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ARTICLE

# IMF lending in sovereign default

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**Abstract**

This paper proposes that an International Monetary Fund (IMF) policy shift was the reason behind major changes in sovereign debt negotiation outcomes observed in 1989. The new policy, in marked departure from past policy, allowed the IMF to lend to nations in default. The paper highlights the stark improvements in debt forgiveness and post-negotiation debt servicing ability coincident with the IMF policy shift. A theoretical framework is proposed in which the IMF policy shift causes the observed changes in negotiation outcomes. The model highlights the policy's potential to improve a country's outside option during negotiations of defaulted debt. In the model, this improvement leads to increased debt forgiveness which in turn leads to less post-negotiation debt servicing difficulties. The model is then used to address an important question regarding the nature of post-negotiation default risk. The case is made that countries face persistent, rather than temporary, default risk after such negotiations. To avert such risk, they moderate their borrowing.

**Keywords:** IMF lending policy; lending into arrears; haircuts; repeat default

## 1. Introduction

The European debt crisis underscored the importance of refining our understanding of the role of international financial institutions, such as the International Monetary Fund (IMF), in sovereign debt markets. This paper focuses on a particular role of the IMF, that of lending to defaulted nations. In 1989, the IMF changed its lending policy toward countries in default.[1] The new policy allowed the IMF to lend to nations in default. This was in marked departure from past policy which prevented the IMF from extending loans to such nations. This paper studies the impact of this policy shift on both sovereign debt negotiations and the aftermath of such negotiations.

In the 1980s, many emerging markets faced severe debt servicing difficulties. Undercapitalization of US banks, the main emerging market creditors at the time, meant that negotiations came soon after defaults as banks tried to avoid late payments from appearing in their books. By the end of the 1980s, banks had built capital, reducing their urgency to negotiate. The IMF became concerned with this diminished urgency to negotiate coupled with the bargaining power granted to banks by its policy of not lending to countries in default. In 1989, the IMF changed its long-standing policy of not lending and started lending to nations in default.[2] Díaz-Cassou et al. (2008) provide case studies where the IMF program of lending to countries in default has been applied. These include Argentina's 2001 and Russia's 1998 defaults.[3]

This paper relates major shifts in sovereign debt negotiation outcomes to the IMF policy shift. Emphasis is placed on two observed shifts: an increase in average debt forgiveness and a decrease in the fraction of countries with post-negotiation debt servicing difficulties. Specifically, countries after 1989 repay on average 50 cents on each dollar of defaulted debt compared to 75 cents before 1989. The fraction of countries with post-negotiation debt servicing difficulties fell from 98% before 1989 to 51% after.[4]

The main contribution of the paper is a theoretical framework to analyze the impact of the policy shift on sovereign debt negotiations and the aftermath of these negotiations. The model proposed here emphasizes the policy's potential to improve the negotiating countries' outside option. This improvement leads to increased debt forgiveness which in turn leads to less post-negotiation debt servicing difficulties. This theoretical framework also produces testable implications regarding the persistence of post-negotiation default risk. In particular, specifications with both short lived and persistent default risk are considered. The two specifications produce distinct implications on the impact of the policy shift on the post-negotiation cost of borrowing. The persistent default risk framework is most consistent with the data, as shown in the last section.

The model concerns a country that has already defaulted on its debts and is currently negotiating with creditors. The model captures several salient features of sovereign debt renegotiation in the data. First, during this negotiation period, the country has no access to private international financial markets (i.e., it is excluded). This exclusion is costly as the sovereign is assumed to have a borrowing need (in the form of a recession[5]).

Second, in the model, direct costs of default (in addition to financial exclusion) vary stochastically and generate post-negotiation default risk.[6] These post-negotiation default costs are key to determining the value of bonds and the dynamics of debt after a successful renegotiation.[7] As a result, after a successful negotiation with creditors, the country emerges with a new outstanding stock of debt. However, such a stock of debt is not necessarily risk free.

Third, creditors make in unison a take-it-or-leave-it offer. This simplifying assumption is utilized for three reasons. First, it emphasizes the effect of the policy shift on countries' rather that lenders' outside option. Second, it prevents delays that may arise due to the holdout problem studied in Pitchford and Wright (2012). Third, it allows for tractable exposition of the effect of the policy shift on the outside option of the sovereign. This negotiation structure is studied in Fernandez and Rosenthal (1990) and is implicitly or explicitly utilized frequently in the literature.[8]

The final ingredient is the specification of IMF policy. In the model, as was the case in reality, the IMF does not lend to defaulted countries with the pre-1989 policy. In contrast, the post-1989 policy is modeled as loans offered by the IMF to defaulted nations. The IMF is assumed to break even in expectation on such loans. On the equilibrium path studied here, the sovereign does not borrow from the IMF. However, off-equilibrium IMF borrowing and potential default on IMF debt shape negotiation outcomes.[9]

The second component of the post-1989 policy is the cost to the country of participating in the IMF program. In real-world applications of its lending policy, the IMF imposes time constraints on the availability of its funds as well as economic targets that have to be met by the country. Further, it requires that countries remain in good terms with their creditors by actively pursuing a successful negotiation. These costs are captured in the model in two salient assumptions. First, during the IMF program the country incurs a per period cost.[10] Second, to terminate the IMF program the country must either successfully negotiate with its creditors or default on the IMF.

The first result from the model is that the policy shift leads to increased debt forgiveness. The sovereign's outside option with the pre-1989 policy is autarky, a daunting outcome due to the recession. The post-1989 IMF program has the potential to improve on autarky by allowing the sovereign to smooth the impact of the recession using IMF funds. As a result, the country's outside option improves resulting in increased debt forgiveness.

The second result from the model is that increased debt forgiveness lowers the post-negotiation default likelihood. To see this, suppose negotiations have concluded and the sovereign has to repay creditors. Immediate repayment would produce a lumpy spending stream. Instead the sovereign would prefer to repay in part by borrowing from financial markets. The more the country borrows, the greater the default risk. It turns out that the less the sovereign has to compensate creditors, the less it has to borrow. Since the 1989 policy resulted to increased debt forgiveness, the likelihood of a post-negotiation default decreases.

The model is then used to shed light on the persistence of post-negotiation default risk. That is, given the increase in debt forgiveness, is short-run or long-run post-negotiation default risk most impacted? To get testable implications from the model, two specifications of default risk are considered. The first specification, termed transient default risk, generates only short-run default risk. The decrease in default risk resulting from the policy shift reduces the cost of borrowing since default risk and prices are tightly connected in the short run.

The second specification, termed persistent default risk, builds on Aguiar et al. (2019) and features long-lived default risk. The persistence of default risk makes borrowing costly. To escape this, the country finds it optimal to reduce its debt sufficiently to eliminate the default risk. Debt reduction is achieved in multiple periods with the sovereign balancing its preference for smooth spending with its need to escape the default risk[11]. Default risk is present for the duration of debt reduction. This specification also yields a reduction in default risk as a result of the policy shift. However, this reduction in default risk is achieved by shortening the duration of debt reduction. Therefore, long-term risk falls while short-term risk remains unchanged. This implies that the borrowing cost remains unchanged.

The final section investigates empirically the shift in debt forgiveness as well as the implications on persistence of post-negotiation borrowing costs. The increase in haircuts is not statistically explained by alternative theories such as the emergence of delays in negotiation of defaulted debts, changes in indebtedness and other economic indicators at the time of negotiations, and effects of geography. Finally, it is shown that it is impossible to reject the hypothesis that the post-negotiation borrowing cost, as quantified by post-negotiation "exit" yields, remains unchanged at the time of the policy shift. This last finding combined with additional empirical observations is taken as evidence for the persistent default risk specification described above.

The rest of the paper is structured as follows. Section 2 places the paper in the literature. Section 3 documents the two empirical regularities, namely larger haircuts and less debt servicing difficulties after negotiations concurrent with the policy shift. Section 4 introduces the model with both policies nested in the countries ability to borrow in default. Section 5 presents the equilibrium outcome in the transient and persistent default risk specifications. Section 6 presents results from a numerical exercise that can be found in Appendix C. Section 7 provides empirical support for the main results in the paper. Finally, Section 8 concludes.

## 2. Literature review

Cruces and Trebesch (2013) provide the most comprehensive database of haircuts and document the increase in haircuts after the 1980s. Their explanation for the increase in haircuts follows from the observation that in the 1980s the overwhelming majority of sovereign debt renegotiations did not result in face value reductions (only maturity extensions). They argue that this could explain the lower haircuts observed in the 1980s. However, they do not investigate the root cause of the absence of face value reductions in the 1980s.

Recent quantitative work on face value reduction versus maturity extension investigates the root cause of the absence of face value reductions in the 1980s. For instance, Dvorkin et al. (2021) propose a simple theory for the absence of face value haircuts in the 1980s. They argue that banks (the predominant lender of the 1980s) were averse to "book value haircuts." They model this as an ad hoc cost to face value haircuts. Not surprisingly, they find that the average face value haircut declines the larger this cost is. More interestingly, they also find that haircuts as measured in Cruces and Trebesch (2013) do not change significantly when this cost declines. Therefore, the high cost of "book value haircuts" faced by banks offers an explanation as to why face value reduction was absent from the 1980s. However, this explanation suggests that haircuts, as measured in Cruces and Trebesch (2013), would remain largely unchanged.

The IMF policy shift has also been studied in Wells (1993) with a different focus. She applies the bargaining framework of Admati and Perry (1987) to sovereign debt negotiations. The emphasis is

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on the impact of the IMF policy shift on sovereign debt negotiation delays. In contrast to Wells, I focus on sovereigns' ability to borrow. The assumption of risk neutrality of the sovereign is relaxed and limited commitment concerns are explicitly modeled. Relaxing these assumptions allows me to jointly study of haircuts and post-negotiation debt servicing difficulties.

The emergence of negotiation delays was one of the most pronounced changes coinciding with the policy shift. The work of Pitchford and Wright (2012) provides insight as to why these delays may have arisen in 1989. They introduce a model of holdouts and show that more creditors with greater difficulty in coordinating lead to protracted negotiations. On this theme, a major change that took place in 1989 was the shift from syndicated bank loans to bonds. Syndicated bank loans were owed to a small group of banks with little difficulty in coordinating. Meanwhile, bonds are purchased by a large number of investors a number of which may be particularly unwilling to cooperate in case of default. Therefore, the shift from syndicated bank loans to bonds could explain the emergence of delays. To address this, in Section 7, regression analysis uses the years from the beginning of the default until the negotiation as a regressor and finds that this shift does not explain the 1989 increase in debt forgiveness.

The 1989 policy shift was met with skepticism because of its potential to increase countries' borrowing cost. This concern has since had empirical support in the work of Cruces and Trebesch (2013). They empirically document that more debt forgiveness is associated with higher post-negotiation borrowing cost. If this relationship is taken to be causal, the increase in debt forgiveness observed in 1989 should increase post-negotiation borrowing cost. However, Cruces and Trebesch (2013) study the period from 1993 to 2010, that is, they exclude the pre-1989 years.[12] In Section 7, these years are included. The empirical analysis suggests that the hypothesis that post-negotiation borrowing cost (captured as "exit yields" computed in Cruces and Trebesch (2013)) remains unchanged pre- and post-1989 cannot be rejected. For this reason, the theoretical analysis abstracts from the impact of the policy shift on borrowing costs.

IMF roles other than lending in default have been studied extensively in the sovereign debt literature. Müller et al. (2015) study the role of IMF fiscal austerity programs in the context of a country suffering from debt overhang. In their framework, fiscal austerity acts as a commitment device allowing the sovereign to make costly reforms by reducing the cost of borrowing. Boz (2011) juxtaposes fiscal austerity to IMF lending above market prices. Her framework rationalizes sporadic IMF lending. Superior IMF default punishment technology allows for cheaper borrowing at the expense of ad hoc costly reforms. In contrast to these two frameworks, this paper emphasizes resource penalties of being in an IMF program, that is, interest rate payment on defaulted debts. IMF lending has been studied in a quantitative model of sovereign default in Boz (2011) and Fink and Scholl (2016). Boz (2011) assumes the IMF lends in arrears, whereas Fink and Scholl (2016) assume it does not. However, these papers do not study the policy shift discussed here. Further, neither of these papers model renegotiation of sovereign debt which is the subject of study here.

The link between IMF policy and haircuts has not been investigated in the quantitative sovereign debt renegotiation literature. The seminal paper of Yue (2010) introduces renegotiation to Arellano (2008) to study the cyclical properties of renegotiation. Benjamin and Wright (2009) take this paradigm further by introducing a natural channel for renegotiation delays. In particular, waiting for the sovereign's economy to recover leads to delays in renegotiation. Finally, Asonuma and Joo (2020) highlight the role of the lenders' economic conditions during renegotiations. They find that when lenders' economy is booming they are less inclined to strike a deal with the sovereign, thereby leading to longer delays and smaller haircuts. In this paper, I introduce IMF policy as an additional factor in determining sovereign debt haircuts.

## 3. Negotiation outcomes from 1970 to 2010

Two stark shifts in sovereign debt negotiation outcomes coincided with the 1989 IMF policy shift. These were vast improvements in debt forgiveness and post-negotiation debt servicing ability.

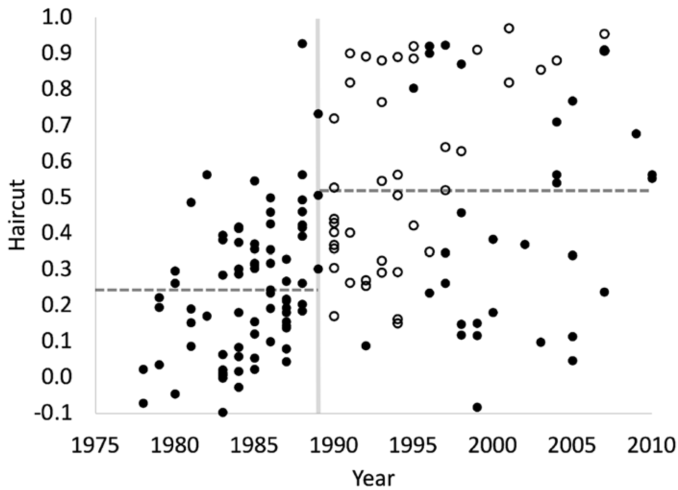


**Figure 1.** Each dot summarizes the outcome of a negotiation between creditors and countries. The summary measure used is the fraction of negotiated debt forgiven, that is, the haircut. The two dashed lines mark the average of this measure for the periods before and after 1989. The hollow dots are negotiations in which the initial default took place before 1989. The vertical faded line marks 1989, the year of the IMF policy change.

This section introduces these regularities by comparing the average value of debt forgiveness and post-negotiation debt servicing ability before and after 1989.

A haircut is defined as the fraction of negotiated debt forgiven. The measure of haircuts used here is the one introduced by Sturzenegger and Zettelmeyer (2008) and measured for the universe of negotiations since 1970 by Cruces and Trebesch (2013). The measure's novelty is that it takes into account forgiveness both in the form of face value forgiveness and loss in value due to rescheduling.

Figure 1 plots on the $x$-axis the year in which a negotiation took place and on the $y$-axis the resulting haircut.[13] The average level of debt forgiveness increases abruptly in 1989 and remains high for the duration of the sample. The dotted horizontal lines mark the average haircut before and after 1989, and the faded vertical line marks 1989. The average increases drastically from around 0.25 to around 0.5. That is, for every dollar of negotiated debt on average 25 cents were forgiven before 1989 and 50 cents were forgiven after 1989.

The increase in haircut variance in 1989 could pose problems in the form of sample selection. To emphasize the limited role of sample selection, the hollow dots mark negotiations for which the initial default took place before 1989. These countries had multiple unsuccessful negotiations in the 1980s and concluded their default episode in the 1990s. For this subsample haircut variance increases in line with the overall post-1989 variance increase. This suggests that sample selection is not a major concern.

Next I restrict the sample of Figure 1 to better reflect certain aspects of this paper's theory. In particular, the theory proposed in this paper relies on the country having access to IMF lending. Further, it is assumed in the theory that the country begins with defaulted debt, that is, the theory does not allow for renegotiations that take place even though no debt payments have been missed. To account for these issues, I restrict the sample by focusing on renegotiations in which the country was in debt to the IMF during the renegotiation. Further, renegotiations are restricted to post-default renegotiations, that is, ones in which renegotiation takes place after debt payments have been missed as discussed in Asonuma and Trebesch (2016). Figure 4 in Appendix D depicts the haircuts for this restricted sample. This figure is similar qualitatively to Figure 1 and the increase in the average haircut in 1989 is almost identical to the one in the unrestricted sample.[14]

Therefore, restricting the sample I can again conclude that haircuts increased in 1989 by about 25 cents for every dollar.

A country is defined as having post-negotiation debt servicing difficulties if within 5 years of negotiating it defaults again. The Standard & Poor's (S&P) definition of default is used.[15] Before the policy shift, 97.5% of negotiations were succeeded by debt servicing difficulties. This statistic fell drastically after 1989 with 45.4% of negotiations being succeeded by debt servicing difficulties.

The unconditional averages of the two variables suggest that 1989 was a significant year for sovereign debt negotiation outcomes. However, the IMF policy shift was not the only significant change in sovereign debt negotiation to take place in 1989. There were drastic changes including a shift from bank loans to bonds and emergence of negotiation delays. Section 7 and Appendix E provide further support for the impact of the policy shift on these two variables by controlling for concurrent major shifts in sovereign debt markets. The theoretical framework introduced in Section 4 and analyzed in Section 5 guides the empirical analysis of Section 7 and provides additional insights that are further investigated in empirical Section 7.

## 4. The environment

Time periods are discrete and denoted $t \in \{0, 1, 2, \ldots\}$. Every period there is a single consumption good. Each period the economy receives a strictly positive endowment of the consumption good, $y_t$. This endowment evolves deterministically and takes two values $\{y_\ell, y_h\}$ with $y_h$ strictly larger than $y_\ell$. Period zero endowment is $y_\ell$, while in all other periods the endowment is $y_h$.

Households in the economy value the stream of consumption goods $\{c_t\}_{t=0}^{\infty}$ using a utility function, given by:

$$U\left(\{c_t\}_{t=0}^{\infty}\right) = \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where the period utility function, $u$, is assumed to be increasing in consumption, strictly concave, and satisfies the Inada conditions, and discount factor $\beta \in (0, 1)$. The sovereign makes consumption decisions for households and is assumed to be benevolent.

### *4.1. Overview of state variables*

The state space consists of maturing debt, $B_M$, arrears, $B_A$, IMF program status, $p_{-1}$, cost of default, $\alpha$, and output, $y$. Collecting these variables, the state space is $(B_M, B_A, p_{-1}, \alpha, y)$. For brevity, I further collect $(p_{-1}, \alpha, y)$ into vector $\mathbf{s}$. Maturing debt incorporates both IMF debt and private debt by making some simplifying assumptions. In particular, I assume IMF and private lending are perfectly substitutable when a country is not in default, that is, when $B_A = 0$. Further, I assume private lenders do not lend to a country in default, that is, when $B_A > 0$. Therefore, if the country is in default maturing debt is owed to the IMF whereas if the country is not in default maturing debt is owed to private creditors. This allows me to combine private debt and IMF debt into maturing debt $B_M$.

Contrary to standard sovereign default models, for example, Arellano (2008), the state space does not feature a variable indicating whether the country is in default. This variable is circumvented by assuming that the presence of arrears determines whether a country is in default. A country that starts without arrears accumulates arrears if it defaults on private lenders. While in arrears, policy permitting, the country may borrow from the IMF and potentially also default on it. If the country defaults on the IMF, IMF debt is added to existing arrears. During renegotiation, all arrears holders negotiate in unison with the country. IMF debt is assumed to be equally senior to private debt. As a result, arrears holders' identity does not matter which is why arrears are captured in one variable $B_A$. In Section 5, the economy begins with arrears. I assume the arrears

**Inherited states:** maturing debt, $B_M$, defaulted debt, $B_A$, and $\mathbf{s} = (p_{-1}, \alpha, y)$

$p_{-1}$: binary, 1 if there is an active IMF program
$\alpha$ : current period cost of default
$y$ : current period endowment

**Timing:**

– Sovereign chooses whether to *Repay* or *Default*

*Default*

Current period borrowing exclusion and default cost, $\alpha$

Maturing debt added to existing arrears, $B'_A = B_A + B_M$

IMF program terminated, $p = 0$

*Repay*

– Negotiation: new level of maturing debt, $B^n_M$, and arrears, $B^n_A$

– Sovereign:

repays maturing debt, $B^n_M$, and interest on arrears when in IMF program, $r^A B^n_A$

borrows, if possible, from financial markets or IMF, $qB'_M$

**Figure 2.** Within period timing of events.

holders are private creditors to match the fact that IMF arrears tend to be very small[16] though as discussed above the identity of the arrears holders does not matter in the benchmark model.

### 4.2. Borrowing, defaulting, and negotiating

Figure 2 illustrates the timing within each period. In the beginning of a given period, the country owes $B_M$ in maturing debt and $B_A$ in defaulted debt. Further, the country may have an active IMF program, $p_{-1} = 1$, or not, $p_{-1} = 0$. The sovereign first chooses whether to *default* or *repay* maturing debt. A *default* results to default cost $\alpha$, maturing debt added to defaulted debt, $B'_A = B_A + B_M$, and IMF program termination, $p = 0$.

If the country *repays* maturing debt, defaulted debt negotiation commences. Negotiation outcomes are negotiated maturing debt, $B^n_M$, that has to be repaid in the current period, and negotiated defaulted debt, $B^n_A$. An active IMF program mandates interest payment on defaulted debt, $r^A B^n_A$. With no defaulted debts, the sovereign can borrow from financial markets. Otherwise, IMF policy permitting, the sovereign is granted access to IMF lending. The IMF program is initiated by borrowing from the IMF and is terminated after a successful negotiation or a default. The stages of defaulting, negotiating, and borrowing are introduced in detail in this section.

**Repay/Default decision.** Each period the sovereign starts by choosing between *Default* and *Repay*. The sovereign's problem is

$$V^s(B_M, B_A, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ d \left[ u(y) - \alpha + \beta \mathbb{E}_{\alpha'|\alpha} V^s(0, B_M + B_A, \mathbf{s}'|_{p=0}) \right] + \right.$$
$$\left. + (1-d)\, V^b\Big( \big(B^n_M, B^n_A\big)(B_M, B_A, \mathbf{s}), \mathbf{s} \Big) \right\}. \tag{1}$$

The period begins with maturing debt, $B_M$, defaulted debt, $B_A$, and variable $\mathbf{s}$ as state variables. Variable $\mathbf{s}$ is comprised of a binary variable specifying whether the country has an active IMF program, $p_{-1}$, current period default cost, $\alpha$, and country's endowment, $y$. Maturing debt is owed

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to the IMF when the IMF program is active, $p_{-1} = 1$ or to financial markets otherwise. The default cost is the same for a default on IMF or financial market debt.[17]

If the sovereign chooses to *Default*, $d = 1$, it incurs default cost $\alpha$, ceases making payments to creditors, and is excluded from borrowing. Current period payoff is then $u(y) - \alpha$. In the subsequent period, the sovereign has to decide again whether to *Repay* or *Default*, $V^s(0, B_M + B_A, \mathbf{s}'|_{p=0})$. Since the sovereign is excluded from borrowing, maturing debt in the subsequent period is zero. Current period maturing debt is added to existing arrears, that is, $B'_A = B_M + B_A$. By defaulting the country dissolves an existing IMF program, that is, $p = 0$. Future payoff is discounted at rate $\beta$ and expectations are formed over next period's default cost $\alpha'$ using the conditional expectations operator $\mathbb{E}_{\alpha'|\alpha}$. The two stochastic processes for $\alpha$ considered in this paper are discussed in detail in Section 5.

If the sovereign chooses to *Repay*, $d = 0$, negotiation ensues. The sovereign takes negotiation outcome, $(B_M^n, B_A^n)$, determined below in problem (3), as given in this stage. This outcome depends on maturing debt, $B_M$, defaulted debt, $B_A$, and state variables, $\mathbf{s}$. The sovereign's payoff, $V^b$, is specified in problem (4) below. This payoff depends on the negotiation outcome, which we assume the sovereign can foresee, and the state of the economy, $\mathbf{s}$.

**The value of defaulted debt.** At the beginning of the period, the arrears holders' value of defaulted debt is[18]

$$V^\ell(B_M, B_A, \mathbf{s}) = \Big(1 - d(B_M, B_A, \mathbf{s})\Big)\, V^n(B_M, B_A, \mathbf{s}) + \\ + d(B_M, B_A, \mathbf{s})\, \beta \mathbb{E}_{\alpha'|\alpha} V^\ell(0, B_A + B_M, \mathbf{s}'|_{p=0})\, \frac{B_A}{B_A + B_M}. \tag{2}$$

Arrears holders take the country's default decision $d(B_M, B_A, \mathbf{s})$, determined in problem (1), as given. If the country chooses to *Repay*, $d = 0$, negotiation ensues. The negotiation value to arrears holders is $V^n(B_M, B_A, \mathbf{s})$ and is determined in problem (3) below. If the sovereign chooses *Default*, $d = 1$, negotiations are delayed for another period. Next period's value of defaulted debt, $V^\ell(0, B_M + B_A, \mathbf{s}'|_{p=0})$, depends on next period's state variables in case of default, discussed in problem (1). Current arrears holders see their claims to future negotiated amount diluted by the introduction of additional creditors. In particular, current arrears holders receive fraction $B_A/(B_A + B_M)$ of future negotiated amount. Arrears holders discount the future at rate $\beta$ and form expectations over next period's default cost $\alpha'$ using the conditional expectations operator $\mathbb{E}_{\alpha'|\alpha}$.

**Negotiation.** The delinquent sovereign and its arrears holders negotiate over outstanding debt in the negotiation stage. Arrears holders in unison make a "take-it-or-leave-it" offer to the sovereign for negotiated maturing debt, $B_M^n$, and negotiated defaulted debt, $B_A^n$. The following problem determines the negotiation outcome:

$$V^n(B_M, B_A, \mathbf{s}) = \max_{\substack{B_M^n \geq 0,\\ B_A^n \geq 0}} \left\{ c + \beta \mathbb{E}_{\alpha'|\alpha} V^\ell(B'_M(B_M^n, B_A^n, \mathbf{s}), B_A^n, \mathbf{s}'|_{p=p(B_M^n, B_A^n, \mathbf{s})}) \right\}$$

$$s.t.$$

$$c = B_M^n - B_M + p(B_M^n, B_A^n, \mathbf{s})\, r^A B_A^n,$$

$$V^b(B_M^n, B_A^n, \mathbf{s}) \geq V^b(B_M, B_A, \mathbf{s}). \tag{3}$$

Negotiated maturing debt $B_M^n$ is divided between maturing debt creditors who receive $B_M$ and arrears holders who receive $B_M^n - B_M$.[19] Interest payment on negotiated arrears is only made when the country is in the IMF program, $p = 1$. The second constraint is the participation constraint for the sovereign. The value to the sovereign of concluding the negotiation with any given amount

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of maturing debts and arrears is denoted as $V^b$ and is determined in problem (4). This constraint guarantees that the sovereign is better off with negotiated debt $(B^n_M, B^n_A)$ than with initial debt $(B_M, B_A)$.[20]

The value to arrears holders in the subsequent period, $V^\ell$, determined in problem (2), depends on the sovereign's choice of borrowing, $B'_M$, negotiated defaulted debt, $B^n_A$, and next period's value of variable $\mathbf{s}$. Borrowing, $B'_M$, and IMF program choice, $p$, are determined in problem (4) below. They depend on negotiated debt amounts, $(B^n_M, B^n_A)$, and state of the economy, $\mathbf{s}$. Arrears holders are risk neutral, discount the future at rate $\beta$, and form expectations over next period's default cost, $\alpha'$, using the conditional expectations operator $\mathbb{E}_{\alpha'|\alpha}$.

**Borrowing.** The sovereign solves the following problem when choosing how much debt to issue and whether to ask for the IMF program:

$$V^b(B^n_M, B^n_A, \mathbf{s}) = \max_{\substack{c \geq 0,\, p \in \{0,1\}, \\ B'_M \geq 0}} \left\{ u(c) + \beta \mathbb{E}_{\alpha'|\alpha} V^s(B'_M, B^n_A, \mathbf{s}') \right\}$$

$$\begin{aligned} &s.t. \\ &c = y + q(B'_M, B^n_A, p, \alpha) B'_M - B^n_M - p\, r^A B^n_A, \\ &p = \mathbb{I}_{B^n_A > 0} \left( \mathbb{I}_{B'_M > 0} + \mathbb{I}_{B'_M = 0}\, p_{-1} \right), \\ &B'_M = 0 \text{ if } B^n_A > 0. \qquad |\text{No Lending into Arrears} \end{aligned} \tag{4}$$

The country decides how much to consume, $c$, borrow, $B'_M$, and whether to enter an IMF program, $p$, subject to three constraints. The first constraint is the sovereign's budget constraint. This depends on endowment, $y$, revenues from debt issuance, $qB'_M$, negotiated maturing debt, $B^n_M$, and interest payments on negotiated defaulted debt when in the IMF program, $p\, r^A B^n_A$.[21] Both with the IMF and in the financial markets the sovereign issues $B'_M$ units of single period debt. The pricing schedule, $q(B'_M, B^n_A, \alpha)$, results from a lender breaking-even condition (5) introduced below. The sovereign takes the schedule as given. The price depends on borrowing, $B'_M$, negotiated defaulted debt, $B^n_A$, the choice of the sovereign to enter the IMF program, $p$, and default cost, $\alpha$.

The second constraint pertains to the rules of the IMF program. The country can exit the IMF program by settling defaulted debt, $\mathbb{I}_{B^n_A > 0} = 0$. An IMF program may be active for two reasons. First, a country with defaulted debt may borrow from the IMF, $\mathbb{I}_{B'_M > 0} = 1$. Second, a country with defaulted debt may inherit an IMF program, $p_{-1} = 1$. Therefore, a country enters the IMF program by taking an IMF loan. It remains in the IMF program until either defaulted debt is settled or it defaults on the IMF.

The final constraint is the "No Lending into Arrears" constraint. This constraint is what separates the "No Lending into Arrears" to the "Lending into Arrears" policy. The model equivalent of the policy shift is the comparative static of moving from the equilibrium with this constraint to the equilibrium without it. Studying the impact of the policy shift on negotiation outcome, post-negotiation default probability, and post-negotiation borrowing cost is the main exercise in this paper.

The sovereign's future value, $V^s$, was defined in problem (1) and depends on borrowing, $B'_M$, negotiated defaulted debt, $B^n_A$, and next period's state variables $\mathbf{s}'$. The sovereign is risk averse, discounts the future at rate $\beta$, and forms expectations over next periods default cost $\alpha'$ using the conditional expectations operator $\mathbb{E}_{\alpha'|\alpha}$.

**Price schedule.** The price schedule the sovereign faces in problem (4) is[22]

$$q(B'_M, B^n_A, p, \alpha) = \beta \mathbb{E}_{\alpha'|\alpha} \left[ 1 - d(B'_M, B^n_A, \mathbf{s}') + d(B'_M, B^n_A, \mathbf{s}') \beta \, \frac{\mathbb{E}_{\alpha''|\alpha'} V^\ell(0, B'_M + B^n_A, \mathbf{s}''|_{p'=0})}{B'_M + B^n_A} \right]. \tag{5}$$

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The price schedule depends on borrowing, $B_M{}'$, negotiated defaulted debt, $B_A^n$, whether the country is in the IMF program, $p$, and current period default cost, $\alpha$. The right-hand side is the net present value of a unit of debt. Both IMF and lenders discount the future at rate, $\beta$, and form expectations over next period's default cost, $\alpha'$, using the conditional expectations operator $\mathbb{E}_{\alpha'|\alpha}$.

The value of debt depends on the sovereign's default decision. If the sovereign chooses to *Repay* in the subsequent period, the unit of debt is fully repaid. If the sovereign chooses to *Default* in the subsequent period, for a unit of debt the lender (IMF or private lenders) receives the net present value of a unit of defaulted debt, discussed in detail in problem (1). Implicit in the break even pricing is that both financial markets and IMF are risk neutral and break even on their lending.[23]

### 4.3. Markov perfect equilibrium

Denote the set of debt positions as $\mathbb{B} = \mathbb{R}_+^2$, the set of default costs as $\mathbb{A}$, and the set of states $\mathbf{s} = (p_{-1}, \alpha, y)$ as $\mathbb{S} = \{0, 1\} \times \mathbb{A} \times \{y_\ell, y_h\}$. A Markov perfect equilibrium is a collection of functions, $V^\ell : \mathbb{B} \times \mathbb{S} \to \mathbb{R}$, $V^s : \mathbb{B} \times \mathbb{S} \to \mathbb{R}$, $V^n : \mathbb{B} \times \mathbb{S} \to \mathbb{R}$, $V^b : \mathbb{B} \times \mathbb{S} \to \mathbb{R}$, $q : \mathbb{B} \times \{0, 1\} \times \mathbb{A} \to \mathbb{R}_+$, $d : \mathbb{B} \times \mathbb{S} \to \{0, 1\}$, $(B_M^n, B_A^n) : \mathbb{B} \times \mathbb{S} \to \mathbb{B}$, $p : \mathbb{B} \times \mathbb{S} \to \{0, 1\}$, and $B'_M : \mathbb{B} \times \mathbb{S} \to \mathbb{R}_+$ that satisfy the following conditions:

i. Given $d$ and $V^n$; $V^\ell$ satisfies (2),
ii. Given $V^b$ and $(B_M^n, B_A^n)$; $V^s$ solves (1) and $d$ is the resulting policy function,
iii. Given $V^\ell$, $V^b$, $B'_M$, and $p$; $V^n$ solves (3) and $(B_M^n, B_A^n)$ is the resulting policy function,
iv. Given $V^s$, and $q$; $V^b$ solves (4) and $B'_M$ and $d$ are the resulting policy functions,
v. Given $d$ and $V^\ell$; $q$ satisfies (5)

## 5. Equilibrium outcomes

This section studies the equilibrium outcome of a country in a recession that starts with defaulted debt. In equilibrium, the country settles its defaulted debts. To understand the negotiation outcome, and in particular the role of IMF policy, the outside option of the country is dissected. After the successful negotiation, the country is exposed to default risk due to the stochastic nature of the default cost. Two specifications of the default cost are considered. First, transient uncertainty over the default cost. Second, persistent uncertainty over the default cost. Both specifications can generate an increase in haircuts and a decrease in probability of a repeat default due to the policy. However, the two specifications produce distinct results in terms of the post-negotiation borrowing cost.

### 5.1. Negotiation outcomes

This section establishes that the country successfully settles defaulted debts in period zero and derives conditions under which the policy shift leads to a strictly larger haircut. There are four negotiation stage results critical in comparing the two IMF policies. First, after period zero, that is, after the recession has ended, a subsequent default leads to defaulted debt being entirely forgiven. This result rests on the assumptions made on the default punishment as well as the fact that country and creditors share the same discount factor. More precisely, if a country defaults it is punished with default cost $\alpha$ and borrowing exclusion both for a period. After the punishment period, negotiations commence. At this stage, the country has no borrowing need. That is, since discount factors of country and creditors are identical the country does not benefit from low interest rates. Further, since the endowment of the country remains constant it has no consumption smoothing motive. Since the country achieves its largest payoff by never repaying its debt, creditors cannot elicit repayment.

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An implication of this result is that the borrowing cost in financial markets does not depend on IMF policy. The potential for the IMF policy shift to affect bond yields is not investigated here (the quantitative model of Appendix C allows for this possibility). Empirical results in Section 7 suggest that it is impossible to reject the hypothesis that the cost of borrowing right after a successful negotiation remains unchanged after the policy shift. This suggests that the effect of the policy shift on post-negotiation cost of borrowing is not of first order.

The second result is that, in the default cost specifications studied here, negotiations always lead to immediate settlement of defaulted debt. The policy functions are guessed to be such that negotiations are settled immediately and negotiated amount $B_M^*(B_M, B_A, \mathbf{s})$ allocates all bargaining surplus to the lenders. This guess is then verified. It is shown that such negotiated amount $B_M^*(B_M, B_A, \mathbf{s})$ does not depend on $\alpha$. This is true because conditional on not defaulting, the country's payoff, $V^b(B_M, B_A, \mathbf{s})$, does not depend on default cost $\alpha$.[24] Suppose, contrary to the initial guess, that in period zero negotiations fail to clear arrears. Instead of the failed negotiation, the country can successfully negotiate by offering the expected value of tomorrow's negotiation outcome, $B'^*_M$,[25] to lenders and rolling over the repaid amount using financial markets. In this case, both lenders and country would be at least as well off.[26]

The third result is with the "No Lending into Arrears" policy the negotiation outcome does not depend on initial defaulted debt, $B_A$ and leads to positive debt repayment. To see this note that since creditors make a "take-it-or-leave-it" offer, repaying the negotiated amount leaves the country indifferent to a failed negotiation, that is, the negotiation outcome $(B_M, B_A)$ in which arrears are not cleared. With "No Lending into Arrears" after a failed negotiation the country cannot borrow to smooth the impact of the recession. The country's payoff is then that of autarky, that is, $u(y_\ell) + \beta u(y_h)/(1-\beta)$. This payoff does not depend on defaulted debt, $B_A$. As a result, the negotiated amount does not depend on $B_A$ either. Further, as will be seen when studying post-negotiation outcomes, a country whose debts are fully forgiven benefits from having access to financial markets when $y_\ell < y_h$. Therefore, full debt forgiveness leaves the country with a surplus that the creditors can capture by demanding some positive repayment.

The final result provides bounds for the "Lending into Arrears" negotiation outcome. In particular, in addition to the lower bound of zero, discussed above, an upper bound is found for the "Lending into Arrears" repayment amount. The "No Lending into Arrears" repayment amount is the first bound. The outside option with "Lending into Arrears" is at least as good as with "No Lending into Arrears." That is, with "Lending into Arrears" the country can always choose to remain in autarky. Therefore, the "Lending into Arrears" repayment amount cannot be larger than the "No Lending into Arrears" one.

The upper bound is in some cases smaller. In particular, the "Lending into Arrears" repayment amount is bounded above by $r^A B_A/(1-\beta)$. This amount is the net present value of interest payments $r^A B_A$ if the country remains in the IMF program indefinitely. To see why this serves as an upper bound, suppose the negotiation is successful and repaid amount is $r^A B_A/(1-\beta)$. This amount can be thought of as being rolled by the country. This is achieved using market prices, that is, the country makes interest payments $(1-q)r^A B_A/(1-\beta)$. The presence of default risk makes these interest payments larger than the ones that would have to be paid in the IMF program, $r^A B_A$. This implies that the country can do at least as well by remaining in arrears indefinitely. Therefore, repayment with "Lending into Arrears" is at most $r^A B_A/(1-\beta)$.

These four results suggest that negotiations take place at time zero and the negotiation outcome is repayment amount $B_M^*$. Further, IMF policy only affects post-negotiation outcomes through repayment amount $B_M^*$. When the cost of remaining in the IMF program, $r^A$, is small enough, "Lending into Arrears" results to strictly less repayment than "No Lending into Arrears." The next section studies the post-negotiation outcomes for two countries that only differ in their initial repayment amount, $B_M^*$.

### *5.2. Post-negotiation outcomes*

Above I established conditions under which the haircut increases as a result of the policy shift. Next, I study the impact of a larger haircut on the post-negotiation default probability. In particular, I establish conditions under which a larger haircut leads to lower probability of default in two different default cost specifications.

**Transient default risk.** The first specification for the stochastic process of the default cost is that of a country that starts with very weak political institutions. The quality of political institutions is reflected in the default cost. The initial value of this cost is the lowest possible, $\alpha = 0$. The political institutions are expected to improve in the subsequent period. However, the extent of the improvement is uncertain. In particular, in the second period the default cost can take any value from 0 to $\alpha_g$. The probability of any realization is determined by c.d.f. $F(a)$ with p.d.f. $f(a)$ that is differentiable and non-decreasing in $(0,\alpha_g)$. That is, higher default cost values are more likely than lower ones. Finally, from the third period onwards the political institutions are strong. This is modeled as a deterministically high default cost with value $\alpha_g$.

After a successful negotiation, the country re-enters financial markets. The country has to repay creditors negotiated amount $B^*_M$ as well as endure a recession. These leave the country with a borrowing need that financial markets can accommodate. The country solves the following problem in period zero, when deciding how much to borrow from financial markets:

$$\max_{c\geq 0, B'_M\geq 0} u(c) + \beta\left(\int_0^{\alpha\left(B'_M\right)} \frac{u(y_h)}{1-\beta} - \alpha'\, dF(\alpha') + \int_{\alpha\left(B'_M\right)}^{\alpha_g} \frac{u(y_h-(1-\beta)B'_M)}{1-\beta}\, dF(\alpha')\right)$$

$$s.t.$$

$$c = y_\ell + \beta\Big(1 - F\big(\alpha\,(B'_M)\big)\Big)B'_M - B^*_M$$

In period one, the country is faced with a choice to repay its debt or default. As described above, the country's debt is forgiven after a post-negotiation default and autarky ensues. Therefore, default results to autarky payoff and default cost, $u(y_h)/(1-\beta) - \alpha'$. After the first period there is no default risk. Therefore, if the country repays it can roll over its debt indefinitely at risk-free rate $\beta$ and receive payoff $u(y_h - (1-\beta)B'_M)/(1-\beta)$.

For a given level of borrowing, $\alpha(B'_M)$ summarizes the period one default decision. In particular, period one default cost values lower than $\alpha(B'_M)$ lead to default whereas period one default cost values above $\alpha(B'_M)$ lead to repayment. The expression for this threshold is $u(y_h)/(1-\beta) - u(y_h-(1-\beta)B'_M)/(1-\beta)$. This expression contrasts the country's payoff from autarky to that of debt repayment. It caputres the benefit to the country of entirely eliminating debt. If the default cost is larger than the benefit of eliminating the debt, the country repays its debt otherwise it defaults.

Finally, in period zero the country can borrow from financial markets but is faced with period one default risk. The default risk impacts the cost at which the country borrows. In particular, the country faces default risk premium $1 - F(\alpha(B'_M))$. This premium compensates the risk neutral financial markets for a potential default. The following theorem summarizes the results of borrowing with transient default risk:

***Theorem* 1.** *Suppose $r^A$ is small enough. Then the policy shift features:*

*i. increased debt forgiveness,*

*ii. a reduction in the likelihood of default in period 1,*

*iii. a reduction in post-negotiation cost of borrowing.*

*Proof.* See Appendix. □

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Increased debt forgiveness impacts period zero borrowing. In particular, increased debt forgiveness lightens the country's borrowing need. Assumptions on preferences and period one default cost imply that borrowing need reduction leads to a reduction in period zero borrowing, that is, less period one debt. In turn, less period one debt decreases the benefit of fully eliminating debt. Therefore, period one default threshold decreases leading to a reduction in period one default probability. Finally, period one default probability reduction directly leads to period zero borrowing cost reduction.

**Persistent default risk.** The persistent default risk specification is one of a country that starts with strong political institutions. Political institutions are reflected again in the default cost. Strong institutions are captured by high default cost $\alpha_g$. Every period there is a risk that political institutions deteriorate. That is, there is a probability, denoted $\pi_{\alpha_b|\alpha_g}$, that the default cost switches from $\alpha_g$ to the lower value of $\alpha_b$. A potential deterioration in the country's political institutions is assumed to be everlasting. That is, default cost $\alpha_b$ is absorbing.

After a successful negotiation, the country decides how much to borrow. In contrast to the transient default risk specification, the country may face default risk for multiple periods. A small amount of borrowing carries no risk. Therefore, the country is able to borrow a small amount at the risk-free rate, $\beta$. In contrast, large borrowing may result to a default if the default cost switches from its high value of $\alpha_g$ to the lower value of $\alpha_b$. The threshold debt level above which there is default risk and below which there is no default risk is denoted $\underline{B}$.

Borrowing less than $\underline{B}$ allows the country to roll over its debt indefinitely at the risk-free price, $\beta$. In contrast, borrowing more than $\underline{B}$ carries a risk premium because of the default risk. In particular, for such debt levels borrowing price is $\beta\pi_{\alpha_g|\alpha_g}$, that is, it takes into account that repayment only takes place in the contingency that the default cost does not switch.[27] Instead of directly determining the country's borrowing, the number of periods for which the country chooses to maintain debt level above $\underline{B}$ is determined.[28] The payoff to the country of reducing its debt to a level below $\underline{B}$ in $T$ periods is

$$\max_{\substack{\{c_t\}_{t=0}^{T-1}\geq 0,\\ B_{M,T}\geq 0}} \sum_{t=0}^{T-1} \beta^t \pi_{\alpha_g|\alpha_g}^t u(c_t) + \sum_{t=1}^{T-1} \pi_{\alpha_b|\alpha_g} \pi_{\alpha_g|\alpha_g}^{t-1} \beta^t V_b^D + \pi_{\alpha_g|\alpha_g}^{T-1} \beta^T \frac{u\left(y_h - (1-\beta)B_{M,T}\right)}{1-\beta}$$

$$s.t.$$

$$\sum_{t=0}^{T-1} \beta^t \pi_{\alpha_g|\alpha_g}^t c_t = y_\ell + \sum_{t=1}^{T-1} \beta^t \pi_{\alpha_g|\alpha_g}^t y_h + \beta^{T-1} \pi_{\alpha_g|\alpha_g}^{T-1} \beta B_{M,T} - B_M^*,$$

$$B_{M,T} \leq \underline{B}.$$

The first $T$ periods are separated into histories where the default cost remains $\alpha_g$ and ones where the cost decreases to $\alpha_b$. Consumption conditional on the default cost being $\alpha_g$ in period $t$ is denoted $c_t$. The payoff to the country of the high default cost histories is the first term of the objective function. The first constraint is the budget constraint for the $T$ period high default cost history. This constraint takes into account that the cost of borrowing is $\beta\pi_{\alpha_g|\alpha_g}$ for the first $T-1$ periods and $\beta$ for the final period when borrowing is $B_{M,T}$. Further, the second constraint guarantees that in period $T$ borrowing is less than $\underline{B}$.

The second term of the objective function is the country's payoff in histories where the cost switches to $\alpha_b$. In these histories, the country defaults and its continuation value is $V_b^D = u(y_h)/(1-\beta) - \alpha_b$. Further, the probability that the cost switches in period $t$ and not any period before that is $\pi_{\alpha_b|\alpha_g}\pi_{\alpha_g|\alpha_g}^{t-1}$. The final term of the objective function is the country's payoff conditional on $\alpha_g$ having been realized for the first $T-1$ periods. In this case, the country has reduced its debt to $\underline{B}$ and it can roll over its debt using the risk-free price, $\beta$, indefinitely. The following theorem summarizes the results of borrowing with persistent default risk:

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***Theorem* 2.** *Suppose $\pi_{\alpha_b|\alpha_g}$ is small enough. Then for each $T > 1$ there exists an interval $(\underline{y}_T, \bar{y}_T)$ so that if $y_\ell \in (\underline{y}_T, \bar{y}_T)$ there is an $r^A$ small enough so that the policy shift features:*

i. *increased debt forgiveness,*
ii. *unchanged likelihood of default in the first $T$ periods,*
iii. *a reduction in the post-period-$T$ likelihood of default,*
iv. *unchanged post-negotiation cost of borrowing, $\beta\pi_{\alpha_g|\alpha_g}$.*

*Proof.* See Appendix. □

Consider a country in a large recession. The country faces the following tradeoff when contemplating its choice of $T$. The longer it takes to reduce its debt down to $\underline{B}$, that is, the larger the $T$ is, the more it can smooth the impact of the recession. However, a longer debt reduction results to more wasted resources due to costly borrowing.[29] The country balances its need to smooth consumption with the cost of expensive borrowing. If $\pi_{\alpha_b|\alpha_g}$ is small enough, the need to smooth consumption dominates.[30] As a result, the country enters a multi-period debt reduction at the conclusion of which it reduces its debt to $\underline{B}$ and no longer faces default risk.

The duration of debt reduction depends on the negotiated amount. In particular, the greater the debt forgiveness, the less time debt reduction takes. The IMF policy shift leads to an increase in debt forgiveness. The theorem states that, for certain depths of recession, increased debt forgiveness leads to a reduction in exit time $T$. Suppose debt reduction takes $\bar{T}$ periods with "No Lending into Arrears" and $\underline{T}$ periods with "Lending into Arrears" with $\bar{T} > \underline{T}$. The probability a country defaults in the first $\underline{T}$ periods, the "short run," is $\sum_{t=1}^{\underline{T}-1} \pi_{\alpha_b|\alpha_g}\pi^{t-1}_{\alpha_g|\alpha_g}$ independently of IMF policy. Further, the post-negotiation cost of borrowing, that is, the cost of borrowing the period of the successful negotiation, is $\beta\pi_{\alpha_g|\alpha_g}$ for both policies. However, the probability that the country defaults in the first $\bar{T}$ periods, the "long run," is $\sum_{t=1}^{\underline{T}-1} \pi_{\alpha_b|\alpha_g}\pi^{t-1}_{\alpha_g|\alpha_g}$ with "Lending into Arrears" and $\sum_{t=1}^{\bar{T}-1} \pi_{\alpha_b|\alpha_g}\pi^{t-1}_{\alpha_g|\alpha_g}$ with "No Lending into Arrears." Therefore, the policy shift increases the long-run post-negotiation probability of default while leaving the short-run post-negotiation probability of default and post-negotiation cost of borrowing unchanged.

### 5.3. Comparing the default risk specifications

The results in this section can be separated in two categories. First are results consistent between the two settings of default risk. In both settings, the policy shift results to an increase in debt forgiveness and a decrease in post-negotiation default probability. These findings are consistent with the changes in averages discussed in Section 3.

The model takes many negotiation primitives as given. In reality, these may be affected by the policy shift. The size of arrears, severity of recession, characteristics of lenders, and desire to delay negotiations are all taken as primitives in the model and do not depend on the IMF policy. In the data, these variables may actually change in 1989 both for reasons related to the policy shift and not. For this reason, in Section 7 the change in averages of haircuts controls for these variables.

The second set of results differ between transient and persistent default risk. First is the impact of the policy shift on post-negotiation borrowing cost. The policy shift decreases the borrowing cost with transient default risk whereas with persistent default risk it leaves it unchanged. Second is the impact of the policy shift on the timing of the post-negotiation default risk. With transient default risk, the policy shift reduces short-run default risk. In contrast, with persistent default risk the policy shift leaves the short-run default risk unchanged. However, in the same specification the policy shift reduces long-run default risk. Section 7 provides empirical support for the theory of persistent default risk.

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## 6. Numerical exercise

This section outlines the numerical exercise performed in Appendix C. The stylized model introduced in Section 4 offers a stark prediction. The IMF policy shift impacts renegotiation outcomes only through the effect on the outside option. The quantitative model in Appendix C incorporates this idea to an otherwise standard model of renegotiation. The objective is to quantify the increase in haircuts as a result of the improvement in the outside option (due to the IMF policy shift).

The quantitative model of Yue (2010) is incorporated with IMF debt. IMF lending on the equilibrium path is kept very simple. In particular, it is assumed that on the equilibrium path the country maintains a constant level of IMF debt $\bar{B}_I$ and is not subject to conditionality. In contrast, IMF borrowing in the outside option is actively managed and similar to that in Boz (2011) and Fink and Scholl (2016) in that it is not defaultable and it comes with conditionality. In particular, in the outside option IMF debt is used to smooth AR(1) GDP shocks and the repayment of outstanding IMF debt $\bar{B}_I$. IMF conditionality is added to the outside option to generate IMF borrowing consistent with the data.

The numerical exercise performed in Appendix C starts by calibrating the model described in the previous paragraph to salient sovereign debt moments of the post-1989 Mexican economy. The average haircut is taken to be 50% to match the unconditional average haircut in the post-1989 sample. Next, the option to borrow from the IMF in the outside option is eliminated to mimic the pre-1989 IMF policy. The outside option deteriorates and the haircut declines to approximately 42%. The 8 percentage point increase in haircuts generated by the model by introducing lending into arrears in the outside option represents a quantitatively significant fraction of the 24 percentage point increase in the average haircut seen in the data in 1989. More details as well as a robustness exercise can be found in Appendix C.

This setup is different to the model presented in Section 4 where IMF lending was used only to smooth an endowment shock. In the quantitative model of Appendix C, being able to smooth the repayment of past IMF debt $\bar{B}_I$ is the primary reason haircuts increase as a result of the IMF policy shift. In particular, quantitative sovereign debt models following Arellano (2008), including Yue (2010), use an AR(1) process to model GDP fluctuations. A well-known result that goes back to Lucas (1987) highlights that the welfare benefit of smoothing AR(1) GDP fluctuations of the magnitude seen in the data is small. Therefore, access to IMF lending that only smooths AR(1) GDP fluctuations does not alter significantly consumers' payoff in the outside option and as a result haircuts. In this sense, the exercise performed here gives a lower bound for the impact of the IMF policy shift on haircuts. For instance, models of the GDP process with a disaster risk, such as the one introduced in Barro (2009), would result to a larger effect of the policy shift on the outside option and as a result haircuts.

## 7. Empirical support

The objective of this section is twofold. The first objective is to provide additional empirical support for the impact of the policy on haircuts. To that end, the haircut achieved in a renegotiation is regressed against a 1989 dummy and a number of control variables. The second objective is to investigate the effect of the policy shift on post-negotiation borrowing cost. The goal is to provide insight into the nature of the default risk faced by countries after negotiations.

**The data.** The data used derive from the World Bank's World Development Indicators (WDI), Global Development Finance (GDF), S&P, the IMF's International Financial Statistics, Global Financial Data (GFD), and from the datasets of Cruces and Trebesch (2013) and Asonuma and Trebesch (2016). We restrict attention to countries with an outstanding balance to the IMF at the time of their sovereign debt renegotiation. The first set of control variables involve the economic fundamentals of the country at the time of the negotiation. The cyclical and trend components of GDP are captured by the deviation of GDP from its linear trend and GDP per capita at the year of negotiation.

The second set of control variables pertain to the indebtedness of the country. These variables can be further broken into aggregate measures of debt and composition of debt. For the aggregate, total debt is taken the year before the negotiation, that is, before any haircuts are applied. The scope of debt is only external debt, as measured by the WDI. The two variables considered are total debt as a fraction of the nation's linear GNI trend and negotiated debt as a fraction of total debt. The second variable controls for the fact that negotiations do not always involve the entire debt stock.

Composition of debt is a breakdown of long-term debt by type of debt instrument. In particular, it measures the fraction of long-term debt in bonds, bank loans, and the official sector. This measure is taken the year the country defaults and not the year of the negotiation. Ideally, one would want this break down at the time of the negotiation. However, this break down is only available for long-term debt and there are issues with the measurement of defaulted long-term debt. A big transition that took place around 1989 was the shift from bank loans to bonds. These variables control for this transition.

The third set of control variables control for the timing of the negotiations. A default episode is defined as the consecutive years for which a country is in default. The first variable is the number of years in the current default episode at the time of negotiation and the second is the number of negotiations in the current default episode at the time of negotiation. The first variable controls for the effect of negotiation delays on negotiation outcomes. This variable is also interacted with the 1989 dummy to allow for the possibility that negotiation delays affected differently negotiation outcomes before and after 1989. The second variable controls for the possibility that failed negotiations affect negotiation outcomes, possibly through pressure from international financial institutions.

The fourth set of control variables are dummy variables whether the negotiation was part of the Brady deal, whether it was donor funded, whether the negotiation took place after defaulted payments, and region dummies. The Brady deal negotiations took place in the end of the 1980s debt crisis with 16 countries participating. The USA recognized that to end the 1980s debt crisis drastic measures had to be taken. Collateralized debt, multiple options for different creditors, and other measures were part of this era. Donor-funded negotiations involve poor countries for which the restructuring is either funded or supported by bilateral or multilateral funds, for example, World Bank. As reported in Cruces and Trebesch (2013), the haircuts associated with these restructurings tend to be higher, potentially due to pressure applied to creditors to forgive more debt. The timing of the negotiation, that is, whether the negotiations takes place before or after missed debt payments, has been shown to matter for negotiation outcomes, see Asonuma and Trebesch (2016).[31] In particular, preemptive restructurings are shown to be associated with lower haircuts. To control for this, dummy variable post-default, which takes the value of one for negotiations which take place after missed payments, is included. Finally, region dummies are considered as specified by the World Bank.[32]

The haircut measure is the one defined in Section 3. The second dependent variable considered is the post-negotiation borrowing spread. Data on yields that encompass the entire dataset are particularly elusive. For this reason, Cruces and Trebesch (2013) constructed their own measure of "exit yields" discussed in detail in their paper and utilized here. Further, the 10 year US bond, constant maturity, yield is used from GFD to construct a measure of the spread.

**Regression results.** The regression coefficients in the first two columns of Table 1 provide further support of the effect of the policy on haircuts.[33] The 1989 dummy has a positive and statistically significant coefficient for both the haircut and repeat default variable. Haircuts increased by 13 percentage points in 1989 even when accounting for all other factors listed in the previous subsection.[34]

The second column of Table 1 reports the change in the post-negotiation borrowing spread in 1989. As described in Section 5.3, the transient default risk theory predicts a decrease in spreads

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**Table 1.** The coefficients of linear regressions with dependent variables: (1) the haircut measure and (2) the spread countries face in borrowing after the negotiation and the control variables described in the text

| Variables | Haircut | "Exit Spread" |
|---|---|---|
| 1989 Dummy | 0.13** | 0.02 |
| | (0.05) | (0.02) |
| GDP | −0.03 | −0.08 |
| *(% dev. from trend)* | (0.15) | (0.06) |
| GDP per capita | −0.05** | −0.02** |
| *(log)* | (0.02) | (0.01) |
| Debt to GNI | 0.15*** | 0.04** |
| *(trend GNI)* | (0.05) | (0.02) |
| Negotiated debt | 0.05 | −0.02 |
| *(% of total)* | (0.08) | (0.03) |
| Episode duration | 0.01* | −0.00 |
| | (0.01) | (0.00) |
| Negotiations | −0.02* | 0.00 |
| | (0.01) | (0.01) |
| Brady deal | 0.01 | −0.01 |
| | (0.06) | (0.02) |
| Donor funded | 0.32*** | −0.04 |
| | (0.08) | (0.03) |
| Post-default | 0.08** | 0.00 |
| | (0.03) | (0.01) |
| 1989 Dummy x | 0.02* | 0.01** |
| Episode duration | (0.01) | (0.00) |
| Debt composition | Y | Y |
| Region dummies | Y | Y |
| Constant | Y | Y |
| Observations | 111 | 111 |
| Adjusted $R^2$ | 0.58 | 0.39 |
| *F* test | 0 | 0 |

*Note*: Standard errors in parentheses.
*** $p<0.01$, ** $p<0.05$, * $p<0.1$.

while the persistent default risk theory predicts no chance as a result of the policy shift. The regression in the second column of Table 1 suggests that the point estimate is slightly positive, that is, spreads increase slightly in 1989, but not statistically significant. That is, one cannot reject the null hypothesis that the spreads do not change at all in 1989. This result provides evidence against the theory of transient default risk.

## 8. Conclusion

This paper investigates the role of a 1989 IMF policy shift in explaining major changes in negotiation outcomes of defaulted sovereign debt. The stark shifts observed in 1989 are more debt forgiveness and less post-negotiation debt servicing difficulties. A theoretical framework is introduced highlighting the IMF policy shift as the culprit behind the two empirical regularities. The theoretical framework emphasizes the potential of the policy shift to improve the country's outside option at the time of the negotiation of defaulted debts. The model provides stark predictions regarding the effect of the policy on the risk premium on borrowing under different specifications of default risk. The final section finds empirical support for the specification where post-negotiation default risk is persistent and the country moderates its debt to reduce this risk.

An additional channel, not investigated in this paper, is the effect of the policy shift on borrowing cost. That is, the country repays a smaller fraction of its debt after 1989. This should have a direct effect on borrowing cost. Lenders, after the policy shift, should require a larger compensation to break even in lending, resulting in a spreads increase. This effect is silenced in the model by making all negotiations after the initial one fail. The reason for silencing it is that for the time frame considered here, close to a successful negotiation, this effect does not seem to be important. That is, empirical Section 7 suggests that there is no evidence of the spread having changed.[35] However, this additional channel could provide insights into changes in borrowing away from default episodes.

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## Notes

**1** Between 1989 and 1998, the IMF's lending in arrears policy only applied to bank loan defaults, not sovereign bond defaults. However, this distinction is complicated by the fact that the majority of defaults in the 1990s were on bank loans. Beers and Chambers (2003) decompose defaults into foreign currency bond and foreign currency bank loan defaults. Between 1990 and 2000, the fraction of sovereign issuers in default in either bonds or bank loans was at its highest 22.5% and at its lowest 12.5%. In the same decade, the largest fraction of sovereign issuers in default on bonds was 2.5%. That is, bond default represented a small fraction of total defaults in the 1990s. Therefore, the IMF policy of lending into arrears applied to the debt countries was predominantly defaulting on in the 1990s.

**2** The narrative behind the motivation of the IMF policy draws entirely from the document "Fund Policy on Lending into Arrears to Private Creditors– Further Consideration of the Good Faith Criterion" published by the IMF.

**3** For instance, Argentina entered two IMF programs after its 2001 default. The first was a transitory program that lasted 7 months and the second a stand-by-arrangement that was scheduled to be active for 3 years but was suspended a bit less than a year after its conception. These programs mainly benefited Argentina in rolling over IMF debts, an option countries would not have had prior to the 1989 policy shift.

**4** A country is defined as having debt servicing difficulties if within 5 years of negotiating defaulted debts it defaults again.

**5** The country here ends up negotiating when output is below trend. Benjamin and Wright (2009) document that around 50% of negotiations happen when GDP is above trend. Here the assumption of a short-lived recession can be substituted for any assumption generating the borrowing need. Further, an important implication of the borrowing need is that after the negotiation the country may be more indebted than it was when it defaulted. This has empirical support in the work of Benjamin and Wright (2009) and Arellano et al. (2013). The former documents that countries tend to exit default more indebted than when they defaulted while the latter documents that countries borrow while in default.

**6** One interpretation is that the politician in power faces a time-varying cost of defaulting.

**7** The work of Borensztein and Panizza (2009) is one of the papers in support of the hypothesis that there are significant short-lived costs associated with default.

**8** For a recent application, see Müller et al. (2015).

**9** Beers and Mavalwalla (2016) document that countries do default on the IMF, albeit in small amounts.

**10** To simplify exposition, this cost is made proportional to outstanding defaulted debts. This is motivated by interest payments on arrears. There is no direct evidence of interest payments on arrears; however, IMF lending is conditional on what is known as Good Faith Criteria. One of the conditions is that the sovereign is not litigated by creditors for the duration of the program. Benjamin and Wright (2009) point out that creditors go to courts for interest payments on arrears with favorable outcomes since 1997. This is taken as evidence that there is pressure for interest payment on defaulted debt for the duration of the IMF program.

**11** Cruces and Trebesch (2013) document that countries on average face a positive spread after negotiating defaulted debt and this average decreases monotonically over time. This observation is consistent with the mechanism in this paper. That is, the positive spreads are consistent with countries facing default risk after negotiations. Further, the decreasing spreads are consistent with countries taking actions to reduce the risk of default.

**12** Presumably, this is done because the frequency of post-negotiation defaults before 1989 greatly interferes with their analysis of post-negotiation financial market re-entry (i.e., the first time the country increases its debt after the negotiation), a variable not studied here.

**13** From the universe of negotiations reported in Cruces and Trebesch (2013), the only observations disqualified are the ones for which additional data from the GDF, described below, are not available.

**14** The average haircut is larger both before and after the policy change. This is consistent with the findings of Asonuma and Trebesch (2016) who find post-default negotiation haircuts to be larger.

**15** According to this definition, "a sovereign default occurs when the central government fails to pay scheduled debt service on the due date or tenders an exchange offer of new debt with less-favorable terms than the original issue."

**16** In the numerical exercise of Appendix C, this assumption is further justified by assuming the country cannot default on IMF debt.

**17** In equilibrium, the country never emerges from default again. Therefore, in equilibrium, the continuation value of default is simply $\beta \mathbb{E}_{\alpha'|\alpha} V^s(0, B_M + B_A, \mathbf{s}'|_{p=0}) = \beta u(y_h)/(1-\beta)$.

**18** In equilibrium, the country never emerges from default after the first period. Therefore, in such a case it never repays its debts, and $V^\ell(0, B_M, \mathbf{s}'|_{p=0})$ is always zero, that is, the second line is zero.

**19** The interpretation of $B^n_M - B_M < 0$ is that the sovereign repays the entire $B_M$ with part of it, $B_M - B^n_M$, coming from the arrears holders. In practice, in Section 5 (in particular Lemma 1 in Appendix A) it is shown that negotiations always feature arrears clearance and the repayment amount is at least as large as maturing debt, that is, $B^n_M - B_M \geq 0$.

**20** The negotiation structure here is flexible, in that arrear repayment can be made at any rate. It will be assumed that if there are multiple solutions to the creditors' problem and one involves arrears being extinguished then this will be the offer chosen by the creditors. In the equilibrium studied here, there is always a solution to problem (3) with $B^n_A = 0$. However, there may also be solutions with small $B^n_A > 0$. The tie is broken by clearing arrears, $B^n_A = 0$, for the following reason. Suppose the sovereign suffers a resource cost $\epsilon$ each period spent in arrears (an assumption made in many papers in the literature, see for instance Yue (2010)). Then in equilibrium arrears are extinguished. As $\epsilon$ tends to zero, the only equilibrium that survives is the one studied here.

**21** Interest payment on arrears captures one of the possible costs of being in an IMF program. Other costs may be austerity programs or other economic reform programs imposed by the IMF.

**22** In equilibrium, the country never emerges from default again. Therefore, the $d = 1$ outcome drops out of the price, that is, the schedule becomes $q(B'_M, B^n_A, p, \alpha) = \mathbb{E}_{\alpha'|\alpha}[1 - d(B'_M, B^n_A, \mathbf{s}')]/(1+r)$.

**23** For financial markets, this assumption is tantamount to financial markets being competitive. That borrowing from the IMF takes place at actuarially fair prices is supported by Zettelmeyer and Joshi (2005). They compute implicit subsidies in IMF lending, that is, how far IMF lending price is from the actuarily fair price. They find that for emerging economies these transfers are very small.

**24** This condition is trivially satisfied when the default cost $\alpha$ is iid. It is also satisfied in the two specifications studied here despite the stochastic process for $\alpha$ displaying persistence.

**25** Where $B'^*_M = B^*_M(B'_M, B^n_A, \mathbf{s}'|_{p,\alpha'})$, $p := p(B^n_M, B^n_A, \mathbf{s})$ is the IMF policy decision assuming remaining arrears, and $B'_M := B'_M(B^n_M, B^n_A, \mathbf{s})$ is the borrowing decision assuming remaining arrears.

**26** This replication argument derives from Aguiar et al. (2019).

**27** There is an upper bound to how much a country is willing to repay even with the high default cost $\alpha_g$. This upper bound is denoted as $D$ in the appendix. The price of debt is zero for debt levels higher than $D$. Therefore, the country never borrows more than $D$.

**28** This equivalence is also utilized in Cole and Kehoe (2000) and Aguiar et al. (2019).

**29** The price of borrowing is actuarily fair. That is, it reflects repayment of $B'_M$ for the high cost and 0 for the low one. However, when the cost is low the country defaults and incurs the default cost. This is equivalent to a partial repayment which no one receives (since the cost is deadweight). From the point of view of the sovereign borrowing is overpriced.

**30** This argument derives from Cole and Kehoe (2000).

**31** The variable regarding the timing of the negotiation is taken from this paper.

**32** The regions are Europe and Central Asia, Middle East and North Africa, Latin America and Caribbean, Sub-Saharan Africa, South Asia, and East Asia and Pacific.

**33** In Appendix E, similar regressions suggest that the impact of the IMF policy on post-negotiation default likelihood reduction is significant. Further, multiple windows for post-negotiation default are considered finding further support for the theory that post-negotiation default risk is persistent rather than transient.

**34** This number is smaller than the 25 percentage point increase of Section 3. The main reason behind this drop is the selection of the sample due to data availability. That is, out of 150 countries 120 remained for which all the data listed in the previous subsection were available. The countries that dropped out tended to be poorer countries who happened to have had larger haircuts after 1989.

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**35** One could argue that combining the theory on transient default risk with this channel, the effect on the spreads could be ambiguous. However, the evidence on the timing of default is not supportive of the transient default risk specification making such a claim less plausible.
**36** $\tilde{V}^b(B_M, B_A, \mathbf{s}) = V^b(B_M + zB_A, 0, \mathbf{s})$, $\tilde{q}(B_M, B_A, p, \alpha) = q(B_M + zB_A, 0, p, \alpha)$, $\tilde{d}(B_M, B_A, \mathbf{s}) = d(B_M + zB_A, 0, \mathbf{s})$, $\tilde{V}^s(B_M, B_A, \mathbf{s}) = V^s(B_M + zB_A, 0, \mathbf{s})$, and $B'_M = \tilde{B}'_M + zB_A$
**37** This rests on the fact that the payoff under no default is the same in $\tilde{V}^b$ and $V^b$ and Assumption 2 dictates that the payoff has to be the same for all other repayment states and those states have to be the same.
**38** Suppose not, then consumption in the first period is less than $y_\ell$. Further, the largest continuation value possible is $u(y_h)/(1-\beta)$; therefore, the value is strictly less than $u(y_\ell) + u(y_h)/(1-\beta)$ which is a contradiction to the condition of the negotiation outcome.
**39** A set being smaller than another set is defined as all elements of one set being smaller than those of the other set.
**40** The fact that $V^b(y_h - y_\ell, \alpha_g) > u(y_\ell) + \beta u(y_h)/(1-\beta)$ for $y_\ell < y_h$ implies that $B^n_M > 0$. Lemma 5 implies that this negotiation amount increases as $y_\ell$ decreases. Therefore, as $y_\ell$ approaches $y_h - \bar{I}_T$ the value of $B^n_M + y_h - y_\ell$ becomes larger than $\bar{I}_T$ and remains larger as $y_\ell$ decreases even further.
**41** The sovereign is forbidden from borrowing for price less than $\bar{Q}$. This is a standard assumption for quantitative models of renegotiation imposed to guarantee that sovereigns do not sell debt cheaply when faced with certain default.
**42** Noise is added to the default and private borrowing decisions as in Dvorkin et al. (2021). The CDF process dictating the noise is$F(\epsilon) = \exp\left[-\left(\sum_{j=1}^{J} \exp\left(\frac{\epsilon_j - \mu}{\rho\sigma}\right)^{\rho} + \exp\left(\frac{\epsilon_{J+1} - \mu}{\rho\sigma}\right)\right)\right]$where $j = 1, \ldots, J$ are extreme value shocks to the value of borrowing the $j$'th element of the private borrowing grid and $J+1$ is an extreme value shock to the value of default. The values for $\rho$ and $\sigma$ are 0.005 and 0.015, respectively. These are chosen to be the smallest values that guarantee convergence and are kept the same through the exercise of altering the outside option.
**43** Rounding down average IMF debt makes the impact of the policy shift more muted. This conservative approach is in line with considering this exercise as a lower bound for the potential impact of the IMF policy shift.
**44** As can be seen in Table 6 most variables are statistically insignificant in explaining post-negotiation defaults. Post-negotiation defaults are more complicated than regular defaults. In particular, as shown in Appendix C, standard quantitative sovereign default models grossly underestimate the occurrence of such defaults. As a result, it is not surprising that standard measures fail to predict their occurrence.

## References


Admati, A. R. and M. Perry (1987) Strategic delay in bargaining. *The Review of Economic Studies* 54(3), 345–364.

Aguiar, M., M. Amador, H. Hopenhayn and L. Werning (2019) Take the short route: Equilibrium default and debt maturity. *Econometrica* 87(2), 423–462.

Aguiar, M., S. Chatterjee, H. L. Cole and Z. Stangebye (2016) Quantitative model of sovereign debt crises. *Handbook of Macroeconomics* 2, 1697–1755.

Amador, M. and C. Phelan (2021) Reputation and sovereign default. *Econometrica* 89(4), 1979–2010.

Arellano, C. (2008) Default risk and income fluctuations in emerging economies. *American Economic Review* 98(3), 690–712.

Arellano, C., X. Mateos-Planas and J. V. Rios-Rull (2013). Partial default. *Manuscript, University of Minnesota*.

Asonuma, T. and H. Joo (2020) Sovereign debt restructurings: Delays in renegotiations and risk averse creditors. *Journal of the European Economic Association* 18(5), 2394–2440.

Asonuma, T. and C. Trebesch (2016) Sovereign debt restructurings: Preemptive or post-default. *Journal of the European Economic Association* 14(1), 175–214.

Barro, R. J. (2009) Rare disasters, asset prices, and welfare costs. *American Economic Review* 99(1), 243–264.

Beers, D. and J. Chambers (2003). Sovereign defaults: Heading lower into 2004. *New York: Standard and Poor's.*

Beers, D. T. and J. Mavalwalla (2016). Database of sovereign defaults. *Bank of Canada.*

Benjamin, D. and M. L. J. Wright (2009). Recovery before redemption: A theory of delays in sovereign debt renegotiations. *CAMA Working Papers,* 2009—2015.

Bianchi, J., J. C. Hatchondo and L. Martinez (2018) International reserves and rollover risk. *American Economic Review* 108(9), 2629–2670.

Borensztein, E. and U. Panizza (2009) The costs of sovereign default. *IMF Staff Papers* 56(4), 683–741.

Boz, E. (2011) Sovereign default, private sector creditors, and the IFIs. *Journal of International Economics* 83(1), 70–82.

Cole, H. L. and T. J. Kehoe (2000) Self-fulfilling debt crises. *The Review of Economic Studies* 67(1), 91–116.

Cruces, J. J. and C. Trebesch (2013) Sovereign defaults: The price of haircutst. *American Economic Journal: Macroeconomics* 5(3), 85–117.

Díaz-Cassou, J., A. Erce and J. J. Vázquez-Zamora (2008). The role of the IMF in recent sovereign debt restructurings: Implications for the policy of lending into arrears. *Banco de Espana Occasional Paper,* No. 0805.

Dvorkin, M., J. M. Sanchez, H. Sapriza and E. Yurdagul (2021) Sovereign debt restructurings. *American Economic Journal: Macroeconomics* 13(2), 26–77.


https://doi.org/10.1017/S136510052200075X Published online by Cambridge University Press

Fernandez, R. and R. W. Rosenthal (1990) Strategic models of sovereign-debt renegotiations. *The Review of Economic Studies* 57(3), 331–349.
Fink, F. and A. Scholl (2016) A quantitative model of sovereign debt, bailouts and conditionality. *Journal of International Economics* 98, 176–190.
Fourakis, S. (2022). Sovereign default and government reputation. Unpublished manuscript.
Lucas, R. E. (1987) *Models of Business Cycles*. New York: Basil Blackwell.
Müller, A., K. Storesletten and F. Zilibotti (2015). Sovereign debt and structural reforms. *CEPR Discussion Paper,* No. DP10588.
Paluszynski, R., G. Stefanidis (2023) Borrowing into debt crises. *Quantitative Economics* 14(1), 277–308.
Pitchford, R. and M. L. J. Wright (2012) Holdouts in sovereign debt restructuring: A theory of negotiation in a weak contractual environment. *The Review of Economic Studies* 79(2), 812–837.
Richmond, C. and D. A. Dias (2009). Duration of capital market exclusion: An empirical investigation. Unpublished manuscript.
Sturzenegger, F. and J. Zettelmeyer (2008) Haircuts: Estimating investor losses in sovereign debt restructurings, 1998–2005. *Journal of International Money and Finance* 27(5), 780–805.
Wells, R. (1993) Tolerance of arrearages: How IMF loan policy can effect debt reduction. *The American Economic Review* 83(3), 621–633.
Yue, V. Z. (2010) Sovereign default and debt renegotiation. *Journal of international Economics* 80(2), 176–187.
Zettelmeyer, J. and P. Joshi (2005). Implicit transfers in IMF lending, 1973—2003. *IMF Working Paper,* No. 05/8.

## Appendix A – Outside option

The guess for negotiation problem (3) is $(B_M^n, B_A^n) = (B_M^*, 0)$ where $B_M^*$ satisfies $V^b(B_M^*, 0, \mathbf{s}) = V^b(B_M, B_A, \mathbf{s})$. That is, arrears are cleared and repayment amount $B_M^*$ leaves the country indifferent between negotiating or not. This guess will be verified.

***Lemma.*** *The outside option in the No Lending into Arrears policy is autarky. That is,* $V^b(0, B_A, \mathbf{s}) = u(y) + \frac{u(y_h)}{1-\beta}$.

*Proof.* Given the guess for the negotiation outcome the country starts every period with $B_A$ in arrears. As a result, it is never able to borrow and the resulting outcome is autarky. □

***Lemma.*** *The solution to default problem (1) for state* $(0, B_A, \mathbf{s}|_{p_{-1}=0})$ *is* $d = 0$.

*Proof.* Suppose the state is $(0, B_A, \mathbf{s}|_{p_{-1}=0})$ and the country defaults. The payoff is $u(y) - \alpha + \beta \mathbb{E}_{\alpha'|\alpha} V^s(0, B_A, \mathbf{s}'|_{p=0})$.

Suppose instead the country does not default. The payoff is $V^b((B_M^n, B_A^n)(0, B_A, \mathbf{s}), \mathbf{s}|_{p_{-1}=0}) = V^b(0, B_A, \mathbf{s}|_{p_{-1}=0})$. In $V^b(0, B_A, \mathbf{s}|_{p_{-1}=0})$ a possible choice is $B'_M = 0$ whose payoff is $u(y) + \beta \mathbb{E}_{\alpha'|\alpha} V^s(0, B_A, \mathbf{s}'|_{p=0})$. The payoff to the country from not defaulting is at least as large as this choice. Therefore, the country is better off not defaulting. □

***Lemma.*** *The sovereign never repays arrears outside of the IMF program after the first period. That is,* $V^b(0, B_A, \mathbf{s}|_{p_{-1}=0,y_h}) = \frac{u(y_h)}{1-\beta}$, *and as a result* $B_M^*(0, B_A, \mathbf{s}|_{p_{-1}=0,y_h}) = 0$.

*Proof.* Define history $h_t = (\alpha_0, \alpha_1, \ldots, \alpha_t)$. Using the guess and the previous lemma, the net present value of consumption satisfies $c_0 + \sum_{t=1}^{\infty} \beta^t \pi_{h_t} c_t(h_t) = \frac{y_h}{1-\beta} - B_M^*$. This follows from break even debt pricing. Guessing that $V^s(0, B_A, \mathbf{s}|_{p_{-1}=0,y_h}) = u(y_h)/(1-\beta)$, if the country chooses $B'_M = 0$ in $V^b(0, B_A, \mathbf{s}|_{p_{-1}=0,y_h})$, then the payoff is $u(y_h)/(1-\beta)$. This is the best choice since it achieves the largest possible lifetime consumption that is constant, verifying the guess. Further, this implies that no repayment of arrears takes place, that is, $B_M^* = 0$. □

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Given the guess and the previous two lemmas, the problem simplifies to the following:

$$V^s(B_M, B_A, \mathbf{s}|_{y_h}) = \max_{d\in\{0,1\}} \left\{ d\left[\frac{u(y_h)}{1-\beta} - \alpha\right] + (1-d)\, V^b(B_M, B_A, \mathbf{s})\right\}. \tag{A1}$$

$$V^b(B_M, B_A, \mathbf{s}) = \max_{\substack{c\geq 0,\, p\in\{0,1\},\\ B'_M\geq 0}} \left\{ u(c) + \beta \mathbb{E}_{\alpha'|\alpha} V^s(B'_M, B_A, \mathbf{s}') \right\}$$

$$\begin{aligned} &s.t. \\ &c = y + q(B'_M, B_A, p, \alpha)B'_M - B_M - p\, r^A B_A, \\ &p = \mathbb{I}_{B_A>0}\left(\mathbb{I}_{B'_M>0} + \mathbb{I}_{B'_M=0}\, p_{-1}\right), \\ &B'_M = 0 \text{ if } B^n_A > 0. \qquad |\text{No Lending into Arrears} \end{aligned} \tag{A2}$$

$$q(B'_M, B_A, p, \alpha) = \beta \mathbb{E}_{\alpha'|\alpha}\left[1 - d(B'_M, B_A, \mathbf{s}')\right]. \tag{A3}$$

***Assumption* 1.** *For any given $(B_M, B_A)$ and $h_t$, with $t \geq 0$, suppose there exists an $\alpha_{t+1}$ with $\pi_{\alpha_{t+1}|\alpha_t} > 0$ for which:*

$$V^b_*(B_M, B_A) := V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1, y_h, \alpha_{t+1}}) > \frac{u(y_h)}{1-\beta} - \alpha_{t+1}.$$

*Then given any other $\tilde{\alpha}_{t+1}$ with $\pi_{\tilde{\alpha}_{t+1}|\alpha_t} > 0$ as long as $V^b_*(B_M, B_A) > \frac{u(y_h)}{1-\beta} - \tilde{\alpha}_{t+1}$ it has to be that $V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1, y_h, \tilde{\alpha}_{t+1}}) = V^b_*(B_M, B_A)$*

**Lemma 1.** *Suppose assumption 1 holds and $(B^n_M, B^n_A)$ solve negotiation problem (3). Then there exists a $(\tilde{B}^n_M, 0)$ that is also a solution to (3).*

*Proof.* Define $B'_M := B'_M(B^n_M, B^n_A, \mathbf{s})$, and $p := p(B^n_M, B^n_A, \mathbf{s})$. Further, consider $\alpha'$ such that $d(B'_M, B^n_A, \mathbf{s}'|_{p,\alpha'}) = 0$. Define $B^*_M := B^*_M(B'_M, B^n_A, \mathbf{s}'|_{p,\alpha'})$ (if no such $\alpha'$ exists set $B^*_M = 0$). Notice that if $p = 0$ then $B'_M = 0$. As a result, $B^*_M$ does not depend on $\alpha'$ as it is always zero. If $p = 1$, then Assumption 1 implies that $B^*_M$ does not depend on which $\alpha'$ is chosen from the ones that satisfy $d(B'_M, B^n_A, \mathbf{s}'|_{p,\alpha'}) = 0$. Set $\tilde{B}^n_M$ as follows:

$$\tilde{B}^n_M = B^n_M + pr^A B^n_A + \beta \mathbb{E}_{\alpha'|\alpha}[1 - d(B'_M, B^n_A, \mathbf{s}'|_{p,\alpha'})](B^*_M - B'_M).$$

Lenders are indifferent between $\tilde{B}^n_M - B^n_M$ in the current period or the gamble of arrear repayment in the subsequent period. Note that since lenders are indifferent it must be that $\tilde{B}^n_M \geq B^n_M$.

Suppose the negotiation outcome is $(\tilde{B}^n_M, 0)$ and the country borrows $B^*_M$. If $\alpha'$ leads to repayment absent arrears clearing then since $V^b(B'_M, B_A, \mathbf{s}|_{y_h,p,\alpha'}) = V^b(B^*_M, 0, \mathbf{s}|_{y_h,\alpha'})$ it also leads to repayment when arrears are cleared. Suppose $\alpha'$ leads to default absent arrears clearing. Then the payoff can only improve by clearing arrears since default is still an option. Therefore, $V^s$ for the subsequent period is at least as large with $(B^*_M, 0, \mathbf{s}|_{y_h,\alpha'})$ as with $(B'_M, B_A, \mathbf{s}|_{y_h,p,\alpha'})$.

Current period consumption is $y + q(B^*_M, 0, p, \alpha)B^*_M - \tilde{B}^n_M$. Substituting in for $\tilde{B}^n_M$ and $q$ this expression becomes:

$$y + q(B'_M, B_A, p, \alpha)B'_M - B^n_M - pr^A B^n_A + \beta \mathbb{E}_{\alpha'|\alpha}[d(B'_M, B_A, \mathbf{s}|_{y_h,p,\alpha'}) - d(B^*_M, 0, \mathbf{s}|_{y_h,\alpha'})]B^*_M.$$

As argued above whenever the country defaults under $(B^*_M, 0, \mathbf{s}|_{y_h,\alpha'})$ it also defaults under $(B'_M, B_A, \mathbf{s}|_{y_h,p,\alpha'})$. Therefore, consumption is at least as large with $(B^*_M, 0, \mathbf{s}|_{y_h,\alpha'})$.

Summarizing, changing debt to $(\tilde{B}^n_M, 0)$ makes both lenders and the country at least as well off proving the lemma. □

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This lemma suggests that we can restrict attention to negotiations that clear arrears. To verify the guess note that $V^b(B_M, 0, \mathbf{s})$ is decreasing and continuous in $B_M$. Therefore, $B_M^*$ is set as per the guess.

The final lemmas of this section assume the policy of lending into arrears is followed. Define $s_t(h_t) = \prod_{j=0}^{t}(1 - d_t(h_t))$. This takes the value of zero if the country defaulted at any point in the past otherwise it is one. Further, define $\alpha_t(h_t)$ as the $t$'th element of $h_t$. Finally, define history $h_t$ as succeeding $h_0$, denoted $h_t \geq h_0$, when $\pi_{h_t|h_0} > 0$.

**Lemma 2.** *For given* $(B_M, B_A)$*, and* $\alpha$*:*

i. *Define* $\underline{\alpha} := \inf_{j\geq 0} \inf_{h_j \geq h_0 = \{\alpha\}} \alpha_j(h_j)$*. Suppose the following condition holds:*

$$\frac{u(y_h - r^A B_A - (1-\beta)B_M)}{1-\beta} \geq \frac{u(y_h)}{1-\beta} - \underline{\alpha},$$

*then the value function has the following upper bound:*

$$V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1, y_h}) \leq \frac{u(y_h - r^A B_A - (1-\beta)B_M)}{1-\beta}.$$

ii. *Suppose instead that:*

$$\frac{u(y_h - r^A B_A - (1-\beta)B_M)}{1-\beta} < \frac{u(y_h)}{1-\beta} - \underline{\alpha},$$

*Then:*

$$V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1, y_h}) \leq \frac{u(y_h)}{1-\beta} - \underline{\alpha}.$$

*Proof.*

i. The value of borrowing can be written as:

$$V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1, y_h}) = u(c_0) + \sum_{t=1}^{\infty} \beta^t \mathbb{E}_{h_t} s_t(h_t) u(c(h_t)) + \\ + \sum_{t=1}^{\infty} \beta^t \mathbb{E}_{h_t} d_t(h_t) \left( \frac{u(y_h)}{1-\beta} - \alpha_t(h_t) \right)$$

and the resulting consumption sequence satisfies

$$c_0 + \sum_{t=1}^{\infty} \beta^t \mathbb{E}_{h_t} s_t(h_t) c(h_t) = y_h + \sum_{t=1}^{\infty} \beta^t \mathbb{E}_{h_t} s_t(h_t)(y_h - r^A B_A) - B_M - r^A B_A$$

Consumption stream $\{c(h_t)\}_{t,h_t}$ provides at best the same payoff as maintaining constant consumption $c^* = y_h - r^A B_A - (1-\beta)B_M$. Further, the condition in the statement bounds the payoff under default. As a result, the largest possible payoff is indeed $u(c^*)/(1-\beta)$.

ii. Define $\underline{B}$ as $u(y_h - (1-\beta)\underline{B})/(1-\beta) = u(y_h)/(1-\beta) - \underline{\alpha}$. Then by the condition of this lemma $c^* < y_h - (1-\beta)\underline{B}$. Since the default cost is bounded above by $u(y_h - (1-\beta)\underline{B})/(1-\beta)$ maintaining constant consumption equal to $y_h - (1-\beta)\underline{B}$ is preferable to payoff $V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1, y_h})$ proving the result. □

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***Corollary.*** *For a given* $(B_M, B_A)$*, and* $\alpha$*, suppose the condition of i. in the previous lemma holds, then:*

$$V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1, y_h}) = \frac{u(y_h - r^A B_A - (1-\beta)B_M)}{1-\beta}$$

*Proof.* Guessing that $V^s(B_M, B_A, \mathbf{s}|_{p_{-1}=1}) = u(y_h - r^A B_A - (1-\beta)B_M)/(1-\beta)$ and $q(B_M, B_A, \alpha, p=1) = \beta$ if $B'_M = B_M$ then the upper bound is achieved verifying the guess. □

***Assumption 2.*** Suppose that for any $B_M$, $y = y_\ell$ *and* $B_A = 0$*, there exists a lower bound on borrowing, that is,* $B_{min} > 0$ *such that* $B_{M,t+1}(h_t) \geq B_{min}$ *for all* $t \geq 0$ *and* $h_t$ *where the country has not defaulted, that is,* $s_t(h_t) = 1$.

**Lemma 3.** *Define* $z := r^A/(1-\beta)$*. Suppose that* $z < B_{min}/B_A$ *and in* $V^b(B_M + zB_A, 0, \mathbf{s}|_{y_\ell})$ *there exists an* $h_T$ *where the country has not defaulted, that is,* $s_T(h_T) = 1$*, and:*

$$\frac{u(y_h - (1-\beta)B_{M,T+1}(h_T))}{1-\beta} \geq \frac{u(y_h)}{1-\beta} - \inf_{j \geq T} \inf_{h_j \geq h_T} \alpha_j(h_j).$$

*Then, the negotiation outcome,* $B^*_M$*, in state* $(B_M, B_A, \mathbf{s}|_{p_{-1}=0, y_\ell})$ *satisfies*

$$B^*_M \in \Big[B_M, B_M + zB_A\Big].$$

*Proof.* Rewrite the problem starting from state $(B_M + zB_A, 0, \mathbf{s})$ as follows:[36]

$$\tilde{V}^s(B_M, B_A, \mathbf{s}|_{y_h}) = \max_{\tilde{d} \in \{0,1\}} \left\{\tilde{d}\left[\frac{u(y_h)}{1-\beta} - \alpha\right] + (1-\tilde{d})\ \tilde{V}^b(B_M, B_A, \mathbf{s})\right\}. \tag{A4}$$

$$\tilde{V}^b(B_M, B_A, \mathbf{s}) = \max_{\substack{c \geq 0,\, p=0,\\ \tilde{B}'_M \geq -zB_A}} \left\{u(c) + \beta \mathbb{E}_{\alpha'|\alpha} \tilde{V}^s(\tilde{B}'_M, B_A, \mathbf{s}')\right\}$$

s.t.

$$c = y + \tilde{q}(\tilde{B}'_M, B_A, p, \alpha)\tilde{B}'_M - B_M - \big(1 - \tilde{q}(\tilde{B}'_M, B_A, p, \alpha)\big)\, zB_A, \tag{A5}$$

$$\tilde{q}(B'_M, B_A, p, \alpha) = \beta \mathbb{E}_{\alpha'|\alpha}\Big[1 - \tilde{d}(B'_M, B_A, \mathbf{s}')\Big]. \tag{A6}$$

Since $z < B_{min}/B_A$, $B'_M \geq B_{min}$, and $\tilde{B}'_M = B'_M - zB_A$ it follows that $\tilde{B}'_M \geq 0$. Consider $\tilde{V}^b(B_{M,T+1}(h_T) - zB_A, B_A, \mathbf{s}|_{\alpha_{T+1}(h_{T+1})})$. Debt is rolled over since it is rolled over in $V^b(B_{M,T+1}(h_T), 0, \mathbf{s}|_{\alpha_{T+1}(h_{T+1})})$. The payoff is $u(y_h - (1-\beta)B_{M,T+1}(h_T))/(1-\beta) \geq u(y_h)/(1-\beta) - \min_{j \geq T+1} \min_{h_j \geq h_{T+1}} \alpha_j(h_j)$.

Consider $V^b(B_{M,T+1}(h_T) - zB_A, B_A, \mathbf{s}|_{p_{-1}=1, \alpha_{T+1}(h_{T+1})})$. Since $u(y_h - (1-\beta)B_{M,T+1}(h_T))/(1-\beta) \geq u(y_h)/(1-\beta) - \min_{j \geq T+1} \min_{h_j \geq h_{T+1}} \alpha_j(h_j)$ it follows that:

$$V^b(B_{M,T+1}(h_T) - zB_A, B_A, \mathbf{s}|_{p_{-1}=1, \alpha_{T+1}(h_{T+1})}) = u(y_h - (1-\beta)B_{M,T+1}(h_T))/(1-\beta).$$

Consider $\tilde{V}^b(B_{M,T}(h_{T-1}) - zB_A, B_A, \mathbf{s}|_{\alpha_T(h_T)})$. The country chooses $B_{M,T+1}(h_T) - zB_A$. Consider $\tilde{h}_{T+1} = (h_T, \alpha) \neq h_{T+1}$. By Assumption 2 as long as rolling over debt $u(y_h - (1-\beta)B_{M,T+1}(h_T))/(1-\beta)$ is preferred to default $u(y_h)/(1-\beta) - \alpha$ the country rolls over its debt. In $V^b(B_{M,T}(h_{T-1}) - zB_A, B_A, \mathbf{s}|_{p_{-1}=1, \alpha_T(h_T)})$ the country can choose $B_{M,T+1}(h_T) - zB_A$ and get the same future payoff as in $\tilde{V}^b$[37]. Further, with this choice of debt the price is not affected since the default states are the same. Finally, since $(1-\tilde{q})z \geq r^A$ consumption in the current period is unchanged. Therefore, $\tilde{V}^b \leq V^b$ for this state.

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The same reasoning can be applied to $(B_{M,T-1}(h_{T-2}) - zB_A, B_A, \mathbf{s}|_{\alpha_{T-1}(h_{T-1})})$. Continuing this argument until $(B_{M,1}(h_0) - zB_A, B_A, \mathbf{s}|_{\alpha_1(h_1)})$ we find that:

$$\tilde{V}^b(B_{M,1}(h_0) - zB_A, B_A, \mathbf{s}|_{\alpha_1(h_1)}) \leq V^b(B_{M,1}(h_0) - zB_A, B_A, \mathbf{s}|_{p_{-1}=1,\alpha_1(h_1)}).$$

In $V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=0,\alpha,y_\ell})$, the country can choose $p = 1$ and $B'_M = B_{M,1}(h_0) - zB_A$. In doing so, it achieves a greater payoff than in $\tilde{V}^b(B_M, B_A, \mathbf{s}|_{\alpha,y_\ell})$ in the subsequent period. Further, the improvement in the future value combined with Assumption 2 guarantees that the country is less likely to default improving the price associated with this choice of $B'_M$. Finally, since $(1-\tilde{q})z \geq r^A$ consumption in the current period can only improve. Since the actual choice makes the country at least as well off as setting $p = 1$ and $B'_M = B_{M,1}(h_0) - (1+r)B_A$ the following is true:

$$V^b(B_M + zB_A, 0, \mathbf{s}|_{\alpha,y_\ell}) = \tilde{V}^b(B_M, B_A, \mathbf{s}|_{\alpha,y_\ell}) \leq V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=0,\alpha,y_\ell}). \qquad \square$$

*Definition* $\underline{B}$ is the solution to $u(y_h - (1-\beta)\underline{B})/(1-\beta) = u(y_h)/(1-\beta) - \alpha_b$.

**Lemma 4.** *Assumption 1 holds under both transient and persistent default risk.*

*Proof.* With transient default risk after period one the only default cost possible is $\alpha_g$. Therefore, $V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1,y_h,\alpha_{t+1}})$ does not depend on $\alpha_{t+1}$. Therefore, Assumption 1 is satisfied.

With persistent default risk, there are two possible states after period zero, $\{\alpha_b, \alpha_g\}$. Since $\alpha_b$ is absorbing there is nothing to check regarding Assumption 1. In contrast, with $\alpha_g$ both states are possible. Suppose that $r^A B_A - (1-\beta)B_M \leq (1-\beta)\underline{B}$. Lemma 2 and its corollary imply that $V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1,y_h}) = u(y_h - rB_A - (1-\beta)B_M)/(1-\beta)$ for both values of $\alpha$. This follows from the fact that the condition for Lemma 2 is satisfied by definition of $\underline{B}$. Suppose instead that $r^A B_A - (1-\beta)B_M > (1-\beta)\underline{B}$. Then the condition of the second part of 2 is satisfied implying that $V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1,y_h}) \leq \frac{u(y_h)}{1-\beta} - \alpha_b$ for both values of initial $\alpha$. Therefore, even if $V^b(B_M, B_A, \mathbf{s}|_{p_{-1}=1,\alpha_g,y_h})$ is larger than $\frac{u(y_h)}{1-\beta} - \alpha_g$ it is smaller than $\frac{u(y_h)}{1-\beta} - \alpha_b$ and as a result the condition of Assumption 1 ($V^b_*(B_M, B_A) > \frac{u(y_h)}{1-\beta} - \tilde{\alpha}_{t+1}$) is never satisfied making Assumption 1 true. $\square$

The dependence of value functions, policy functions, and prices on arrears, $B_A$, and policy, $p$, is dropped since both are set to zero for the remainder of the appendix. Further, the following relationship $V^b(B_M, \mathbf{s}|_{y_\ell}) = V^b(B_M + y_h - y_\ell, \mathbf{s}|_{y_h})$ is used to drop the dependence on output with the understanding that $y = y_h$ and that in the first period $\tilde{B}_M = B_M + y_h - y_\ell$ is the maturing debt. The problem post-negotiation then becomes

$$V^s(B_M, \alpha) = \max_{d \in \{0,1\}} \left\{ d \left[ \frac{u(y_h)}{1-\beta} - \alpha \right] + (1-d)\, V^b(B_M, \alpha) \right\}. \tag{A7}$$

$$V^b(B_M, \alpha) = \max_{\substack{c \geq 0,\, p \in \{0,1\}, \\ B'_M \geq 0}} \left\{ u(c) + \beta \mathbb{E}_{\alpha'|\alpha} V^s(B'_M, \alpha') \right\}$$

*s.t.*

$$c = y_h + q(B'_M, \alpha)B'_M - B_M, \tag{A8}$$

$$q(B'_M, \alpha) = \beta \mathbb{E}_{\alpha'|\alpha} \left[ 1 - d(B'_M, \alpha') \right]. \tag{A9}$$

**Lemma 5.** *In "No Lending into Arrears," negotiation amount $B^n_M$ is decreasing in $y_\ell$.*

*Proof.* Consider $\bar{y}_\ell > \underline{y}_\ell$. Suppose $\underline{B}^n_M$ is the outcome of negotiation with $\underline{y}_\ell$ and $\bar{B}^n_M$ is the outcome of negotiation with $\bar{y}_\ell$. By definition of $\bar{B}^n_M$, $V^b(\bar{B}^n_M + y_h - \bar{y}_\ell, \alpha) = u(\bar{y}_\ell) + \beta u(y_h)/(1-\beta)$. Consider the surplus to the country starting with $\underline{y}_\ell$ when the negotiation outcome is $\bar{B}^n_M$:

$$
\begin{aligned}
&V^b(\bar{B}^n_M + y_h - \underline{y}_\ell, \alpha) - u(\underline{y}_\ell) - \beta u(y_h)/(1-\beta) = \\
&\quad = V^b(\bar{B}^n_M + y_h - \underline{y}_\ell, \alpha) - V^b(\bar{B}^n_M + y_h - \bar{y}_\ell, \alpha) + u(\bar{y}_\ell) - u(\underline{y}_\ell) \geq \\
&\qquad \geq u(\underline{y}_\ell + q(\bar{B}'_M, \alpha)\bar{B}'_M - \bar{B}^n_M) - u(\bar{y}_\ell + q(\bar{B}'_M, \alpha)\bar{B}'_M - \bar{B}^n_M) - \Big(u(\underline{y}_\ell) - u(\bar{y}_\ell)\Big) \geq 0
\end{aligned}
$$

where $\bar{B}'_M$ is the debt choice when the state is $\bar{B}^n_M + y_h - \bar{y}_\ell$. The last inequality follows from the fact that $q(B'_M, \alpha)B'_M - \bar{B}^n_M \geq 0$[38] and the concavity of $u$. Therefore, the surplus when the negotiation outcome is $\bar{B}^n_M$ is not negative. Since the lenders extract all the surplus repayment can only increase. □

## Appendix B – Post-negotiation outcome

**Transient default risk.** Lemma 2 implies that after period 1 if $u(y_h - (1-\beta)B_M)/(1-\beta) \geq u(y_h)/(1-\beta) - \alpha_g$ the country rolls over its debt otherwise it defaults. During period one the same lemma suggests that, for any given $\alpha \in (0, \alpha_g)$, if $u(y_h - (1-\beta)B_M)/(1-\beta) \geq u(y_h)/(1-\beta) - \alpha$ the country repays and rolls over its debt, otherwise it defaults. The value in period zero, $V^b(B_M, \alpha_g)$, is then determined by the following problem:

$$
\max_{c \geq 0, B'_M \geq 0} \; u(c) + \beta \left( \int_0^{\alpha\left(B'_M\right)} \frac{u(y_h)}{1-\beta} - \alpha' \, dF(\alpha') + \int_{\alpha\left(B'_M\right)}^{\alpha_g} \frac{u(y_h - (1-\beta)B'_M)}{1-\beta} \, dF(\alpha') \right)
$$

*s.t.*

$$
c = y_\ell + \beta\Big(1 - F\big(\alpha\,(B'_M)\big)\Big)B'_M - B_M
$$

where $\alpha(B'_M) = u(y_h)/(1-\beta) - u(y_h - (1-\beta)B'_M)/(1-\beta)$ is the threshold default cost below which the country defaults and above which it repays.

***Theorem* 3.** *Suppose $r^A$ is small enough. Then the policy shift features:*

i. *increased debt forgiveness,*
ii. *a reduction in the likelihood of default in period 1,*
iii. *a reduction in post-negotiation cost of borrowing.*

*Proof.* Assumption 2 holds because in the Supplementary Appendix it is shown that the country borrows a positive amount independent of $B_M$. Further, borrowing is increasing in $B_M$; therefore, $B_{min}$ can be set as the debt that is chosen when $B_M = 0$. Further, after period 0 as long as the country does not default it rolls over its debt indefinitely.

The condition for Lemma 3 holds since the country borrows a positive amount for any $B_M$ and it never borrows to the point where it defaults for sure in period 1. Therefore, there always exist a state where the payoff from rolling over is greater than the default payoff.

These two imply that Lemma 3 can be applied and for $r^A$ small enough $B^*_M \in [0, B_A r^A/(1-\beta)]$ in "Lending into Arrears." With "No Lending into Arrears" $B^*_M > 0$ since the country always benefits from borrowing some positive amount. Therefore, as long as $r^A$ is small enough debt forgiveness with "Lending into Arrears" is greater than "No Lending into Arrears." Since borrowing is increasing in $B_M$ and prices are decreasing in $B'_M$ the post-negotiation results follow. □

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**Persistent default risk.** Lemma 2 and its corollary imply that under both high and low default cost in the region of debt $B_M \leq \underline{B}$ the country repays and rolls over its debt. For values of debt larger than $\underline{B}$, the same lemma suggests that the country defaults under the low default cost state. In the same region in the high default cost state dynamics of debt emerge. To characterize the dynamics, in the Supplementary Appendix the equivalence between the solution to the Markov perfect problem and a problem where the government can commit to the path of debt but not on repayment is shown.

The government commits to reducing its debt to $\underline{B}$ in $T$ periods. For the duration of these periods, the price of borrowing is $\beta\pi_{\alpha_g|\alpha_g}$ and the country defaults if the state switches to the low default cost. The $T$ period problem can be expressed as a "lifetime problem" in the following way:

$$X_T(B_M) = \max_{\{B_{M,T},\{c_t\}_{t=0}^{T-1}\}} \sum_{t=0}^{T-1} \beta^t \pi^t_{\alpha_g|\alpha_g} u(c_t) + \sum_{t=1}^{T-1} \pi_{\alpha_b|\alpha_g} \pi^{t-1}_{\alpha_g|\alpha_g} \beta^t V_b^D + \pi^{T-1}_{\alpha_g|\alpha_g} \beta^T \frac{u(y_h - (1-\beta)B_{M,T})}{1-\beta}$$

$s.t.$

$$B_M = \sum_{t=0}^{T-1} \beta^t \pi^t_{\alpha_g|\alpha_g} (y_h - c_t) + \beta^T \pi^{T-1}_{\alpha_g|\alpha_g} B_{M,T}$$

$$B_{M,T} \leq \underline{B}$$

$$c_t \geq 0 \qquad t = 0, 1, \ldots, T-1$$

where $V_b^D = u(y_h)/(1-\beta) - \alpha_b$. The no Ponzi condition is imposed on borrowing $\lim_{t\to\infty} \beta^t \pi^t_{\alpha_g|\alpha_g} B_{M,t} = 0$ and $T = \infty$ is included as an option. The value function for the high default cost state can be expressed as follows:

$$V^b(B_M, \alpha_g) = \begin{cases} u(y_h - (1-\beta)B_M)/(1-\beta) & \text{If } B_M \leq \underline{B} \\ \max_T X_T(B_M) & \text{If } B_M \in (\underline{B}, D] \end{cases}.$$

where $D$ is defined as the solution to $\max_T X_T(D) = u(y_h)/(1-\beta) - \alpha_g$.

***Theorem* 4.** *Suppose $\pi_{\alpha_b|\alpha_g}$ is small enough. Then for each $T > 1$ there exists an interval $(\underline{y}_T, \bar{y}_T)$ so that if $y_\ell \in (\underline{y}_T, \bar{y}_T)$ there is an $r^A$ small enough so that the policy shift features:*

*i. increased debt forgiveness,*
*ii. unchanged likelihood of default in the first T periods,*
*iii. a reduction in the post period T likelihood of default,*
*iv. unchanged post-negotiation cost of borrowing, $\beta\pi_{\alpha_g|\alpha_g}$.*

*Proof.* In the Supplementary Appendix, it is shown that, for $\pi_{\alpha_b|\alpha_g}$ small enough, the set $(\underline{B}, D]$ can be partitioned into $\{I_t\}_{t=1}^{\infty}$ intervals with $I_t < I_{t+1}$ for all $t \geq 1$. For any given $T$, interval, $I_T$, is characterized by the fact that it is optimal to exit in $T$ periods, that is, $T = \arg\max_T X_T(B_M)$.

Consider interval $I_T = (\underline{I}_T, \bar{I}_T]$ for $1 < T < \infty$. Suppose $y_h - y_\ell = \underline{I}_T$, that is, $y_\ell = y_h - \underline{I}_T$. Consider $B_M^n$ defined as $V^b(B_M^n + y_h - y_\ell, \alpha_g) = u(y_\ell) + \beta u(y_h)/(1-\beta)$, that is, the negotiation outcome with "No Lending into Arrears." Starting with state $B_M^n + y_h - y_\ell$ there are two possibilities. The time it takes to reduce debt to $\underline{B}$ is either more or equal to $T$. If it is more than $T$, then choose $\underline{y}_T = y_h - \underline{I}_T$. If not, then there must be a $\underline{y}_T \in (y_h - \underline{I}_T, y_h - \bar{I}_T)$ so that the exit time is less than $T$ below $\underline{y}_T$ and larger than $T$ above $\underline{y}_T$.[39] The upper bound $\bar{y}_T = y_h - \bar{I}_T$. Therefore, for any $y_\ell \in (\underline{y}_T, \bar{y}_T)$ the time it takes to reduce debt to $\underline{B}$ is longer that $T$ in the "No Lending into Arrears" regime.

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Consider a $y_\ell \in (\underline{y}_T, \bar{y}_T)$ where the interval is specified as described above. Further, suppose arrears satisfy $B_A r^A/(1-\beta) + y_h - y_\ell < \bar{I}_T$. In this case, the conditions for Lemma 3 are satisfied since $h_T = (\alpha_g, \alpha_g, \ldots, \alpha_g)$ results to debt equal to $\underline{B}$ and this debt satisfies the condition in Lemma 3. Further, Assumption 2 is satisfied by setting $B_{min} = \underline{B}$. Therefore, $B_M^* \in [0, B_A r^A/(1-\beta)]$. As a result, $y_h - y_\ell \leq B_M^* + y_h - y_\ell \leq B_A r^A/(1-\beta) + y_h - y_\ell < \bar{I}_T$ and the exit time is $T$ periods. □

## Appendix C – Numerical exercise

In this appendix, I present a quantitative model of sovereign debt renegotiation in which the IMF policy of Lending into Arrears impacts the sovereign's outside option. In the numerical exercise, the IMF policy shift is shown to have a quantitatively significant impact on the size of the average haircut.

### *Economic environment*

*Endowment process.* Each period the economy receives a stochastic endowment $Y_t$. This process has the following autoregressive structure:

$$\log\ Y_{t+1} = \rho \log Y_t + \epsilon_{t+1}. \tag{C1}$$

The innovation term, $\epsilon_{t+1}$, is iid and is drawn from a normal distribution with zero mean and $\sigma_\epsilon$ standard deviation. Parameter $\rho$ is the usual autoregressive coefficient.

*Preferences.* The government values the uncertain stream of consumption $\{\{C_t(s^t)\}_{s^t}\}_{t=0}^{\infty}$ using a utility function, given by:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_t(s^t)).$$

$\mathbb{E}$ denotes the expectations on $s^t$ which includes the history of all random variables discussed below. We assume the function $U(\cdot)$ is strictly increasing, concave, and twice continuously differentiable. The discount factor is given by $\beta \in (0, 1)$.

*Debt, default, and renegotiation.* The country begins each period, $t$, with debt $B_{p,t}$. A $\delta$ fraction of the debt matures and has to be repaid. Outstanding debt, $(1-\delta)B_{p,t}$, receives coupon $\kappa$. Finally, the government decides on debt issuance $B_{p,t+1} - (1-\delta)B_{p,t}$ and risk neutral competitive lenders price this debt issuance. Parameters $\delta$ and $\kappa$ are specified in the quantitative analysis.

Default allows the country to renegotiate debt $B_{p,t}$. However, there are two costs associated with default. The first cost of defaulting is that the government is temporarily excluded from financial markets. Re-entry occurs stochastically with per period probability $\theta$. At the end of the exclusion period defaulted debt is renegotiated. Renegotiations are modeled as in Yue (2010). In particular, in the *outside option*, that is, in the off-equilibrium path where negotiations break down, the sovereign never repays defaulted debt, is permanently excluded from financial markets, and incurs endowment loss $Y - Y^o(Y)$ where $Y^o(Y) = \min(Y, Y^o)$. Parameter $Y^o$ is calibrated in the quantitative analysis. Depending on the IMF policy, the IMF may or may not lend to the country in the *outside option*. Creditors are assumed to capture the entire surplus of bargaining.

The second cost of default is resource cost $Y - Y^d(Y)$ where $Y^d(Y) = \min(Y, Y^d)$. That is, endowment can at most be $Y^d$ as a result of the default Arellano (2008). The cost is linearly increasing in $Y$ for values of $Y$ larger than $Y^d$ and zero for values of $Y$ less than $Y^d$. Parameter $Y^d$ is calibrated in the quantitative analysis.

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*IMF lending.* In the *outside option*, that is, if negotiations break down, the country starts with exogenously determined value $\bar{B}_I$ of IMF debt. As in Boz (2011), the cost of borrowing from the IMF is determined by the following equation:

$$q^I(B'_I) = \frac{1}{1 + r + \kappa B'^I} \tag{C2}$$

That is, the country pays a premium above the risk-free rate to borrow from the IMF. This premium is assumed to depend linearly on the amount of borrowing. Finally, IMF conditionality is modeled in the outside option as an ad hoc upper limit $\bar{C}$ on consumption when IMF borrowing is positive, $B'_I > 0$. On the equilibrium path, I assume the country rolls over IMF debt $\bar{B}_I$ and is not subject to conditionality.

*Decision problem*

In this section, I formalize the economic environment by stating the problem faced by market participants in recursive form. The government enters a period with debt $B_p$ and endowment realization $Y$.

*Government.* The government that is current on its debt obligations decides between repayment or default. The value function is given by:

$$W(B_p, Y) = \max_{d \in \{0,1\}} \left\{ d\, V^D(B_p, Y) + (1-d) V^R(B_p, Y) \right\} \tag{C3}$$

Repayment ($d = 0$) allows the government to borrow. The value function is given by:[41]

$$\begin{aligned} V^R(B_p, Y) &= \max_{B_p' \geq 0, C \geq 0} \left\{ U(C) + \beta \mathbb{E}_{Y'|Y} W(B_p', Y') \right\} \\ &\textit{subject to} \\ C &= Y - \left(\delta + (1-\delta)\kappa\right) B_p + Q(B_p', Y)\left(B_p' - (1-\delta)B_p\right) - \left(1 - q^I(\bar{B}_I)\right)\bar{B}_I, \\ Q(B_p', Y) &\geq \bar{Q} \text{ if } B_p' \geq (1-\delta)B_p. \end{aligned} \tag{C4}$$

A sovereign that defaults ($d = 1$) is excluded from international credit markets and has probability $\theta$ of being readmitted every subsequent period with renegotiated debt $B_N$. The associated value is

$$\begin{aligned} V^D(B_p, Y) &= \max_{C \geq 0} \left\{ U(C) + \beta \mathbb{E}_{Y'|Y}\left[\theta\, W(B_N(B_p, Y'), Y') + (1-\theta) V^D(B_p, Y')\right]\right\} \\ &\textit{subject to} \\ C &= Y^d(Y) - \left(1 - q^I(\bar{B}_I)\right)\bar{B}_I. \end{aligned} \tag{C5}$$

*Renegotiation.* The lenders are assumed to make a take-it-or-leave-it offer to the sovereign. The associated renegotiation outcome is determined as follows:

$$\begin{aligned} B_N(B_p, Y) &= \arg \max_{B_p \geq B_N \geq 0} \{S_L(B_N, Y)\} \\ &\textit{subject to} \\ W(B_N, Y) &\geq V^O(\bar{B}_I, Y). \end{aligned} \tag{C6}$$

The value of a unit of debt when a country is not in default is determined by the following equation:

$$\frac{S_L(B_p, Y)}{B_p} = \left(1 - d(B_p, Y)\right)\left(\delta + (1-\delta)\kappa + (1-\delta) Q\left(B_p'(B_p, Y), Y\right)\right) + d(B_p, Y)\Phi(B_p, Y). \tag{C7}$$

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Similarly, the value of a unit of debt when a country is in default is determined by the following equation:

$$\Phi(B_p, Y) = \frac{1}{1+r}\mathbb{E}_{Y'|Y}\left[\theta\,\frac{S_L\Big(B_N\big(B_p, Y'\big), Y'\Big)}{B_p} + (1-\theta)\,\Phi(B_p, Y')\right]. \tag{C8}$$

The value to the sovereign of the outside option, that is, being excluded from financial markets but policy permitting receive IMF lending, is

$$\begin{aligned} V^O(B_I, Y) = &\max_{B_I{}' \geq 0, C \geq 0} \Big\{U(C) + \beta \mathbb{E}_{Y'|Y} V^O(B'_I, Y')\Big\} \\ &\textit{subject to} \\ &C = Y^o(Y) - B_I + q^I(B_I{}')B_I{}', \\ &C < \bar{C} \text{ if } B_I{}' > 0|\text{Austerity}, \\ &B'_I = 0 \qquad |\text{No Lending into Arrears}. \end{aligned} \tag{C9}$$

*International lenders.* Lenders are assumed to be risk neutral and perfectly competitive. The actuarially fair bond price that compensates them for default risk is

$$Q(B_p{}', Y) = \frac{\mathbb{E}_{Y'|Y}\Big[\Big(1 - d(B_p{}', Y')\Big)\Big(\delta + (1-\delta)\kappa + (1-\delta)Q\big(B_p{}'(B_p{}', Y'), Y'\big)\Big) + d(B_p{}', Y')\Phi(B'_p, Y')\Big]}{1+r} \tag{C10}$$

A Markov perfect equilibrium for this economy consists of the government value functions $W(B_p, Y)$, $V^R(B_p, Y)$, $V^D(B_p, Y)$, $V^O(B_I, Y)$; debt valuation functions $S_L(B_p, Y)$, $\Phi(B_p, Y)$; policy functions $B_p{}'(B_p, Y)$, $B_N(B_p, Y)$, $d(B_p, Y)$,$B_I{}'(B_I, Y)$; and bond price schedule $Q(B_p{}', Y)$ such that:

1. $W(B_p, Y)$ is the value function in equation (C3) with policy function $d(B_p, Y)$.
2. $V^R(B_p, Y)$ is the value function in equation (C4) with policy function $B_p{}'(B_p, Y)$.
3. $V^D(B_p, Y)$ is the value function in equation (C5).
4. $B_N(B_p, Y)$ is the policy function resulting from equation (C6).
5. $S_L(B_p, Y)$, $\Phi(B_p, Y)$ are the debt valuation functions calculated in equations (C7) and (C8).
6. $V^O(B_I, Y)$ is the value function in equation (C9) with policy function $B_I{}'(B_I, Y)$.
7. Bond price function $Q$ solves equation (C10).

*Calibration*

The model in which the IMF lends into arrears, that is, without the “No Lending into Arrears” constraint in (C9), is calibrated to match salient post-1989 Mexican sovereign debt moments.[42]

*Parameters set externally.*

**Preferences** Each period is assumed to be 1 year. We assume a constant relative risk aversion utility function of the form $U(G) = \frac{G^{1-\gamma}}{1-\gamma}$, with the risk aversion parameter $\gamma$ set to 2.

**Endowment** The persistence $\rho$ of Mexico’s annual GDP is 0.65, estimated using data from 1980, while the standard deviation of innovations $\sigma$ is 0.03.

**Sovereign debt** The risk-free interest rate is set to 4% (annual value) and the probability of re-entry after default is fixed at 0.33, following Richmond and Dias (2009) who find that the median time to re-enter the credit market was 3 years in 1980–2005. To select the values for parameters that describe Mexico’s debt structure, I follow related papers, such as Aguiar et al. (2016) or Bianchi

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**Table 2.** Parameters calibrated externally

| Parameter | Meaning | Value |
|---|---|---|
| $\gamma$ | Risk aversion | 2 |
| $\theta$ | Prob. of exiting excl. | 0.333 |
| $\delta$ | Bond maturity prob. | 0.285 |
| $\kappa$ | Coupon rate | 0.05 |
| $r$ | Risk-free rate | 0.04 |
| $\bar{Q}$ | Price cap | 0.7 |
| $\bar{B}_I/E(Y)$ | IMF debt to GDP (%) | 3 |

et al. (2018). The maturing probability $\delta$ is set to 0.285, while the (annual) coupon rate $\kappa$ is 5%. Parameters calibrated externally are summarized in Table 2.

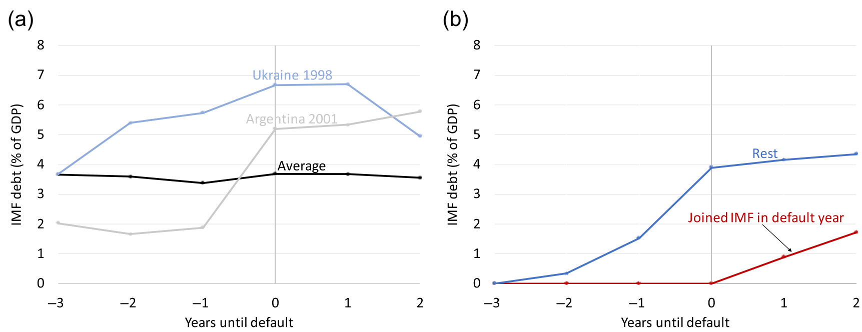
*Parameters calibrated in the model.* The remaining five parameters, $(\beta, Y^d, Y^o, \bar{C}, \kappa)$, are calibrated using the simulated method of moments. The economy's endowment is simulated for 2 million periods, with the first 100 observations dropped. I also drop observations for periods where the country is either in default or was in default less than 5 years prior (for the on-equilibrium moments). Five moments are used to identify the parameters: average debt to GDP, average spread, cross-sectional average haircut post-1989, average IMF debt to GDP in the outside option, and average IMF lending rate in the outside option.

**Sovereign debt** External debt to GDP is chosen to be 16% as reported in Aguiar et al. (2016) for 1993–2014. The data measure for spread is the EMBI spread which was 3% on average for Mexico from 1994 to 2014. The average haircut is set at the post-1989 average haircut of roughly 50%.

**IMF lending** The quantitative model presented above makes a stark assumption regarding IMF lending. IMF debt is rolled over on equilibrium. This choice is based on two data observations discussed here. First, countries entered default with similar levels of IMF debt pre- and post-1989, approximately 3.5% of GDP on average. Second, post-1989 countries did not drastically increase IMF borrowing in default. That is, lending in arrears more likely impacted their outside option rather than the on-equilibrium borrowing in default. To substantiate these two claims, I take default episodes in which the country was in good credit standing for 3 years prior to the default episode, the default episode lasted at least 3 years, and the country held IMF debt for at least 1 year in this 6-year window. Post-1989, 17 default episodes satisfy this criterion. Of the 17 episodes, 9 were of countries that owed a positive amount to the IMF 3 years prior to the default episode. Average IMF borrowing for these nine countries can be seen in the left panel of Figure 3. As can be seen, average IMF borrowing in default is quite stable. IMF borrowing paths for two prominent defaults are also depicted in Figure 3 to make this point, that of the 1998 Ukrainian default and the 2001 Argentinian default.

Of the remaining eight default episodes, that is, the ones in which the country held no IMF debt 3 years prior to the default, four joined the IMF in the period of their default and the remaining four were legacy members. The right panel of Figure 3 depicts average IMF borrowing for these eight countries divided into the group of new and legacy IMF members. The assumption of constant IMF debt does not fit as well for these countries. In particular, countries that joined the IMF the period of default borrowed aggressively from the IMF in default. The legacy members are characterized by a large increase in borrowing in the period of default and stable IMF debt thereafter. The four countries that already had access to the IMF were hit by extreme shocks, for example, the Algerian war in 1991 and the Asian financial crisis that hit Indonesia particularly hard in 1998. Since Mexico is a long-standing member of the IMF and its income shocks considered here are not quite as extreme, assuming IMF debt is constant in equilibrium is not as extreme an assumption.

I assume that on the equilibrium path the country maintains IMF debt to GDP of 3%. In particular, I round down the 3.5% average IMF debt to GDP at the period of default observed for




Episodes with IMF debt 3 years prior to default

Episodes without IMF debt 3 years prior to default

**Figure 3.** The two figures plot average IMF borrowing around defaults for 17 default episodes. The sample is separated into defaults in which the country held IMF debt 3 years prior to the default (left panel) and ones in which the countries did not hold IMF debt 3 years prior to default (right panel). The right panel further separates the sample between countries that became IMF members at the time of default and ones that were legacy members at the time of default.

**Table 3.** Parameters calibrated in the model

| Parameter | | |
|---|---|---|
| Discount factor, $\beta$ | | 0.911 |
| Max default endowment, $Y^d$ | | 0.939 |
| Max outside option endowment, $Y^o$ | | 1.029 |
| IMF Conditionality, $\bar{C}$ | | 0.973 |
| IMF premium, $\kappa$ | | 0.180 |
| **Target** | **Data post 89** | **LIA model** |
| Avg. debt/GDP (%) | 16.00 | 16.00 |
| Avg. spread (%) | 3.03 | 3.00 |
| Avg. haircut (%) | 50.00 | 50.02 |
| Avg. IMF debt/GDP* (%) | 3.00 | 3.04 |
| Avg. IMF lending premium* (%) | 0.54 | 0.54 |
| **Untarget** | **Data post 89** | **LIA model** |
| $\sigma$(C)/$\sigma$(Y) | 1.00 | 1.01 |
| $\sigma$(spread) (%) | 2.21 | 1.80 |
| Prob. repeat default (%)** | 45.00 | 18.49 |

* Calculated for the outside option.
** Probability of default within 5 years of renegotiation.

the nine countries that held positive IMF debt 3 years prior to default post-1989 discussed above. Further, average IMF borrowing to GDP pre-1989 in the period of default was also around 3.5%.[43] In the outside option average, IMF borrowing is assumed to remain the same, that is, 3% of GDP (a robustness exercise below considers the lower amount of 1.6%). Finally, the IMF lending premium is set to 0.54% following Fink and Scholl (2016). The results of the calibration can be seen in Table 3. All targeted parameters are well matched.

*Results.* The model matches salient untargeted moments of the data quite well. In particular, as can be seen in the last rows of Table 3 the model matches the variability of consumption relative to the variability of GDP almost perfectly. This is a moment emphasized in Bianchi et al. (2018). Further, the model gets close to the variability of the spread, a moment that has proven to be elusive as highlighted in Aguiar et al. (2016) and further studied in Paluszynski and Stefanidis (2023).

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**Table 4.** Changing the lending into arrears policy in the model

| Statistics | Data pre-1989 | Data post-1989 | No LIA model | LIA model |
|---|---|---|---|---|
| Avg. haircut (%) | 26.30 | 50.02 | 41.97 | 50.02 |
| Prob. repeat default (%)** | 97.00 | 45.00 | 20.02 | 18.49 |
| Avg. debt/GDP (%) | | | 18.72 | 16.00 |
| Avg. spread (%) | | | 2.56 | 3.00 |
| Avg. IMF debt/GDP* (%) | | | 0.00 | 3.04 |
| Avg. IMF lending premium* (%) | | | 0.00 | 0.54 |

* Calculated for the outside option.
** Probability of default within 5 years of renegotiation.

**Table 5.** Robustness exercise

| Statistics | Data pre-1989 | Data post-1989 | No LIA model | LIA model |
|---|---|---|---|---|
| Avg. haircut (%) | 26.30 | 50.02 | 44.76 | 50.00 |
| Prob. repeat default (%)** | 97.00 | 45.00 | 19.56 | 18.67 |
| Avg. debt/GDP (%) | | | 18.72 | 16.00 |
| Avg. spread (%) | | | 2.56 | 3.00 |
| Avg. IMF debt/GDP* (%) | | | 0.00 | 1.68 |
| Avg. IMF lending premium* (%) | | | 0.00 | 0.66 |

* Calculated for the outside option.
** Probability of default within 5 years of renegotiation.

An untargeted moment this model misses is the probability of repeat default 5 years after renegotiation. In the data, post-1989 45% of renegotiations lead to defaults within 5 years. This moment is significantly lower in the model at 18.49%. The issue of serial defaults has been documented in the literature and recently there has been an effort to model this behavior using reputation, Amador and Phelan (2021), and quantified in Fourakis (2022). As a result, here I do not place emphasis in matching this moment and subsequently in quantifying its increase with the no lending into arrears policy. However, as will be seen below, qualitatively this moment changes as it does in the data.

Table 4 presents the results from moving from the no lending into arrears to the lending into arrears policy in the model (last two columns) and contrasts it to the cross-sectional averages in the data (first two columns). The increase in haircuts is approximately 8 percentage points compared to the 24 percentage point increase seen in the data. The model captures a quantitatively significant fraction of the increase in haircuts. The probability of repeat default declines, which is qualitatively consistent with what happens in the data. However, the decline only represents a very small fraction of the decline seen in the data.

In Table 5, the assumption that average IMF debt remains at 3% in the outside option is altered. Instead, it is assumed that IMF debt in the outside option is 10% of average private debt when not in default, that is, approximately 1.6%. Boz (2011) reports that IMF debt was 10% of private debt for her sample of countries. As expected, with IMF lending becoming more limited the haircut does not increase as much in this specification, approximately five percentage points compared to eight percentage points in the previous specification. However, the increase remains quantitatively significant relative to the 24 percentage point increase seen in the data.

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**Table 6.** The coefficients of linear regressions with dependent variables and binary variables that takes the value of one if the country defaults again within 1, 2, 3, 4, and 5 years of the negotiation and zero otherwise and the control variables described in the text

| Variables | | | Repeat default | | |
|---|---|---|---|---|---|
| | *(1 year)* | *(2 year)* | *(3 year)* | *(4 year)* | *(5 year)* |
| 1989 Dummy | −0.14 | −0.10 | −0.28** | −0.32** | −0.28** |
| | (0.14) | (0.14) | (0.13) | (0.13) | (0.12) |
| GDP | −0.52 | −0.59 | −0.28 | −0.26 | −0.12 |
| *(% dev. from trend)* | (0.42) | (0.40) | (0.38) | (0.38) | (0.36) |
| GDP per capita | −0.04 | 0.02 | 0.07 | 0.06 | 0.08 |
| *(log)* | (0.06) | (0.06) | (0.06) | (0.06) | (0.05) |
| Debt to GNI | 0.11 | 0.12 | 0.06 | 0.06 | 0.08 |
| *(trend GNI)* | (0.13) | (0.13) | (0.12) | (0.12) | (0.11) |
| Negotiated debt | −0.19 | −0.22 | −0.30 | −0.23 | −0.23 |
| *(% of total)* | (0.21) | (0.20) | (0.19) | (0.19) | (0.18) |
| Episode duration | 0.01 | 0.01 | 0.00 | 0.01 | 0.01 |
| | (0.02) | (0.02) | (0.01) | (0.01) | (0.01) |
| Negotiations | −0.04 | −0.05 | −0.03 | −0.02 | −0.04 |
| | (0.04) | (0.04) | (0.03) | (0.04) | (0.03) |
| Brady deal | −0.34** | −0.28* | −0.21 | −0.18 | −0.13 |
| | (0.16) | (0.15) | (0.14) | (0.15) | (0.14) |
| Donor funded | −0.04 | 0.14 | 0.18 | 0.13 | 0.04 |
| | (0.23) | (0.22) | (0.20) | (0.20) | (0.19) |
| Post-default | −0.03 | −0.12 | −0.02 | −0.02 | −0.07 |
| | (0.09) | (0.09) | (0.08) | (0.09) | (0.08) |
| ’89 Dummy x | 0.05* | 0.05** | 0.04 | 0.03 | 0.02 |
| Episode duration | (0.02) | (0.02) | (0.02) | (0.02) | (0.02) |
| Debt composition | Y | Y | Y | Y | Y |
| Region dummies | Y | Y | Y | Y | Y |
| Constant | Y | Y | Y | Y | Y |
| Observations | 111 | 111 | 111 | 111 | 111 |
| Adjusted $R^2$ | 0.39 | 0.40 | 0.45 | 0.42 | 0.43 |
| *F* test | 0 | 0 | 0 | 0 | 0 |

*Note*: Standard errors in parentheses.
*** $p<0.01$, ** $p<0.05$, * $p<0.1$.

## Appendix D – Further empirical evidence

Table 6 provides more evidence for the persistent default risk theory. Section 5.3 suggests that this theory has implications about the timing of the change in default risk as a result of the policy shift. In particular, the short-run default risk should not change as a result of the policy shift whereas the long-run default risk should decrease. To test this prediction, five different windows for the default risk variable are considered.

The first column in Table 6 reports the change in the probability a country defaults a year after the negotiation takes place. Countries were only 14 percentage points less likely to default again within a year of the negotiation after 1989. Moreover, statistically one cannot reject the null hypothesis that there was no change in the likelihood of a default in this window in 1989[40].

The point estimate is even smaller for a 2-year window and again statistically insignificant. The improvement in the probability of default only becomes statistically significant after the second year with the magnitudes of the point estimates increasing drastically. These results are seen as additional evidence for the theory of persistent default risk which emphasizes the role of the dynamics of borrowing in the aftermath of negotiations.[44]

## Appendix E – Additional figures

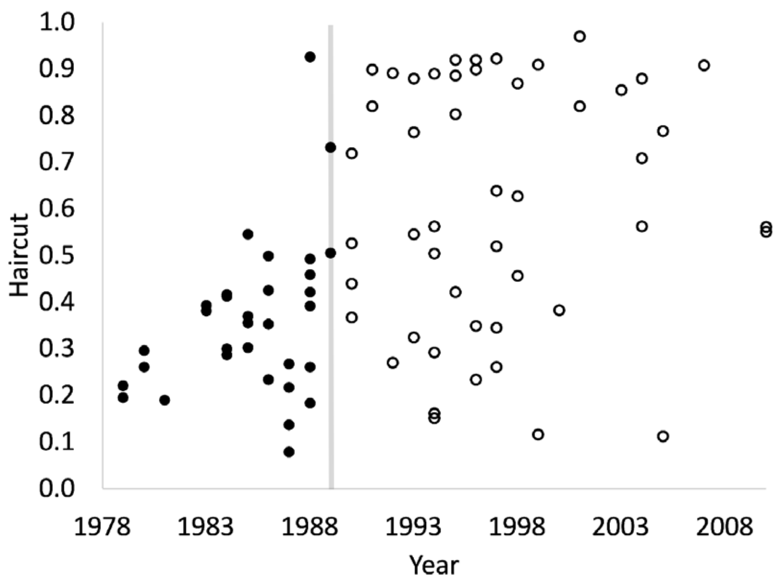


**Figure 4.** Each dot summarizes the outcome of a negotiation between creditors and countries. The summary measure used is the fraction of negotiated debt forgiven, that is, the haircut. The vertical faded line marks 1989, the year of the IMF policy change. The sample in this figure is restricted to include only post-default negotiations (i.e., negotiations in which a debt payment has been missed) in which the country owed to the IMF during the negotiation.

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