giving an introduction to ring theory. This is a splendid book for the expert, however, and because of its comprehensiveness certainly one for the university library. The main criticism concerns the arrangement of some of the material. For example, Goldie's Theorem does not appear in the chapter on classical rings of quotients (Chapter 15) but in Chapter 2. Also it would be more natural to proceed more directly from the theory of Abelian and Grothendieck categories (Chapters 4 and 5) to the definition of quotient categories and their relationship with Grothendieck categories (Chapters 9 and 10). Instead the author interposes a discussion of torsion theories including hereditary torsion theories and their relationship to Gabriel topologies, fully bounded Noetherian rings, commutative Noetherian rings and Artinian rings (Chapters 6, 7 and 8). The first three chapters give a general introduction to modules, rings of fractions and modular lattices. Chapters 10-15 are concerned with particular rings of quotients—the maximal flat epimorphic ring of quotients, the maximal ring of quotients and the classical ring of quotients. A short preliminary version of this book was published in the Springer series "Lecture Notes in Mathematics" under the title "Rings and modules of quotients" (Volume 237, 1971).

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BLATTER, C., *Analysis* I, II and III (Heidelberger Taschenbücher, Springer-Verlag, Berlin-Heidelberg-New York, 1974), 204+180+184 pp., each volume $6.10.

This is a comprehensive first rigorous course on analysis of one and more real variables. In the first volume Dedekind sections are used to complete the rationals and form the real numbers $\boldsymbol{R}$, from which the complex numbers $\boldsymbol{C}$ are constructed in the usual way. Metric spaces are then introduced and the author achieves generality by working wherever possible in a metric space X, which can be either $\boldsymbol{R}$, $\boldsymbol{C}$ or $\boldsymbol{R}^n$. Thus continuity is defined for functions whose domain and range are subsets of the same space X. After a chapter on convergence the exponential, trigonometric and hyperbolic functions are defined by means of the series for exp z ($z \in \boldsymbol{C}$). The logarithmic function is defined on the positive reals as the inverse of the exponential function and the volume concludes with the usual theorems of differentiability, including Taylor's theorem.

The second volume is concerned with the Riemann integral including the usual techniques for evaluating definite and indefinite integrals. Curves and functions of bounded variation are discussed as well as uniform convergence and power series. The last two chapters are devoted to the study of functions from $\boldsymbol{R}^m$ to $\boldsymbol{R}^n$, including differentiability and the finding of extrema.

The third volume continues the discussion of functions of several variables with proofs of the inverse function, implicit function and immersion theorems, with applications to higher dimensional surfaces and Lagrange multipliers. Jordan content in $\boldsymbol{R}^m$ is introduced and multiple Riemann integrals. The final chapters deal with vector fields and the theorems of Green, Stokes and Gauss. The vector product $\boldsymbol{p} \times \boldsymbol{q}$ is introduced by noting that the determinantal function $\varepsilon(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{x}) = [\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{x}]$ is a linear functional and so uniquely determines a vector such that $\varepsilon(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{x}) = \boldsymbol{a} . \boldsymbol{x}$ for all $\boldsymbol{x} \in \boldsymbol{R}^3$. The vector product $\boldsymbol{p} \times \boldsymbol{q}$ is then defined to be this vector $\boldsymbol{a}$. It is also of interest that the Heine-Borel Theorem (so called by the author) appears first on p. 126 of the third volume.

The book is beautifully produced and clearly written. It would be excellent as a textbook for a university course on analysis but would have to be supplemented by numerous examples and exercises as these are not included in the text.

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