient than simple linearity. Furthermore, some non-Newtonian fluids have a shear stress, or viscosity, that depends not only on velocity gradient but also on time or deformation history.

1.2.2   Surface Tension of Liquids

Surface tension is an amazing force, often used in science shows to draw a crowd, such as through coins floating on water and oversized soap bubbles. Figure 1.8 shows one of the most common surface tension phenomena. Figure 1.8(a) shows an ellipsoidal droplet of water on a plane, and Figure 1.8(b) shows the diagram of forces on water molecules near the droplet's surface. As can be seen, any water molecules inside the
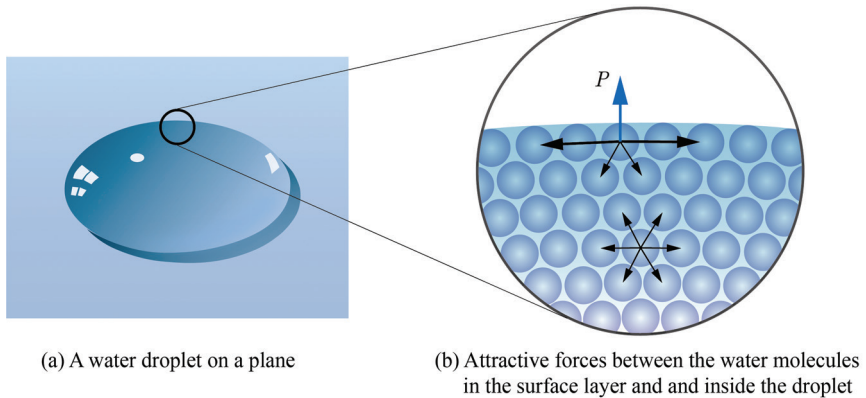
(a) A water droplet on a plane

(b) Attractive forces between the water molecules
in the surface layer and and inside the droplet

Figure 1.8 An ellipsoidal droplet of water and the diagram of forces on water molecules inside it.

droplet do not experience a net force because the weak attractive forces exerted by the neighboring molecules all cancel out. On the other hand, since the water molecules in the surface layer are a bit farther apart than those inside the droplet, they are more attracted to each other and to the layers of water below. They are not pulled upwards, but only from the sides and downwards. Surface tension is viewed as the net force of attraction among the water molecules at the surface.

Surface tension seeks to minimize the surface area, so that all water droplets tend to be pulled into a spherical shape. Due to the phenomenon of surface tension, the pressure inside the droplet will always be higher than the atmospheric pressure on the outside. Therefore, in addition to bonding to each other, the water molecules at the surface are also subjected to a repulsive force exerted by the internal water molecules. The repulsive force P is directed outwards, away from and perpendicular to the surface, as shown in Figure 1.8.

Water is especially suitable for demonstrating surface tension since it has the greatest surface tension of any liquid other than mercury. The reason is that the attractive forces between water molecules include an unusually strong type of hydrogen bond between the hydrogen and oxygen atoms, which is stronger than the other intermolecular forces. Organic solvents have lower surface tension than water. For example, the surface tension coefficient of alcohol is only one-third of that of water, thereby forming much smaller droplets. Some surfactants, such as soaps and detergents, can significantly reduce the surface tension of water, playing an important role in human life and production. There is a common misconception that soapy water has greater surface tension than pure water. But in fact, the soap added to water decreases the pull of surface tension. The principle behind a soap bubble is that a water film does not quite have the same thickness all over. With soap added, where the soap film is thinner, there will be less soap and the surface tension will be higher, and thus the film is stronger; where it is thicker, there will be more soap and lower surface tension, and thus the film would be stretched thinner. In other words, soapy water forms a bubble with more uniformed thickness, allowing the soap bubble to last a longer time.
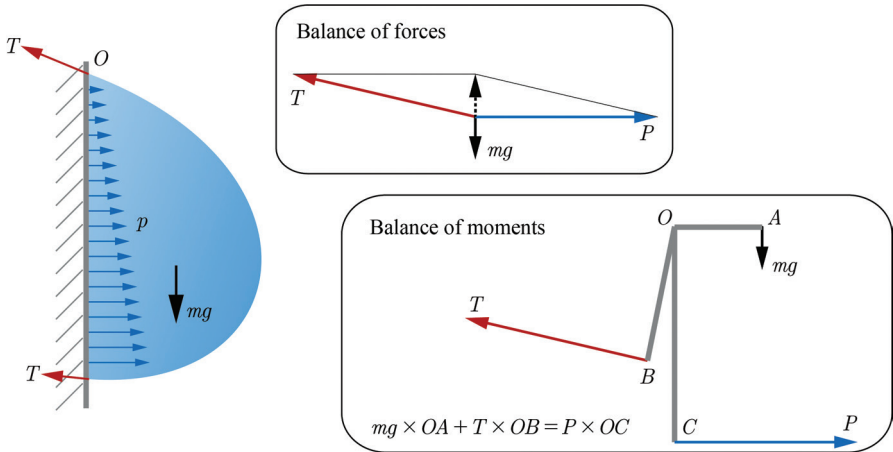
Figure 1.9 Force field analysis on the water droplet positioned on a vertical glass surface.

Take a look at the water droplets in Figure 1.3. Force analysis is carried out as shown in Figure 1.9. The water molecules in the surface layer play a key role in shaping the droplet. There won't be any attractive force, on average, on any single molecule inside the droplet from the surrounding molecules. Simply wrapped by a surface film of water, the droplet can remain at rest on the glass surface. Analogically, if you wrap a piece of cloth around some dry sand and pin all sides of the cloth to the wall, the sand bag will stay at rest. There can be no shear stress between the sand particles as long as the cloth is durable enough and pinned tightly to the wall.

For a water droplet to remain stable on a vertical glass surface, the surface film of water must be shaped as shown in Figure 1.9. Under gravity, the pressure in the lower part of the droplet is greater than that in the upper part. Thus, the curvature at the bottom of the surface film will be greater than that at the top to generate a higher pressure inside the droplet. The total tensile force between the surface film of water and the vertical glass surface, T, must be directed obliquely upward in order to balance the weight of the water droplet without the need for shear stress. The support force exerted upon the water droplet by the glass surface, P, indispensable to achieve balance, is directed horizontally. The most critical reasons why the water droplet can remain attached to the glass surface are the adhesive forces between the surface film of water and the vertical surface, and this surface film holding the droplet together. Through this example we seek to emphasize the highly important fact that *there is no shear force within a static fluid*.

1.2.3   Equation of State for Gases

The equation relating the values of pressure, volume, and temperature of a gas is known as the equation of state. A theoretical gas composed of many randomly moving

point molecules that are not subjected to intermolecular forces is known as an ideal gas. The state of an ideal gas may be expressed by the following formula:

$$p = \rho \frac{R_0}{M} T \,,$$

where p is the pressure of the gas; ρ is its density; T is its absolute temperature; $R_0 = 8.314$ J/(mol·K) is the ideal gas constant; and M is its molar mass (the unit will be expressed in kilograms for use with the International System of Units).

The equation of state for an ideal gas is applicable to most real gases in which the mean free path of a molecule is relatively large at moderate pressure and temperature. The equation of state for air is usually abbreviated as

$$p = \rho RT, \tag{1.2}$$

where $R = 287.06$ J/(kg·K) is the gas constant of air.

Here it is necessary to discuss in depth the three macroscopic physical properties of a gas, namely, density, temperature, and pressure. Further discussion will inevitably involve gas kinetics and statistical physics on a micro-scale. In order to make it clear and concise, the discussion in this sector will be carried out from a qualitative perspective.

First, *the density of a gas is defined as its mass per unit volume*. Strictly speaking, since a gas is composed of molecules in thermal motion that are separated from each other by distances far greater than their own sizes, density cannot be defined at a point in space. The continuum hypothesis, however, considers fluids to be continuous, so the density of a gas at one point in space is actually the average mass of a small volume element at that point divided by its volume, and the element is large enough to contain many molecules. For a particular gas, the magnitude of density represents the number of molecules in a given volume.

Second, *the temperature of a gas is the measure of the average kinetic energy of all the gas molecules*. For a given gas, the magnitude of the average velocity of molecular thermal motion depends directly on temperature. For the gas in a specific confined space (i.e., density is constant), an increase in temperature obviously increases the collision probability between molecules, the number of collisions per second between molecules and walls, and the intensity of single collisions, which will be reflected as an increase in the pressure of the gas.

The pressure of a gas represents a tendency to expand itself, or its thrust against the adjacent solid–liquid interfaces or the vessel walls. The attractive and repulsive forces between gas molecules are negligible, and this expansion and thrust arise from the collisions of some gas molecules with each other and with the vessel walls. The momentum theorem states that the magnitude of pressure depends on the amount of momentum transferred from the gas molecules to the vessel walls per unit time. Two factors influencing the total amount of momentum transfer are the magnitude of the momentum transfer to the wall due to a single collision, and the number of molecules transferring the momentum. The former factor is determined by the temperature of the gas, and the latter depends on both its density and temperature. On the one hand, higher temperature increases the average kinetic energy and therefore the average

velocity of the gas molecules. Thus, the molecules collide with the vessel walls more frequently and with greater momentum transfer in a single collision. On the other hand, higher density increases the number of molecules hitting a unit area of the wall per unit time. Therefore, the pressure of a gas is positively correlated to its density and temperature, reflected as the relation in Equation (1.2).

By the way, unlike gases, there's no unified ideal equation of state for liquids. Liquid density generally decreases with increasing temperature, with different rates of change for different liquids. It is difficult to describe the rates of change in density with a unified theory; these can be affected by many factors and are usually measured by experiments. However, liquid water is an exception to this rule: When its temperature rises from 0°C to 4°C, its density increases instead of decreasing as expected. Water reaches its maximum density at about 4°C. The pressure inside a liquid is independent of its temperature and density, but directly depends on the external forces exerted upon it.

1.2.4   Compressibility of Gases

Compressibility represents the relative change in the volume or density of a certain amount of substance as a response to a pressure change. The molecules in a solid or liquid are so tightly packed together that any attempt to bring them closer will be very hard due to significant intermolecular repulsive forces, so both solids and liquids are very difficult to compress. Consider water, for example. At room temperature (20°C), its density at 1 atm is 998.2 kg/m^3, while at 100 atm it has only a slight increase of less than 0.5% to 1002.7 kg/m^3.

Unlike solids and liquids, gases have larger intermolecular distances and negligible intermolecular forces, making them highly compressible. The equation of state for an ideal gas states that the density of a gas is affected by its pressure and temperature. For an isothermal (constant temperature) compression/expansion of a gas, its density is directly proportional to pressure, while for an isobaric (constant pressure) compression, its density is inversely proportional to temperature.

In fluid mechanics the so-called compressibility refers specifically to the compressibility of flow rather than the compressibility of a fluid. *Compressible flow is defined as a flow in which the volume of a fluid element undergoes a significant change.* Gases can be compressed much more easily than liquids. However, whether a gas would be compressed in a specific flow also depends on the external forces exerted on it. The gas enclosed in a cylinder can be easily compressed with a piston, but most of the time we deal with the flow of gases or air in open space. Being unconfined in all directions, the gas elements can move out of the way in time, thus avoiding being compressed. Air is a compressible fluid that displays as incompressible flow at normal speeds. For example, if a car is moving at a speed of 120 km/h, the air hit by the car is compressed very slightly (with a 0.5% or lower increase in density) and then disperses in all directions.

As we can see, the gas in open space won't be compressed as long as its escape velocity keeps up with the velocity of the moving object compressing the gas, so whether or not the gas is compressed depends on both the compression and escape velocities. In the previous example, if the compression velocity is that of the car, then what is the escape

velocity of the gas? The escape velocity is related to the thermal velocity of gas molecules, macroscopically reflected in the speed of sound in the gas. *In general, when the velocity of a gas flow is below 0.3 times the speed of sound, the relative change in gas density remains negligible and the flow can generally be treated as incompressible.* Of course, this standard can be raised or lowered appropriately depending on the required accuracy. (The relationship between the relative change in gas density, the velocity of the gas flow, and the speed of sound in the gas will be discussed in detail in Chapter 7.)

The compression caused by air deceleration can also be understood as the effect of the inertial force of the gas behind on the gas in front during deceleration. The inertial force arises from a change in the magnitude or direction of the velocity of the gas flow. In a high-speed, rotating vessel, the pressure of the gas varies along the radial direction, correspondingly creating a radial density gradient. Even if the gas flows slowly from the center to the periphery of the vessel, there will be a significant change in its density. This type of compressible flow is caused by the effect of the centrifugal force.

Gravity can and does affect density, such as the apparent variations of atmospheric density with altitude. The gravitational effect is generally neglected when the height difference is small. For example, in some problems the air density at the top of a 10-story building is generally assumed to be the same as that at ground level, while there is in fact a slight difference of a few thousandths between them. For the gas rising from the ground to a cloud, the density change by gravity must be taken into account. The effect of temperature on the density of gases is also not negligible. If a gas element conducts heat to its environment while flowing, its temperature decreases. Then, the gas is compressed, and work is done on it. If a gas element receives heat from its environment while flowing, its temperature increases. In that case, the gas expands and does work on its environment.

If flow is incompressible, there is no volume work, then temperature has a negligible effect on the fluid flow. As a consequence, the flow problem can be significantly simplified. The incompressible flow model can be used to calculate the slow flow of a gas with small temperature gradient and weak centrifugal force, which is known as incompressible flow theory. Incompressible flow has been the most extensively studied and widely used flow theory in fluid mechanics.

1.2.5   Thermal Conductivity of Gases

Thermal conductivity is the property of a material that describes the ability to conduct heat from hot to cold places. The flow of heat through a unit area per unit time satisfies Fourier's law of heat conduction, that is:

$$\dot{q} = -\lambda \frac{\partial T}{\partial n}, \tag{1.3}$$

where $\dot{q}$ is the conductive heat flux; λ is the coefficient of thermal conductivity; and n is the direction of the temperature gradient.

The mechanisms of heat conduction in gases and solids are quite different. In solids, the molecules, on average, vibrate but stay in fixed positions, so the transfer of

molecular kinetic energy is due to their vibrations (or electron flow in some substances). In gases, heat conduction also depends on diffusion of the molecules on account of thermal motion. Heat conduction caused by molecular diffusion still exists even if the macroscopic velocity of the gas is zero; thus, molecular diffusion is different from the mixing in a flow. The heat transfer by the macroscopic mixing of gases is known as convective heat transfer. Although facilitated by the molecular diffusion effect, the thermal conductivity of gases is still much smaller than that of solids and liquids due to the large distances between molecules, so that it can usually be ignored.

1.3   The Concept of Continuum

Matter is composed of elementary particles, and the concepts of molecules and atoms have been extensively used in the previous analysis. In classical solid and fluid mechanics it is not necessary to study the behavior of individual molecules or atoms. Even for gases composed of molecules that are separated from each other by large distances, macroscopic properties, such as pressure and temperature, can be used to describe their microscopic mechanical behavior. Therefore, *in classical mechanics, a matter is always considered to be continuously separable, which is known as the continuum hypothesis.* Both solid mechanics and fluid mechanics are branches of continuum mechanics.

When describing the properties of a fluid at a certain point, it actually refers to a small region of the fluid. The region should be small enough to be approximated as a point on the macro-scale, but still be large enough to contain a large number of molecules on the micro-scale, so that the macroscopic quantities of interest are not affected by the microscopic randomness of the molecular thermal motion.

The only criterion in determining whether the continuum hypothesis is applicable is the relationship between the length scale of the flow and the molecular size (solids and liquids) or mean free path (gases). For gases at room temperature and pressure, the molecular mean free path is of the order of 7×10^{-5} mm. As long as the investigated object is far larger than the molecular mean free path, the continuum hypothesis holds. Common flows satisfy the continuum hypothesis. However, tiny pollen grains suspended in still water move randomly due to the thermal motions of water molecules colliding with them. This so-called Brownian motion does not satisfy the continuum hypothesis. Furthermore, a rocket or spacecraft moving at an altitude of 120 km does not satisfy the continuum hypothesis either, because the local mean free path of air molecules becomes as large as 0.3 m.

1.4   Forces in a Fluid

In fluid mechanics, the forces acting on an object are traditionally classified into two main categories according to their type of action: (1) a force exerted on the fluid
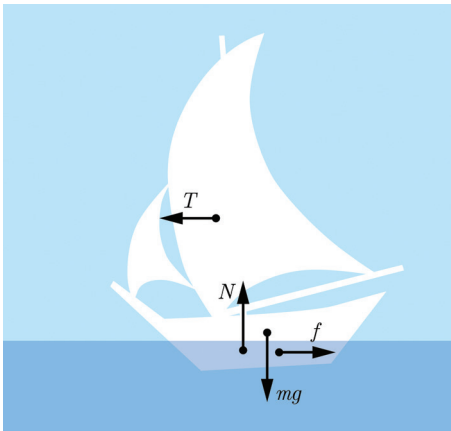
Figure 1.10 Forces exerted upon a sailboat.

without touching it, and therefore acting uniformly on the whole body of a fluid parcel, is called a body force; (2) a force appreciable only if fluid elements are in contact, and therefore existing only on the surface, is called a surface force.

Both gravitational force (universal gravitation) and magnetic force are body forces, and so is inertia force in non-inertial coordinate systems. Both pressure and viscous forces are surface forces. Pressure, as a normal stress, always acts inward and normal to any surface in a fluid. Shear stress inside a fluid is completely generated by viscous force, which also contributes to the normal stress. In most cases, the viscous normal stress is negligible compared to the pressure, so it is generally considered that viscosity only makes the fluid experience shear stress.

The pressure at any point in a stationary fluid or in a moving inviscid fluid is independent of direction. This property makes pressure a scalar quantity that can be regarded as a state parameter for fluids. Pressure is direction-independent because it is the only surface force in a stationary or moving inviscid fluid. Body forces (gravitational and inertial) at a point within a fluid tend to zero, and surface forces from all directions have to be identical in order to cancel out.

Figure 1.10 shows all the forces exerted upon a sailboat, among which the wind gives the sailboat a thrust of T, and f is the drag exerted mostly by the water. Because the sailboat is floating in static equilibrium, the magnitude of the buoyant force, N, must be equal to the magnitude of the weight of the boat, mg. Among these forces, gravity is the only body force, while buoyancy, thrust, and drag are the resultant forces on surfaces. A sudden increase in wind strength increases the thrust, which is not balanced by the drag, resulting in the forward acceleration of the sailboat. Then, the thrust needs to overcome both the drag and the inertia of the vessel. Taking the moving sailboat as a reference, the inertia is reflected as the inertial force, which is a body force.

Expanded Knowledge

States of Matter

In fluid mechanics a fluid is distinguished from a solid by its mechanical properties, while in general physics books the differences between solid, liquid, and gas are due to the arrangement of molecules or atoms. In fact, in modern scientific classification there are more than three states of matter, and the differences between them are not entirely clear. Solids can also be divided into two categories: crystalline and amorphous solids. Crystalline solids, such as ice, salt, and solid metals, have an ordered long-range arrangement of atoms or molecules within the structure. Amorphous solids, such as glass, rosin, and plastic, lack such an arrangement. Due to the similar configuration of atoms or molecules as a liquid, an amorphous solid is sometimes said to be a liquid with fairly low fluidity. That is to say, amorphous solids have the physical properties of liquids. Started in 1930, the tar drop experiment in Australia is one of the longest continuously running laboratory experiments to measure the flow of a piece of tar over many years. A "solid" piece of tar placed in a funnel at room temperature flows at a very low rate, taking several years to form a single drop. The experiment is conducted by more than one scientific organization, and interested readers can find relevant information online. This experiment shows that tar is actually a liquid rather than a solid, although it looks like one. What about glass, which looks more like a solid? Some claim that glass is also a liquid, and point to the fact that the glass in medieval church windows is thicker at the bottom than at the top, suggesting that the solid glass has flowed. However, it is not known what state the glass was in when the windows were installed, and most scientists agree that glass should be a solid.

Compressibility of Water

Since water is more resistant to compression, it is usually treated as an incompressible fluid. However, there are at least two cases in which the compressibility of water needs to be considered. One is the presence of extremely high-pressure conditions. For example, the water pressure in a hydraulic mining system that adopts the impact force of a high-pressure waterjet is up to 4,000 atm, and the corresponding water density is 15% higher than that at standard atmospheric pressure. The other case is when the problem being investigated is very sensitive to the change in water density, such as the calculation for the speed of sound in water. Sound is a wave consisting of alternating cycles of compression and expansion. If water were totally incompressible, the theoretical value of the speed of sound in water would be infinite. In reality, the measured speed of sound in water at room temperature and pressure is about 1,500 m/s. The magnitude of compression and expansion caused by the sound wave traveling through the water is very small. Such a weak compressibility is sufficient to allow the sound waves carrying very little energy to travel through the water.

Compressibility of Solids

The compressibility of solids is a problem rarely encountered. Compressibility is not very important for solids. The primary reason is, of course, that solids are just as significantly difficult to compress as liquids. Besides this, since a stationary solid can experience shear stresses, it requires no volume change to resist external pressures acting on it. For example, if a cylindrical object is compressed along its axis, it gets thicker and shorter, but its volume remains essentially the same. Such a stress state is commonly encountered in the study of solid mechanics. The interaction between a solid and the external forces acting on it is usually little affected by the change in its volume, but mainly related to the elastic forces arising from any combination of fundamental load types: tension, compression, shear, bending, and torsion. When the external forces significantly change the volume of the solid, the working stress reaches approximately the limit that causes the material to yield or fail. An exception occurs when a solid is under uniform external pressure. For example, it is necessary to take into account the compressibility of the rocks on the ocean floor, or deep inside Earth's crust, that are subjected to pressure coming from all directions.

Questions

1.1 You put a bottle of water in outer space, and the bottle itself suddenly disappears. Imagine the variations of the water and its final state. What happens if you use air instead of water in this experiment?

1.2 Look up the coefficient of viscosity for air, and use Newton's law of viscosity to discuss why air viscosity is generally not perceived in daily life.

1.3 What happens if you fill a glass bottle with water, tighten the lid, and let it sink to a depth of 5,000 meters under the surface of the sea? For an open glass bottle filled with seawater at a depth of 5,000 meters, what happens if you tighten the lid and bring it to the surface?